1 ADT: Map

A map (also called associative array) is a collection of unique keys and a collection of values, where each key is associated with a single value. Supported operations are:

- creating an empty map;
- querying whether a map contains a given key;
- lookup of a value associated with a given key, if the key is present;
- inserting a key and a value to be associated with it, if the key is not already present;
- removing a key (together with the associated value), if the key is present.

Design an abstract data type MAP that corresponds to the specification given above.

TYPES

MAP [K, V]

FUNCTIONS

- **new**: MAP[K, V]
- **has**(m, k): MAP[K, V] × K → BOOLEAN
- **item**(m, k): MAP[K, V] × K ⫸ V
- **put**(m, k, x): MAP[K, V] × K × V ⫸ MAP[K, V]
- **remove**(m, k): MAP[K, V] × K ⫸ MAP[K, V]

PRECONDITIONS

P1 item(m, k) require has(m, k)
P2 put(m, k, x) require ¬has(m, k)
P3 remove(m, k) require has(m, k)

AXIOMS

A1 ¬has(new, k)
A2 has(put(m, k, x), k)
A3 has(put(m, k, x), l) = has(m, l), if l ≠ k
A4 ¬has(remove(m, k), k)
A5 has(remove(m, k), l) = has(m, l), if l ≠ k
A6 item(put(m, k, x), k) = x
A7 item(put(m, k, x), l) = item(m, l), if l ≠ k ∧ has(m, l)
A8 item(remove(m, k), l) = item(m, l), if l ≠ k ∧ has(m, l)
1.1 Proof of sufficient completeness

Prove that your ADT is sufficiently complete.

For all terms $T$ of type MAP there exist resulting terms not involving any functions of the ADT when evaluating $\text{has}(T,k)$ and $\text{item}(T,k)$.

**Induction basis**

For all creators above holds.

- $\text{has}(\text{new},k) \equiv \text{False}$

We don’t have to check $\text{item}(\text{new},k)$, because the precondition is never satisfied.

**Induction hypothesis**

Assume for any $T_{sub}$ being a subterm of $T$ that this is true.

**Induction step**

- For $T = \text{put}(T_{sub},k,x)$:

  \[
  \text{has}(\text{put}(T_{sub},k,x),l) = \begin{cases} 
  A2 & \text{True if } k = l \\
  A3 & \text{has}(T_{sub},l) \text{ if } k \neq l 
  \end{cases}
  \]

  \[
  \text{item}(\text{put}(T_{sub},k,x),l) = \begin{cases} 
  A6 & x \text{ if } k = l \\
  A7 & \text{item}(T_{sub},l) \text{ if } k \neq l \land \text{has}(T_{sub},l)
  \end{cases}
  \]

- For $T = \text{remove}(T_{sub},k)$:

  \[
  \text{has}(\text{remove}(T_{sub},k),l) = \begin{cases} 
  A4 & \text{False if } k = l \\
  A5 & \text{has}(T_{sub},l) \text{ if } k \neq l 
  \end{cases}
  \]

  \[
  \text{item}(\text{remove}(T_{sub},k),l) \equiv \text{has}(T_{sub},l) \text{ if } k \neq l \land \text{has}(T_{sub},l)
  \]