Separation logic for OO

Stephan van Staden
Introduction

OO languages are popular and widely used

We need to reason about OO programs

Some (problematic) features of OO languages:
• Shared mutable state
• Inheritance (subtyping and overriding)
  – Determining what a method call does is difficult!
    Complicated method lookup scheme that relies on dynamic info. We are interested in static verification...
Reasoning about OO programs

We can use separation logic to reason about shared mutable state

But it is not enough! We need to accommodate and control inheritance
(Many published OO proof systems cannot reason about simple programming patterns, or support them in a very complicated way)

We will look at a state of the art separation logic for OO by Parkinson and Bierman (“Separation Logic, Abstraction and Inheritance”, proceedings POPL 2008)
Outline

1. Shared mutable state
   1. Memory model
   2. Simple statements & proof rules

2. Inheritance
   1. Abstract predicate families
   2. Method specification and verification
1. Shared mutable state
1.1 OO memory model

State =def Stack x DType x Heap
Stack =def Var -> Value
Value =def Ref U Int U ... 
DType =def Ref ->\text{fin} Type (dynamic type info)
Heap =def Field ->\text{fin} Value (location granularity = field)
Field =def Ref x Attributenname

\[(S, D, H) \vdash e.f \leftrightarrow e' =def H([e]_S, f) = [e']_S \quad \text{(different!)}\]
\[(S, D, H) \vdash e : C =def D([e]_S) = C\]
\[(S, D, H) \vdash e = e' =def [e]_S = [e']_S\]
\[(S, D, H) \vdash P * Q =def \exists H_1, H_2 . \ H_1 \perp H_2, \ H = H_1 \cup H_2, \]
\[(S, D, H_1) \vdash P, \ (S, D, H_2) \vdash Q\]
1.2 Simple instructions & proof rules

- Field mutation
  \( \{x.f \mapsto \_\} \ x.f := y \ \{x.f \mapsto y\} \)

- Field lookup
  \( \{x.f \mapsto e\} \ y := x.f \ \{x.f \mapsto e \ast y = e\} \), provided \( y \) is not the same as \( x \), and \( y \) does not occur free in \( e \)

Rules for variable assignment, sequential composition, conditional, loop, etc. are the familiar ones

Later: method calls (a bit complicated due to inheritance)
Structural rules

• Frame:

\[
\begin{array}{c}
\{P\} s \{Q\} \\
\{P*R\} s \{Q*R\}
\end{array}
\]

provided \text{modify}(s) \cap \text{FV}(R) = \{\}

Note: \text{modify}(x.f := y) = \{\}

• Auxiliary Variable Elimination:

\[
\begin{array}{c}
\exists v. P \} s \exists v. Q
\end{array}
\]

provided \(v\) does not occur free in \(s\)

• Consequence, ...
2. Inheritance
2.1 Abstract predicate families (apfs)

OO programming is based on encapsulated data-centered abstractions

Parkinson and Bierman’s system embraces this, resulting in simple reasoning that accommodates OO (esp. inheritance) well

Apfs are heavily used in method specs
Abstract predicates

Abstract predicate: name, definition and scope. Within the scope, one can freely change between the name and definition. Outside the scope, one can use the name only atomically.

Examples: list, tree, etc.

Since sep logic predicates describe data, abstract predicates are encapsulated data abstractions. Fit OO remarkably well!

For simplicity: single class as the scope. Think “interface” and “implementation” of a predicate. Clients use interfaces
The abstraction boundary is a class, but the abstractions themselves are not dictated by classes.

Examples:

• An abstract predicate `List` can be implemented with Node objects.
• class `List` can also implement a `Stack` abstract predicate.
• class `Student` can implement a `Person` predicate.
The “family” part of apfs

Different classes (and in particular subclasses) can implement abstractions differently.

They can provide different definitions, or entries, for the same predicate, hence predicate family. The definition that applies is based on the dynamic type of the first predicate argument. Apfs as “dynamically dispatched predicates”

Example: class Cell defines \( x.\text{Val}_{\text{Cell}}(n) \) as \( x.\text{val} \mapsto n \).

- Val is an apf
- \( \text{Val}_{\text{Cell}} \) is class Cell’s entry of the apf Val
class Cell {
    // apf definitions
    define x.Val_{Cell}(n) as x.val ↦ n
    // field declarations
    val: int
    // methods e.g. get, set
}

Within the scope of class Cell only, we can use the assumptions (in the rule of consequence when verifying the methods of Cell):

• FtoE: ∀x,n . x : Cell ⇒ [x.Val(n) ⇔ x.Val_{Cell}(n)]
• EtoD: ∀x,n . x.Val_{Cell}(n) ⇔ x.val ↦ n
• Arity reduction: ∀x . x.Val() ⇔ x.Val(_)

Arity reduction allows subclasses to add arguments to apfs
class Cell {
    // apf definitions
    define x.Val_{Cell}(n) as x.val \mapsto n
    // field declarations
    val: int
    // methods e.g. get, set
}

class ReCell inherit Cell {
    // apf definitions
    define x.Val_{ReCell}(n, b) as x.Val_{Cell}(n) * x.bak \mapsto b
    // field declarations: a backup value
    bak: int
    // methods: new, overridden, ...
}

- ReCell does not know the definition of x.Val_{Cell}(n), yet it defines its entry of apf Val in terms of it
- ReCell adds an argument to the apf Val. In the scope of ReCell:
  \forall x . x.Val() \leftrightarrow x.Val(_)
  \forall x,n . x.Val(n) \leftrightarrow x.Val(n, _)
2.2 Method specification & verification
Static & dynamic specs

Two types of method calls in OO languages:
  • Statically dispatched/bound calls
    Examples: `super` calls in Java, `x.C::m(a)` in C++
  • Dynamically dispatched/bound calls
    Example: `x.m(a)`

Specify each method with a static and a dynamic spec
  • Use static spec to verify statically dispatched calls. Static spec describes in detail what the body does
  • Use dynamic spec to verify dynamically dispatched calls. Dynamic spec is more abstract – it gives the idea behind the method that all subclasses must respect
class Cell {
    // apf definitions
    define x.Val_{Cell(n)} as x.val \mapsto n
    // field declarations
    val: int

    // methods
    introduce set(x: int)
    dynamic {this.Val(_)}\_\{this.Val(x)}
    static {this.Val_{Cell(_)}\_\{this.Val_{Cell(x)}}
        \{ this.val := x \}

    introduce get(): int
    dynamic {this.Val(v)}\_\{this.Val(v) * Res = v}
    static {this.Val_{Cell(v)}\_\{this.Val_{Cell(v)} * Res = v}
        \{ Res := this.val \}
}
Client reasoning

Assume the constructor `Cell(x: int)` has dynamic spec:
{true}_{this.Val(x)}

```java
class Cell {
    // apf definitions
    define x.Val\_cell(n) as x.val \mapsto n
    // field declarations
    val: int

    // methods
    introduce set(x: int)
    dynamic {this.Val(_)}_{this.Val(x)}
    static {this.Val\_cell(_)}_{this.Val\_cell(x)}
    { this.val := x }

    introduce get(): int
    dynamic {this.Val(v)}_{this.Val(v) \ast Res = v}
    static {this.Val\_cell(v)}_{this.Val\_cell(v) \ast Res = v}
    { Res := this.val }
}
```

{true}

```java
x := new Cell(3)
{x.Val(3)}
y := new Cell(4)
{x.Val(3) \ast y.Val(4)}
x.set(5)
{x.Val(5) \ast y.Val(4)}
n := y.get()
{x.Val(5) \ast y.Val(4) \ast n=4}
m := x.val
{???}
```
Verifying a newly introduced method

Two proof obligations:

1. Body verification
   Verify that the body satisfies the static spec, using the apf assumptions of the containing class and all method specs
   \[
   \begin{align*}
   &\{\text{this} &.\text{Val}_{\text{Cell}}(\_)} \}\{\text{this} &.\text{val} \mapsto \_}\} \\
   &\{\text{this} &.\text{val} \mapsto x\} \\
   &\{\text{this} &.\text{Val}_{\text{Cell}}(x)\}
   \end{align*}
   \]

   \[
   \begin{align*}
   &\text{define } x.\text{Val}_{\text{Cell}}(n) \text{ as } x.\text{val} \mapsto n \\
   &\text{methods} \\
   &\text{introduce } set(x: \text{ int}) \\
   &\text{dynamic } \{\text{this} &.\text{Val}(\_}\}_{'this} &.\text{Val}(x)\} \\
   &\text{static } \{\text{this} &.\text{Val}_{\text{Cell}}(\_}\}_{'this} &.\text{Val}_{\text{Cell}}(x)\} \\
   &\{\text{this} &.\text{val} := x\}
   \end{align*}
   \]

2. Dynamic dispatch
   Verify the consistency of the static and dynamic specs.
   In particular, check under the apf assumptions that the dynamic spec with “this : Cell” added to the precondition follows from the static spec.
   \[
   \begin{align*}
   &\{\text{this} &.\text{Val}_{\text{Cell}}(\_}\}_{'this} &.\text{Val}_{\text{Cell}}(x)\} \implies {\text{this} &.\text{Val}(\_}\}_{'this} &.\text{Val}(x)\}
   \end{align*}
   \]
Specification refinement

\{P\}_{Q} \implies \{P'\}_{Q'} also means that:

• the specification \{P\}_{Q} is stronger than \{P'\}_{Q'}
• whenever a statement satisfies \{P\}_{Q}, it must also satisfy \{P'\}_{Q'}

A proof of \{P\}_{Q} \implies \{P'\}_{Q'} uses the structural rules of sep logic
(\text{Frame}, \text{AuxVarElim}, \text{Consequence}, ...) to establish \{P'\}_{Q'} under the assumption \{P\}_{Q}

For example,
\{\text{this} \cdot \text{Val}_{\text{Cell}}(\_)}_{\text{this} \cdot \text{Val}_{\text{Cell}}(x)} \implies \{\text{this} : \text{Cell} \ast \text{this} \cdot \text{Val}(\_)}_{\text{this} \cdot \text{Val}(x)}

Proof:
Assumption \quad \{\text{this} \cdot \text{Val}_{\text{Cell}}(\_)}_{\text{this} \cdot \text{Val}_{\text{Cell}}(x)}
Frame rule \quad \{\text{this} : \text{Cell} \ast \text{this} \cdot \text{Val}_{\text{Cell}}(\_)}_{\text{this} : \text{Cell} \ast \text{this} \cdot \text{Val}_{\text{Cell}}(x)}
Consequence \quad \{\text{this} : \text{Cell} \ast \text{this} \cdot \text{Val}(\_)}_{\text{this} : \text{Cell} \ast \text{this} \cdot \text{Val}(x)}
Consequence \quad \{\text{this} : \text{Cell} \ast \text{this} \cdot \text{Val}(\_)}_{\text{this} \cdot \text{Val}(x)}

Remember the apf assumption of class Cell:
\forall x,n \cdot x : \text{Cell} \implies [x \cdot \text{Val}(n) \iff x \cdot \text{Val}_{\text{Cell}}(n)]
Subclassing

class Cell {
    // apf definitions
    define x.Val_{\text{Cell}}(n) as x.val \mapsto n
    // field declarations
    val: int

    // methods
    introduce set(x: int)
    dynamic {this.Val(_)}_\{this.Val(x)}
    static {this.Val_{\text{Cell}}(_)}_\{this.Val_{\text{Cell}}(x)}
    { this.val := x }

    introduce get(): int
    dynamic {this.Val(v)}_\{this.Val(v) * Res = v}
    static {this.Val_{\text{Cell}}(v)}_\{this.Val_{\text{Cell}}(v) * Res = v}
    { Res := this.val }
}

class ReCell inherit Cell {
    // apf definitions
    define x.Val_{\text{ReCell}}(n, b) as x.Val_{\text{Cell}}(n) * x.bak \mapsto b
    // field declarations: a backup value
    bak: int

    // methods
    override set(x: int)
    dynamic {this.Val(v, _)}_\{this.Val(x, v)}
    static {this.Val_{\text{ReCell}}(v, _)}_\{this.Val_{\text{ReCell}}(x, v)}
    { local t: int
      t := this.Cell::get(); this.Cell::set(x); this.bak := t }

    inherit get(): int
    dynamic {this.Val(v, b)}_\{this.Val(v, b) * Res = v}
    static {this.Val_{\text{ReCell}}(v, b)}_\{this.Val_{\text{ReCell}}(v, b) * Res = v}
}
Verifying an overridden method

The three proof obligations use the apf assumptions of the child class:

1. **Body verification**

2. **Dynamic dispatch**

3. **Behavioural subtyping**
   Verify that the dynamic spec of the method in the child class is stronger than the one in the parent class

Example: \( \{\text{this.\text{Val}(v, \_)}\}_x\{\text{this.\text{Val}(x, v)}\} \implies \{\text{this.\text{Val}(\_)}\}_x\{\text{this.\text{Val}(x)}\} \)

   **Proof:**
   
   **Assumption:** \( \{\text{this.\text{Val}(v, \_)}\}_x\{\text{this.\text{Val}(x, v)}\} \)
   
   **AuxVarElim:** \( \exists v. \{\text{this.\text{Val}(v, \_)}\}_x\{\exists v. \text{this.\text{Val}(x, v)}\} \)
   
   **Consequence:** \( \{\text{this.\text{Val}(\_)}\}_x\{\text{this.\text{Val}(x, \_)}\} \)
   
   **Consequence:** \( \{\text{this.\text{Val}(\_)}\}_x\{\text{this.\text{Val}(x)}\} \)

Remember the apf assumption of class ReCell:
\( \forall x,n . \text{x.\text{Val}(n)} \iff \text{x.\text{Val}(n, \_)} \)
Verifying an inherited method

The three proof obligations use the apf assumptions of the child class:

1. Behavioural subtyping

2. Dynamic dispatch

3. Inheritance
   Verify that the static specification of the method in the child class follows from the one in the parent class

Example: {this.$Val_{Cell}$\{v\}}_{\{this.$Val_{Cell}$\{v\} * Res = v\}} ==>
{this.$Val_{ReCell}$\{v, b\}}_{\{this.$Val_{ReCell}$\{v, b\} * Res = v\}}

Proof:
Assumption: {this.$Val_{Cell}$\{v\}}_{\{this.$Val_{Cell}$\{v\} * Res = v\}}
Frame: {this.$Val_{Cell}$\{v\} * this.bak $\mapsto$ b}_{\{this.$Val_{Cell}$\{v\} * Res = v * this.bak $\mapsto$ b\}}
Consequence: {this.$Val_{ReCell}$\{v, b\}}_{\{this.$Val_{ReCell}$\{v, b\} * Res = v\}}
Static/dynamic specs - reflection

Only dynamic specs are involved in behavioural subtyping

Apfs are a great enabler of behavioural subtypes

With static specs, child classes never need to see the code of parents. Good for modularity
Copy-and-paste inheritance

```plaintext
class Cell {
    // apf definitions
    define x.ValCell(n) as x.val → n
    // field declarations
    val: int

    // methods
    introduce set(x: int)
    dynamic {this.Val(_)}{this.Val(x)}
    static {this.ValCell(_)}{this.ValCell(x)}
    { this.val := x }

    introduce get(): int
    dynamic {this.Val(v)}{this.Val(v) * Res = v}
    static {this.ValCell(v)}{this.ValCell(v) * Res = v}
    { Res := this.val }
}

class DCell inherit Cell {
    override set(x: int)
    { this.Cell::set(2*x) }
}
```

- Is this a “proper” use of inheritance?
- Can one ever hope to verify such code?
class Cell {
    // field declarations
    val: int

    // methods
    introduce set(x: int)
    dynamic {this.Val(_)}_{this.Val(x)}
    static {this.ValCell(_)}_{this.ValCell(x)}
    { this.val := x }

    introduce get(): int
    dynamic {this.Val(v)}_{this.Val(v) * Res = v}
    static {this.ValCell(v)}_{this.ValCell(v) * Res = v}
    { Res := this.val }
}

class DCell inherit Cell {
    // field declarations
    // methods
    introduce set(x: int)
    dynamic {this.Val(_)}_{this.Val(x)}
    static {this.ValCell(_)}_{this.ValCell(x)}
    { this.val := x }

    introduce get(): int
    dynamic {this.Val(v)}_{this.Val(v) * Res = v}
    static {this.ValCell(v)}_{this.ValCell(v) * Res = v}
    { Res := this.val }
}

define x.Val_{Cell}(n) as x.val \mapsto n
define x.DVal_{Cell}(n) as x.Val_{Cell}(n)

define x.Val_{DCell}(n) as false
define x.DVal_{DCell}(n) as x.Val_{Cell}(n)
The key insight:  \textbf{define} x.\text{Val}_{\text{DCell}}(n) \textbf{as} false

The proof obligations (e.g. behavioural subtyping) are trivialized

DCell ensures that no client will ever have a Val predicate for a Dcell object. Therefore, in the “Val-world”, DCell is not a subtype of Cell (that is, a variable of static type Cell that satisfies Val will not point to a DCell object)

Apfs specify logical inheritance
Conclusion

Separation logic for reasoning about shared mutable state

Apfs and static/dynamic method specs allow flexible handling of inheritance

The combination (Parkinson & Bierman’s system) suits the OO paradigm well. Modular and intuitive. Can verify common design patterns

Implemented in tools: jStar, VeriFast

This was just the basics – there is much more in the paper, and several extensions also exist