# Software Verification – Exam

ETH Zürich

17 December 2012

**Surname, first name**: .....

**Student number**: .....

I confirm with my signature that I was able to take this exam under regular circumstances and that I have read and understood the directions below.

**Signature**: .....

Directions:

- Exam duration: 1 hour 45 minutes.
- Except for a dictionary you are not allowed to use any supplementary material.
- All solutions can be written directly on the exam sheets. If you need more space for your solution ask the supervisors for a sheet of official paper. You are **not** allowed to use other paper. Please write your student number on **each** additional sheet.
- Only one solution can be handed in per question. Invalid solutions need to be crossed out clearly.
- Please write legibly! We will only correct solutions that we can read.
- Manage your time carefully (take into account the number of points for each question).
- Please **immediately** tell the exam supervisors if you feel disturbed during the exam.

**Good luck!**

| Question | Available points | Your points |
|---|---|---|
| 1) Axiomatic semantics | 18 | |
| 2) Separation Logic | 15 | |
| 3) Data flow analysis | 10 | |
| 4) Model checking | 15 | |
| 5) Real time verification | 12 | |
| **Total** | **70** | |

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# 1 Axiomatic semantics (18 points)

Consider the following annotated program, where $A$ is an array indexed from 1 of element type $G$, $n$ is an integer variable storing $A$'s size, $k$ is another integer variable, $v$ is a variable of type $G$ initialized to some fixed value, *found* is a Boolean variable.

```
  { n ≥ 0 }
1   from
2     k := n
3     found := False
4   until found or k < 1 loop
5     if A[k] = v then
6       found := True
7     else
8       k := k − 1
9     end
10  end
  { (found ⟹ 1 ≤ k ≤ n ∧ A[k] = v) ∧ ( ¬found ⟹ k < 1 )}
```

## 1.1 Program semantics (2 points)

Characterize, in plain English, which value of $k$ the program computes from the inputs $A$, $n$, and $v$. In other words: what does the program do?

**Solution:**
As apparent from the postcondition, the program searches for an element with value $v$ in $A$ between positions 1 and $n$. If it finds the element, $k$ stores the element's position; if it does not find the element, $k$ is out of array's bounds.

## 1.2 Partial correctness (15 points)

Prove that the triple (precondition, program, postcondition) is a theorem of Hoare's axiomatic system for partial correctness.

**Solution:**

```
1 { n ≥ 0 }
2   from
3     k := n
4     found := False
5   { 0 ≤ k ≤ n  ∧  (found ⟹ 1 ≤ k ≤ n ∧ A[k] = v) }
6   until found or k < 1 loop
7     { 1 ≤ k ≤ n  ∧  ¬ found  ∧  (found ⟹ 1 ≤ k ≤ n ∧ A[k] = v) }
8     if A[k] = v then
9       { A[k] = v  ∧  1 ≤ k ≤ n  ∧  ¬ found }
10      { 0 ≤ k ≤ n  ∧  1 ≤ k ≤ n  ∧  A[k] = v }
```

11 $found := \textbf{True}$
12 $\{\ 0 \leq k \leq n\ \wedge\ (found \Longrightarrow 1 \leq k \leq n \wedge A[k] = v)\ \}$
13 **else**
14 $\{\ A[k] \neq v\ \wedge\ 1 \leq k \leq n\ \wedge\ \neg\, found\ \}$
15 $\{\ 1 \leq k \leq n+1\ \wedge\ (found \Longrightarrow 2 \leq k \leq n+1 \wedge A[k-1] = v)\ \}$
16 $k := k - 1$
17 $\{\ 0 \leq k \leq n\ \wedge\ (found \Longrightarrow 1 \leq k \leq n \wedge A[k] = v)\ \}$
18 **end**
19 $\{\ 0 \leq k \leq n\ \wedge\ (found \Longrightarrow 1 \leq k \leq n \wedge A[k] = v)\ \}$
20 **end**
21 $\{\ (found \wedge 1 \leq k \leq n \wedge A[k] = v)\ \vee\ (\neg found \wedge k = 0)\ \}$
22 $\{(found \Longrightarrow 1 \leq k \leq n \wedge A[k] = v) \wedge (\ \neg found \Longrightarrow k < 1\ )\}$

## 1.3 Termination (1 point)

Find a suitable *variant* function $V$ to prove termination. $V$ must be such that it decreases along all branches of the loop body, and it is nonnegative after every iteration of the loop. You do *not* have to prove termination, just write a suitable variant and informally argue why it is a suitable variant.

**Solution:**
The variant $V \triangleq k + [found = \textbf{False}]$, where the square brackets denote Iverson bracket (equal to 1 if its arguments holds, and equal to 0 otherwise), is always nonnegative and decrease in both branches ($k$ decreases along the **else** branch, and $[found = \textbf{True}]$ decreases along the **then** branch). Hence, $V$ can be used to build a proof of termination for the loop.

# 2 Separation Logic (15 points)

A well-formed binary tree $t$ is given by the grammar:

$t \stackrel{\text{def}}{=} n \mid (t_1, t_2)$

So a tree value $t$ can be either a leaf, which is a single number $n$, or an internal node with a left subtree $t_1$ and a right subtree $t_2$.

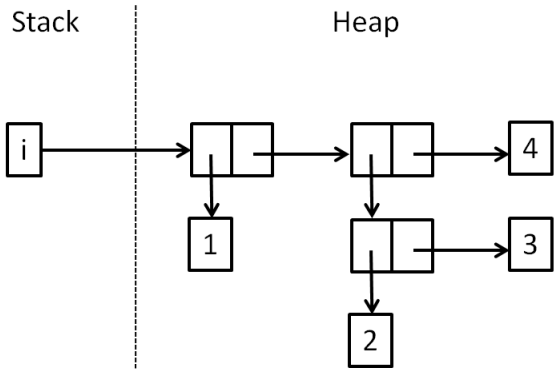
Consider the definition of the recursive predicate *tree* $t$ $i$ which asserts that $i$ is a pointer to a well-formed binary tree $t$:

$\mathit{tree}\ n\ i \stackrel{\text{def}}{=} i \mapsto n$
$\mathit{tree}\ (t_1, t_2)\ i \stackrel{\text{def}}{=} \exists l, r \cdot i \mapsto l, r * \mathit{tree}\ t_1\ l * \mathit{tree}\ t_2\ r$

With these definitions in mind, answer the following questions.

## 2.1 Predicate Satisfaction (6 points)

Consider the following program state:




Indicate in the table whether or not a given assertion is satisfied by this state. Indicate satisfaction with a `T` and non-satisfaction with an `F`.

| | `T` or `F` |
|---|---|
| $\exists z \cdot i \mapsto z$ | `F` |
| $\exists x, y \cdot i \mapsto 1, x * x \mapsto y, 4 * \mathit{true}$ | `F` |
| $\exists j \cdot \mathit{tree}\ (2,3)\ j * \mathit{true}$ | `T` |
| $\exists j, t \cdot i \mapsto j * j \mapsto 1 * \mathit{tree}\ t\ (i+1)$ | `F` |
| $\exists t_1, t_2 \cdot \mathit{tree}\ (t_1, (t_2, 4))\ i$ | `T` |
| $\exists j, k \cdot i \mapsto j * j \mapsto 1 * (i+1) \mapsto k * \mathit{tree}\ ((2,3),4)\ k$ | `T` |

## 2.2 Code Verification (9 points)

Give a brief proof outline of the following triple. There must be at least one assertion between every two sub-commands.
$\{\mathit{tree}\ (1, t)\ i\}$
$x := [i];\ \ [i] := 2;\ \ y := [i+1];\ \ \textbf{dispose}\ i;\ \ \textbf{dispose}\ x;\ \ \textbf{dispose}\ (i+1)$
$\{\mathit{tree}\ t\ y\}$

$\{tree\ (1, t)\ i\}$
$\{\exists l, r \cdot i \mapsto l, r * tree\ 1\ l * tree\ t\ r\}$
$\quad x := [i];$
$\{\exists r \cdot i \mapsto x, r * tree\ 1\ x * tree\ t\ r\}$
$\quad [i] := 2;$
$\{\exists r \cdot i \mapsto 2, r * tree\ 1\ x * tree\ t\ r\}$
$\quad y := [i + 1];$
$\{i \mapsto 2, y * tree\ 1\ x * tree\ t\ y\}$
$\quad \textbf{dispose}\ i;$
$\{(i + 1) \mapsto y * tree\ 1\ x * tree\ t\ y\}$
$\{(i + 1) \mapsto y * x \mapsto 1 * tree\ t\ y\}$
$\quad \textbf{dispose}\ x;$
$\{(i + 1) \mapsto y * tree\ t\ y\}$
$\quad \textbf{dispose}\ (i + 1);$
$\{tree\ t\ y\}$

# 3 Data flow analysis (10 points)

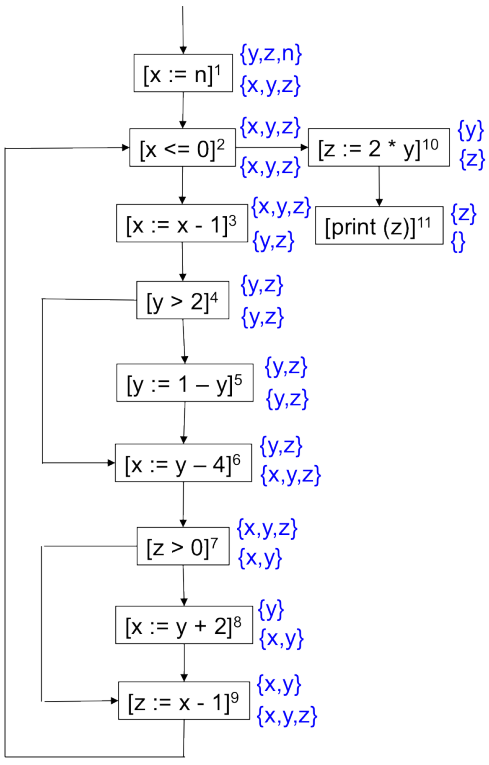
Consider the following program fragment (all variables are of type *INTEGER*):

```
1   from
2     x := n
3   until x ≤ 0 do
4     x := x − 1
5     if y > 2 then
6       y := 1 − y
7     end
8     x := y − 4
9     if z > 0 then
10      x := y + 2
11    end
12    z := x − 1
13  end
14  z := 2 ∗ y
15  print (z)
```

**(1) (3 points)** Draw the *control flow graph* of the program fragment and label each elementary block.

**(2) (5 points)** Annotate your control flow graph with the analysis result of a *live variables analysis* of the program fragment.

**Solution:**



**(3) (2 points)** Explain how the live variables analysis can be used for dead code elimination, and apply this technique to the program fragment; it suffices to state which statement(s), if any, would be removed as dead code.

**Solution:**

An assignment $[x := a]^\ell$ is dead if the value of $x$ is not used before it is redefined. The live variables analysis gives, for each program point, the variables that may be live at the exit from the point. Hence if $x \notin LVexit(\ell)$, the assigment is dead.

This is the case for the block at program point 3, which is the only statement to be be removed.

# 4 Model Checking (15 points)

Recall the semantics of LTL over finite words with alphabet $\mathcal{P}$. For a word $w = w(1)w(2)\cdots w(n) \in \mathcal{P}^*$ with $n \geq 0$ and a position $1 \leq i \leq n$ the satisfaction relation $\models$ is defined recursively as follows (where $p, q \in \mathcal{P}$).

| | | |
|---|---|---|
| $w, i \models p$ | iff | $p = w(i)$ |
| $w, i \models \neg\phi$ | iff | $w, i \not\models \phi$ |
| $w, i \models \phi_1 \wedge \phi_2$ | iff | $w, i \models \phi_1$ and $w, i \models \phi_2$ |
| $w, i \models \mathsf{X}\phi$ | iff | $i < n$ and $w, i+1 \models \phi$ |
| $w, i \models \phi_1 \,\mathsf{U}\, \phi_2$ | iff | there exists $i \leq j \leq n$ such that: $w, j \models \phi_2$ and for all $i \leq k < j$ it is the case that $w, k \models \phi_1$ |
| $w, i \models \Diamond\, \phi$ | iff | there exists $i \leq j \leq n$ such that: $w, j \models \phi$ |
| $w, i \models \Box\, \phi$ | iff | for all $i \leq j \leq n$ it is the case that: $w, j \models \phi$ |
| $w \models \phi$ | iff | $w, 1 \models \phi$ |

## 4.1 Automata and LTL formulas (7 points)

Consider the automaton $\mathcal{A}$ (with states $A, B, C$) in Figure 1, over the alphabet $\{p, q\}$. Notice that $A$ is the initial state and $B$ is final.

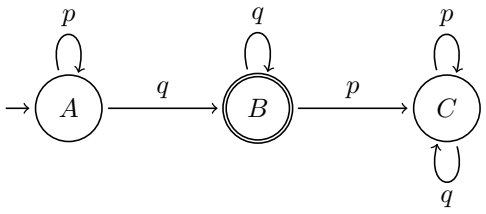



Figure 1: Automaton $\mathcal{A}$ over alphabet $\{p, q\}$.

For each of the following LTL formulas say whether every accepting run of $\mathcal{A}$ satisfies the formula. If it does, argue informally (but precisely) why this is the case; if it does not, provide a counterexample.

**(1)** $\mathcal{A} \models \Diamond\, p$

No: the word $q$ is accepted by $\mathcal{A}$ but does not satisfy $\Diamond\, p$.

**(2)** $\mathcal{A} \models \Diamond\, q$

Yes: every accepting run must reach state $B$, and hence it must include $q$ somewhere.

**(3)** $\mathcal{A} \models \neg p \;\vee\; \mathsf{X}(\neg q)$

No: the word $p\,q$ does not satisfy $\neg p \vee \mathsf{X}(\neg q)$, because $p$ holds in the first position and $q$ holds in the second position; however, it is accepted by $\mathcal{A}$.

**(4)** $\mathcal{A} \models \Box(\, q \Longrightarrow \Box\, q\,)$

Yes: after the first $q$ is read, $\mathcal{A}$ is in state $B$, which no accepting run ever leaves; and the only transition possible in $B$ is reading $q$.

**(5)** $\mathcal{A} \models \Box(\, p \,\mathsf{U}\, q\,)$

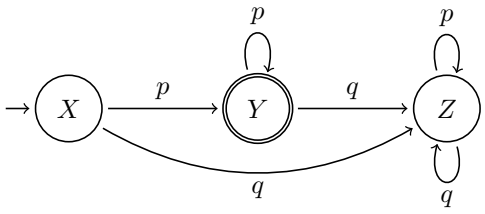
Yes: when $\mathcal{A}$ is in state $A$, $q$ must occur in the future (to reach the accepting state $B$) and only $p$ can occur before it; hence $\mathcal{A} \models p \,\mathsf{U}\, q$. After $\mathcal{A}$ enters state $B$, it does not leave it on any accepting run; the only transition possible in $B$ is reading $q$, which trivially satisfies $p \,\mathsf{U}\, q$ with $q$ occurring at the evaluation instant.

## 4.2 Automata-based model checking (8 points)

Show that $\mathcal{A} \models p \implies \Diamond\, q$ using automata-based model checking as follows.

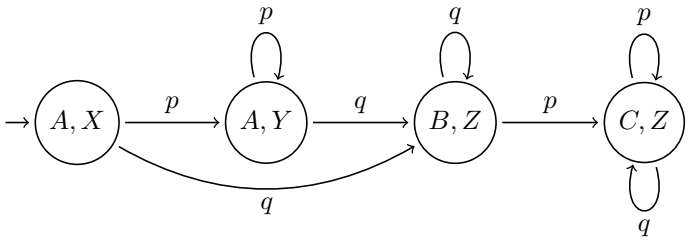
**Property automaton (4 points).** Construct an automaton $\mathcal{F}$ that accepts *precisely* the words that satisfy $\neg(p \implies \Diamond\, q)$, that is, the *complement* of the property we want to verify.

**Solution:**
$\neg(p \implies \Diamond\, q)$ is accepted by the automaton:




**Intersection automaton (4 points).** Construct the intersection automaton $\mathcal{A} \times \mathcal{F}$ that accepts precisely the words accepted by both $\mathcal{A}$ and $\mathcal{F}$ and show that $\mathcal{A} \times \mathcal{F}$ does not accepts any words.

**Solution:**
No accepting state is connected.



# 5 Real Time Verification (12 points)

Recall the semantics of (a subset of) MTL over finite timed words with alphabet $\mathcal{P}$ and time domain $\mathbb{T}$. For a timed word

$$w = [\sigma(1), t(1)][\sigma(2), t(2)] \cdots [\sigma(n), t(n)] \in (\mathcal{P} \times \mathbb{T})^*$$

with $n \geq 0$ and a position $1 \leq i \leq n$ the satisfaction relation $\models$ is defined recursively as follows for $p \in \mathcal{P}$ and $J$ an interval of $\mathbb{T}$ with integer endpoints.

| | | |
|---|---|---|
| $w, i \models p$ | iff | $p = \sigma(i)$ |
| $w, i \models \neg\phi$ | iff | $w, i \not\models \phi$ |
| $w, i \models \phi_1 \wedge \phi_2$ | iff | $w, i \models \phi_1$ and $w, i \models \phi_2$ |
| $w, i \models \Diamond_J \phi$ | iff | there exists $i \leq j \leq n$ such that: $t(j) - t(i) \in J$ and $w, j \models \phi$ |
| $w, i \models \Box_J \phi$ | iff | for all $i \leq j \leq n$: if $t(j) - t(i) \in J$ then $w, j \models \phi$ |
| $w \models \phi$ | iff | $w, 1 \models \phi$ |

As time domain $\mathbb{T}$, we will consider either the natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$ or the nonnegative real numbers $\mathbb{R}_{\geq 0}$.

## 5.1 MTL semantics (4 points)

**(1)** Are the MTL formulas $\psi_1$ and $\psi_2$:

$$\begin{aligned} \psi_1 &\triangleq \Diamond_{[1,1]}\Big(\Diamond_{[1,1]}(p)\Big) \\ \psi_2 &\triangleq \Diamond_{[2,2]}(p) \end{aligned}$$

equivalent over time domain $\mathbb{N}$? If they are, show that their semantics imply each other; if they are not, provide a timed word which is satisfied by one formula and not satisfied by the other.

**Solution:**
They are *not* equivalent: the word $\mu = [p, 0][p, 2]$ is satisfied by $\psi_2$ but not by $\psi_1$ which requires at least *three* positions in a word to satisfy it.

**(2)** Does the answer to the previous question (1) change over the time domain $\mathbb{R}_{\geq 0}$? Explain why it does or does not change.

**Solution:**
The answer is still negative, and the very same word $\mu$ works for the time domain $\mathbb{R}_{\geq 0}$.

## 5.2 Timed automata and MTL formulas (8 points)

In this section, we take $\mathbb{R}_{\geq 0}$ as the time domain $\mathbb{T}$. Consider the timed automaton $\mathcal{T}$ (with locations $A, B, C$) in Figure 2, over the alphabet $\{p, q\}$ and with clocks $x$ and $y$. Notice that $A$ is the initial location and $C$ is final.
For each of the following MTL formulas say whether every accepting run of $\mathcal{T}$ satisfies the formula. If it does, argue informally (but precisely) why this is the case; if it does not, provide a counterexample.

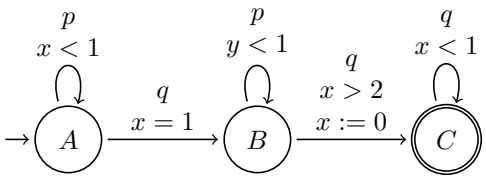



Figure 2: Timed automaton $\mathcal{T}$ over alphabet $\{p, q\}$ with clocks $x$ and $y$.

**(1)** $\mathcal{T} \models \Diamond_{(0,2)}\, q$

No: the timed word $[q, 1][q, 10]$ is accepted by $\mathcal{T}$ but it does not satisfy $\Diamond_{(0,2)}\, q$ because $q$ does not occur in the interval $(0, 2)$ relative to the first position where the formula is evaluated (according to the semantics given above).

**(2)** $\mathcal{T} \models \Box_{[0,\infty)}\Big(\Diamond_{(1,\infty)}\, p\Big)$

No: the timed word $[q, 1]\,[q, 2.2]$ is accepted by $\mathcal{T}$ but it does not satisfy $\Box_{[0,\infty)}\Big(\Diamond_{(1,\infty)}\, p\Big)$ because $p$ never occurs..

**(3)** $\mathcal{T} \models \Box_{[0,\infty)}\Big(q \Longrightarrow \Box_{[0,\infty)}(\neg p)\Big)$

Yes: after the first occurrence of $q$ at time 1 (which appears in every accepted word), $p$ cannot happen anymore, thus trivially satisfying $\Box_{[0,\infty)}(\neg p)$. In fact, clock $y$ has the value 1 when entering location $B$, since it is synchronized with $x$; therefore, the self loop on location $B$ cannot be taken. Finally, there are no accessible other transitions where $p$ may occur.

**(4)** $\mathcal{T} \models \Diamond_{(2,\infty)}(q) \;\wedge\; \Box_{(3,\infty)}(\neg q)$

No: the timed word $[q, 1]\,[q, 10]\,[q, 10.3]$ is accepted by $\mathcal{T}$ but it does not satisfy $\Box_{(3,\infty)}(\neg q)$, and hence neither the whole MTL formula $\Diamond_{(2,\infty)}(q) \;\wedge\; \Box_{(3,\infty)}(\neg q)$, even if it satisfies its first conjunct.