Solution 8: Recursion

ETH Zurich

1 An infectious task

1. Correct. However, this version will call set_flu twice on all reachable persons except the initial one. On the initial person set_flu will be called once in case of a non-circular structure and three times in case of a circular structure.

2. Incorrect. This version results in endless recursion if the coworker structure is cyclic. The main cause is that the coworker does not get infected before the recursive call is made, so with a cyclic structure nobody will ever be infected to terminate the recursion.

3. Incorrect. This version results in an endless loop if the structure is cyclic. The main problem is with the loop’s exit condition that does not include the case when q is already infected.

4. Correct. This version works and uses tail recursion. It will always give the flu to p first, and then call infect on his/her coworker. The recursion ends when either there is no coworker, or the coworker is already infected. Without the second condition the recursion is endless if the coworker structure is cyclic.

Multiple coworkers

class
   PERSON

create
   make

feature -- Initialization

   make (a_name: STRING)
      -- Create a person named ‘a_name’.
      require
         a_name_valid: a_name /= Void and then not a_name.is_empty
      do
         name := a_name
         create {V.ARRAYED_LIST[PERSON]} coworkers
      ensure
         name_set: name = a_name
         no_coworkers: coworkers.is_empty
      end

feature -- Access
name: STRING
   -- Name.

coworkers: V_LIST [PERSON]
   -- List of coworkers.

has_flu: BOOLEAN
   -- Does the person have flu?

feature -- Element change

add_coworker (p: PERSON)
   -- Add 'p' to 'coworkers'.
   require
      p_exists: p /= Void
      p_different: p /= Current
      not_has_p: not coworkers.has (p)
   do
      coworkers.extend_back (p)
   ensure
      coworker_set: coworkers.has (p)
   end

set_flu
   -- Set 'has_flu' to True.
   do
      has_flu := True
   ensure
      has_flu: has_flu
   end

invariant
   name_valid: name /= Void and then not name.is_empty
   coworkers_exists: coworkers /= Void
   all_coworkers_exist: not coworkers.has (Void)
end

infect (p: PERSON)
   -- Infect 'p' and coworkers.
   require
      p_exists: p /= Void
   do
      p.set_flu
      across coworkers as c
      loop
         if not c.item.has_flu then
            infect (c.item)
         end
      end
   end
The coworkers structure is a directed graph. The master solution traverses this graph using \textit{depth-first search}.
2 Short trips

Listing 1: Class SHORT.TRIPS

note
description: "Short trips."

class
SHORT.TRIPS

inherit
ZURICH_OBJECTS

feature -- Explore Zurich

highlight_short_distance (s: STATION)
  -- Highlight stations reachable from 's' within 2 minutes.
require
  station_exists: s /= Void
  do
    highlight_reachable (s, 2 * 60)
  end

feature {NONE} -- Implementation

highlight_reachable (s: STATION; t: REAL_64)
  -- Highlight stations reachable from 's' within 't' seconds.
require
  station_exists: s /= Void
local
  line: LINE
next: STATION
do
  if t >= 0.0 then
    Zurich_map.station_view(s).highlight
    across
    s.lines as li
    loop
      line := li.item
      next := line.next_station (s, line.northterminal)
      if next /= Void then
        highlight_reachable (next, t - s.position.distance(next.position) / line.speed)
      end
      next := line.next_station (s, line.southterminal)
      if next /= Void then
        highlight_reachable (next, t - s.position.distance(next.position) / line.speed)
      end
    end
  end
end
3 N Queens

Listing 2: Class PUZZLE

note
description: "N–queens puzzle."

class
PUZZLE

feature -- Access

size: INTEGER
-- Size of the board.

solutions: LIST [SOLUTION]
-- All solutions found by the last call to 'solve'.

feature -- Basic operations

solve (n: INTEGER)
-- Solve the puzzle for 'n' queens
-- and store all solutions in 'solutions'.
require
n_positive: n > 0
do
size := n
create {LINKED_LIST [SOLUTION]} solutions.make
complete (create {SOLUTION}.make_empty)
ensure
solutions_exists: solutions /= Void
complete_solutions: across solutions as s all s.item.row_count = n end
end

feature {NONE} -- Implementation

complete (partial: SOLUTION)
-- Find all complete solutions that extend the partial solution 'partial'
-- and add them to 'solutions'.
require
partial_exists: partial /= Void
local
c: INTEGER
do
if partial.row_count = size then
solutions.extend (partial)
else
from
c := 1
until
c > size
loop
if not under_attack (partial, c) then
  complete (partial, extended_with (c))
end

end

c := c + 1
end

under_attack (partial: SOLUTION; c: INTEGER): BOOLEAN
-- Is column ‘c’ of the current row under attack
-- by any queen already placed in partial solution ‘partial’?
require
  partial_exists: partial /= Void
  column_positive: c > 0
local
  current_row, row: INTEGER
do
  current_row := partial, row_count + 1
from
  row := 1
until
  Result or row > partial, row_count
loop
  Result := attack_each_other (row, partial, column_at (row), current_row, c)
  row := row + 1
end
end

attack_each_other (row1, col1, row2, col2: INTEGER): BOOLEAN
-- Do queens in positions (‘row1’, ‘col1’) and (‘row2’, ‘col2’) attack each other?

4 MOOC: Design by Contract, recursion

The order in which the questions and the answers appear here in the solution may vary because they are randomly shuffled at each attempt.

Design by Contract: preconditions

• In class KNIGHT you have feature set_reputation (rep: INTEGER). What precondition would you write for it? \( \text{rep} \geq -5 \) and \( \text{rep} \leq 5 \)

• In class KNIGHT you have feature attack_monster (mon: MONSTER; wep: WEAPON). What precondition would you write for it? \( \text{wep} /= \text{Void} \) and \( \text{mon} /= \text{Void} \) and then \( \text{wep}.\text{is_ready} \)
• In class MONSTER you have feature `scan_direction (dir: DIRECTION)`. What precondition would you write for it? No explicit precondition is needed.

• In class WEAPON you have feature `set_ready (wep_ready: BOOLEAN)`. What precondition would you write for it? No precondition is needed here.

• Suppose that in class MONSTER, feature `attack`, you want to add the expression `is_knight_close` to the existing precondition `is_angry`. The true sentence is: The compound precondition `is_angry and is_knight_close` is a stronger precondition than `is_angry`.

• Suppose you know that a knight can only fight in battle if his or her hit points are greater than 10. Which is a reasonable precondition for BOOLEAN feature `is_fit_for_battle` in class KNIGHT? No precondition is needed here.

**Design by Contract: postconditions**

• In class KNIGHT you have feature `set_reputation (rep: INTEGER)`. What postcondition would you write for it? `reputation = rep`

• In class KNIGHT you have feature `attack_monster (mon: MONSTER; wep: WEAPON)`. What postcondition would you write for it? `old_mon.hit_points >= mon.hit_points and not wep.is_ready`

• In class MONSTER you have feature `scan_direction (dir: DIRECTION)`. What postcondition would you write for it? `is_knight_found or is_scanning_complete`.

• In class WEAPON you have feature `set_ready (wep_ready: BOOLEAN)`. What postcondition would you write for it? `is_ready = wep_ready`.

• Suppose that in class KNIGHT, feature `attack`, you want to add to the existing postcondition `old_mon.hit_points >= mon.hit_points and not wep.is_ready` the new clause: `reputation = old reputation + 1 or reputation = 5`. The true sentence is: The compound postcondition: `old_mon.hit_points >= mon.hit_points and not wep.is_ready and (reputation = old reputation + 1 or reputation = 5)` is a stronger postcondition than the pre-existing postcondition.

• Suppose you know that a knight can only fight in battle if his or her hit points are greater than 10. Which is a reasonable postcondition for BOOLEAN feature `is_fit_for_battle` in class KNIGHT? **Result** = `(hit_points > 10)`

**Design by Contract: class invariants**

• Given what you know about class KNIGHT, what invariant would you write? `reputation >= -5 and reputation <= 5 and hit_points >= 0`

• Given what you know about class MONSTER, what invariant would you write? `hit_points >= 0`

• Given what you know about class WEAPON, what invariant would you write? `is_magic implies is_ready and damage >= 1`.

• Given what you know about class DIRECTION, what invariant would you write? `internal_direction = 1 or internal_direction = 2 or internal_direction = 3 or internal_direction = 4`. 
Design by Contract: contracts and inheritance

- Given what you know about class KNIGHT_MAGE, which precondition clause would you write for feature `attack_monster` (mon: MONSTER; wep: WEAPON)? `require` else
  `mana > 0`

- Given what you know about class KNIGHT_MAGE, which postcondition clause would you write for feature `attack_monster` (mon: MONSTER; wep: WEAPON)? `ensure` then
  `mana <= old mana`

- Given what you know about class KNIGHT_MAGE, which class invariant would you write for it? `mana >= 0`.

- Given what you know about class GOBLIN, which precondition would you write for feature `attack_with_weapon` (kni: KNIGHT; wep: WEAPON)? `require` last_knight_found =
  `kni and is_angry and wep.is_ready`.

- Given what you know about class GOBLIN, which postcondition would you write for feature `attack_with_weapon` (kni: KNIGHT; wep: WEAPON)? `ensure` is_angry.

- Given what you know about class GOBLIN, which class invariant would you write for it? No invariant clause is needed.

Design by Contract: putting it all together

- Assume a class FILTER receiving input data from a class INPUT_HANDLER that in turn is used to validate user input. The following statements are true: To check for user input correctness, you should not be using preconditions in class INPUT_HANDLER, but use if statements instead; To check for user input correctness, you should be using preconditions in class FILTER instead of if statements.

- Assume that the correct precondition for a feature \( f \) (s: STRING) is: `pre: s /= Void and then s = "test"` Consider now the following precondition: `pre2: s /= Void and then not s.is_empty` The following statements are true: `pre2` is an over-approximation of `pre`; `pre2` is complete and unsound.

- Assume that the correct precondition for a feature \( f \) (s: STRING) is: `pre: s /= Void and then not s.is_empty` Consider now the following precondition: `pre2: s /= Void and then s = "test"` The following statements are true: `pre2` is an under-approximation of `pre`; `pre2` is incomplete and sound.

- Assume that the correct postcondition for a feature \( f \) is: `post: s /= Void and then not s.is_empty` Where `s: STRING` is an attribute. Consider now the following postcondition: `post2: s /= Void and then s = "test"`. The following statements are true: `post2` is an under-approximation of `pre`; `post2` is too strong; `post2` is sound but incomplete.

- Assume that the correct postcondition for a feature \( f \) is: `post: s /= Void and then s = "test"` Where `s: STRING` is an attribute. Consider now the following postcondition: `post2: s /= Void and then not s.is_empty` The following statements are true: `post2` is an over-approximation of `post`; `post2` is complete and unsound; `post2` is too weak.

Recursion

- The correct way to complete the code of the routine `countdown` is the following:
countdown (n: INTEGER)
    -- Count down from n to 0.
    do
        if n >= 0 then
            print (n.out)
            countdown (n−1)
        else
            -- nothing here
        end
    end

• The following routine, when called with n having value 4, keeps printing consecutive numbers starting from 4, and goes into an infinite loop:

countdown (n: INTEGER)
    do
        if n > 0 then
            print (n.out)
            countdown (n+1)
        else
            print ("Done")
        end
    end

• The following routine, when called with n having value 4, prints “4321Done”:

countdown (n: INTEGER)
    do
        if n > 0 then
            print (n.out)
            countdown (n−1)
        else
            print ("Done")
        end
    end

• If a routine r calls another routine s, which calls another routine t, which finally calls routine s, then routine s is recursive (indirect recursion) and routine t is recursive (indirect recursion).

Programming exercise: recursive algorithm for gcd

Listing 3: Class RECURSIVE_GCD

class RECURSIVE_GCD

note
description: "Encapsulates a recursive algorithm for computing the gcd of two positive integers."
author: "mp"
date: "$Date$"
revision: "$Revision$"
feature -- Basic operations

\[
gcd \ (a, \ b: \text{INTEGER}): \text{INTEGER}
\]

-- Greater common divisor between a and b.

require
\[
a_{\text{positive}}: \ a > 0
\]
\[
b_{\text{positive}}: \ b > 0
\]
do

-- This solution is from Dijkstra.
-- It is based on the observation that if \(a > b\),
-- then \(\text{gcd} \ (a, b) = \text{gcd} \ (a-b, b)\)

if \(a = b\) then

\[
\text{Result} := a
\]
else if \(a > b\) then

\[
\text{Result} := \text{gcd} \ (a-b, b)
\]
else

\[
\text{Result} := \text{gcd} \ (a, b-a)
\]
end

ensure
\[
\text{result}_{\text{positive}}: \ \text{Result} > 0
\]
end