Abstract Data Types

Mathematical description

- An ADT is a mathematical specification
- Describes the properties and the behavior of instances of this type
- Doesn't describe implementation details (therefore its *abstract*)
- An example: STACK

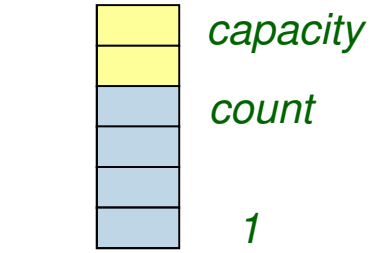
Stack

- LIFO Queue (last in, first-out)
- Operations:
  - put: Put something onto the STACK (Command)
  - remove: Remove the top element of the STACK (Command)
  - item: Return the value of the top item (Query)
  - empty: Is the STACK empty? (Query)

Abstract data types

One data type – many implementations. E.g. for STACK:

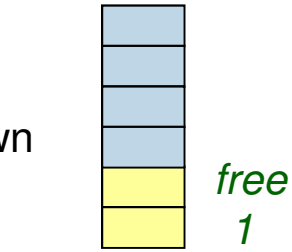
Variant 1: array up



"Push" *x* on stack *representation*:

representation [*count*] := *x*
count := *count* + 1

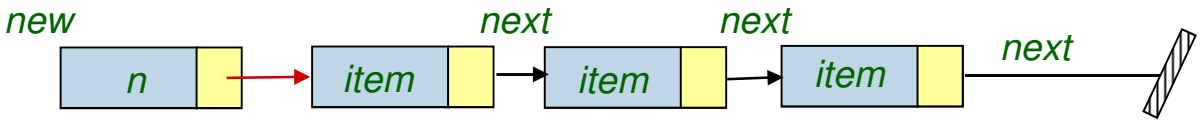
Variant 2: array down



"Push" *x* on stack *representation*:

representation [*free*] := *x*
free := *free* - 1

Variant 3: linked list



"Push" operation:

new (*n*)
n.item := *x*
n.next := *first*
first := *new*

Formal description of a STACK

Types:

The type(s) that are described by the ADT.

Functions:

The functions that can be applied to the ADT.

Preconditions:

Preconditions that need to be fulfilled to apply a feature.

Axioms:

Axioms that the ADT fulfills.

Abstract data types

Creators:

Create a new instance of an ADT.

{OTHERS} ⟶ ADT

Queries:

Functions that have a return value and do not change the instance.

ADT {x OTHERS} ⟶ OTHERS

Commands:

Functions without return value that change the instance.

ADT {x OTHERS} ⟶ ADT

Partial function ↛

There are cases where no valid return value can be given for a function
e.g.. division by 0

Abstract data types

Types

STACK [G]

-- G: Formal generic parameter

Functions

put: STACK [G] $\times$ G $\longrightarrow$ STACK [G]

remove: STACK [G] $\nrightarrow$ STACK [G]

item: STACK [G] $\nrightarrow$ G

empty: STACK [G] $\longrightarrow$ BOOLEAN

new: STACK [G]

Abstract data types

Preconditions

remove (s: STACK [G]) **require** not empty (s)

item (s: STACK [G]) **require** not empty (s)

Axioms For all x: G, s: STACK [G]

1. item (put (s, x)) = x
2. remove (put (s, x)) = s
3. empty (new)
4. not empty (put (s, x))

Well-formed and correct terms

Well-formed: all functions get a right number of arguments of right types

Correct: preconditions of all functions are satisfied

empty (item (put (new, 3))) ill-formed

item (put (new, 3)) 3

item (remove (put (new, 3))) incorrect

empty (remove (put (new, 7))) True

item (put (put (remove (put (new, 4)), 3), 2)) 2

Structural induction

Goal: Prove that a property P is valid for all correct terms T of the ADT.

Induction basis (step 0):

Prove that P holds for all creators of the ADT.

Induction hypothesis (step n-1):

Assume that P holds for any correct term T_{sub}.

Induction step (step n):

Prove for all commands that can be applied correctly to T_{sub} that P will still hold afterwards.

Sufficient completeness

An ADT is sufficiently complete if and only if:

1. For every term you can determine whether it is **correct** or not using the axioms of the ADT.
2. Every correct term where **the outermost function is a query** of the ADT can be reduced, using the axioms of the ADT, into a **term not using any function of the ADT**.

Your turn: Design an ADT (types, functions, preconditions, axioms)

We have the following requirements for a BANK_ACCOUNT class:

1. Every BANK_ACCOUNT has an owner and a balance.
2. The balance is recorded in “Rappen” (as an INTEGER).
3. The owner is recorded with his/her name (as a STRING).
4. It should always be possible to retrieve the balance and owner for any given BANK_ACCOUNT.
5. It is possible to deposit money to and withdraw money from the BANK_ACCOUNT.
6. The balance on the BANK_ACCOUNT is adjusted accordingly.
7. The balance of any BANK_ACCOUNT should never become negative.

Supplementary reading

Object-oriented Software Construction, Second Edition, by Bertrand Meyer, pp. 148-159