Software Verification

Lecture 5: Assertion Inference

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Proving Programs Automatically

The Program Verification problem:

- **Given**: a program $P$ and a specification $S = [\text{Pre}, \text{Post}]$
- **Determine**: if every execution of $P$, for any value of input parameters, satisfies $S$
- **Equivalently**: establish whether $\{\text{Pre}\} P \{\text{Post}\}$ is (totally) correct

- A general and fully automated solution to the Program Verification problem is unachievable because the problem is undecidable

- One of the consequences of this inescapable limitation is the impossibility of computing intermediate assertions fully automatically

  (It is not an obvious consequence: formally, a reduction between undecidable problems)
The Program Verification problem:

- **Given**: a program $P$ and a specification $S = [\text{Pre}, \text{Post}]$
- **Determine**: if every execution of $P$, for any value of input parameters, satisfies $S$
- **Equivalently**: establish whether $\{\text{Pre}\} P \{\text{Post}\}$ is (totally) correct

**Practically**:

Proving the correctness of a computer program requires knowledge about the program that is not readily available in the program text

-- Chang & Leino

In this lecture, we survey techniques to automatically infer assertions in interesting special cases
The Assertion Inference Paradox

Correctness is consistency of implementation to specification

The paradox:

if the specification is inferred from the implementation, what do we prove?
The Assertion Inference Paradox

The paradox:
if the specification is inferred from the implementation, what do we prove?

Possible retorts:

- The paradox only arises for correctness proofs; there are other applications (e.g. reverse-engineering legacy software)
- The result may be presented to a programmer for assessment
- Inferred specification may be inconsistent, thus denoting a problem
The Assertion Inference Paradox

The paradox:
if the specification is inferred from the implementation, what do we prove?

The paradox does not arise if we only infer intermediate assertions and not specifications (pre and postconditions)

- Intermediate assertions are a technical means to an end (proving correctness)
  - tools infer loop invariants
- The specification is a formal description of what the implementation should do
  - programmers write specifications
Invariants

Let us consider a general (and somewhat informal) definition of invariant:

**Def. Invariant:** assertion whose truth is preserved by the execution of (parts of) a program.

\[
\begin{align*}
x &: \text{INTEGER} \\
\text{from } x &= 1 \text{ until } \ldots \text{ loop } x := -x \text{ end}
\end{align*}
\]

Some invariants:
- \(-1 \leq x \leq 1\)
- \(x = -1 \lor x = 0 \lor x = 1\)
- \(x \geq -10\)
Kinds of Invariants

**Def. Invariant:** assertion whose truth is preserved by the execution of (parts of) a program.

We can identify different *types of invariants*, according to what parts of the program preserve the invariant:

- **Location invariant at** $x$: assertion that holds whenever the computation reaches location $x$
- **Program invariant**: predicate that holds in any reachable state of the computation
- **Class invariant**: predicate that holds between (external) feature invocations
- **Loop invariant**: predicate that holds after every iteration of a loop body

{\{P\}} A \{I\}
{\{I \land \neg c\}} B \{I\}
\underline{\{P\}}
from A until c
loop B end
\{I \land c\}
Kinds of Invariants

1: \textit{x: INTEGER}
2: \textit{from x := 1 until ... loop x := - x end}

- Location invariant at 2:
- Loop invariant:
- Program invariant:
Kinds of Invariants

1: \( x: \text{INTEGER} \)
2: 
3: from \( x := 1 \)
4: until ...
5: loop \( x := -x \) end

- Location invariant at 2:
  \( x = 0 \)

- Loop invariant:
  \( x = -1 \lor x = 1 \)

- Program invariant:
  \( x \geq -10 \)
Focus on Loop Invariants

If we have loop invariants we can get (basically) everything else at little cost

- but not vice versa:
  getting loop invariants requires invention

In the following discussion we focus on loop invariants (and call them simply “invariants”)

This focus is also consistent with the Assertion Inference Paradox
Focus on Loop Invariants

The various kinds of invariants are closely related by the inference rules of Hoare logic

- If $L_x$ is a location invariant at $x$ then:
  \[ @x \Rightarrow L_x \]
  is a program invariant

- If $P$ is a program invariant then it is also a location invariant at every location $x$

- If $I$ is a loop invariant of:
  \[ x: \text{from} \ldots \text{until} \ c \ \text{loop} \ldots \text{end} \]
  then $I \land c$ is a location invariant at $x+1$

- If $L$ is a location invariant at $x+1$:
  \[ x: \quad a := b + 3 \]
  then $L [b + 3 / a]$ is a location invariant at $x$

- Etc...
Techniques for Invariant Inference

Classification of invariant inference techniques:

- **Dynamic** techniques
- **Static** techniques
  - statistical techniques
  - exact techniques

Classifying is neither sharp nor complete, yet useful
Exact Static Techniques for Invariant Inference
Static Invariant Inference: classification

Static exact techniques for invariant inference are further classified in categories:

- Direct
- Assertion-based
  - postcondition mutation
- Based on abstract interpretation
  - forward analysis (bottom-up)
  - backward analysis (top-down)
- Constraint-based
  - usually, template-based
Exact Static Techniques for Invariant Inference:

Postcondition-mutation Approach
The Role of User-provided Contracts

Techniques for invariant inference rarely take advantage of other annotations in the program text, such as contracts provided by the user.

- Not every annotation can (or should, cf. Assertion Inference Paradox) be inferred automatically!

However, there is a close connection between a loop's invariant and its postcondition.
The Role of User-provided Contracts

• However, there is a close connection between a loop's invariant and its postcondition

  The invariant is a weakened form of the postcondition
  - It is a larger collection of program states

• Example: \( \text{from } x := 0 \text{ until } x = n \text{ loop } x := x + 1 \text{ end} \)
  - Post: \( x = n \) (for some \( n > 0 \))
  - Invariant: \( 0 \leq x \leq n \)
Invariants by Postcondition Mutation

- In a nutshell:

  Static verification of candidate invariants obtained by mutating postconditions

  - Assume the availability of postconditions
  - Mutate postconditions according to various heuristics
    - the heuristics mirror common patterns that link postconditions to invariants
    - each mutated postcondition is a candidate invariant
  - Verify which candidates are indeed invariants
    - With an automatic program prover such as Boogie
  - Retain all verified invariants

- 2009 - CAF & BM

- Implementation: gin-pink which finds invariants in Boogie programs
Loop invariant inference

Input

Output

Pre | Post | Program

Candidate Invariants

Loop

Loop Invariants

{Pre} Program {Post}

mutate

checking invariance

proving correctness
(possibly using additional info)
Postcondition Mutation Heuristics

Constant relaxation

- replace “constant” by “variable”
  - cannot/may be changed by any of the loop bodies

Uncoupling

- replace subexpression appearing twice by two subexpressions
  - for example: subexpression = variable id

Term dropping

- remove a conjunct

Variable aging

- replace subexpression by another expression representing its previous value
Invariant Inference: the Algorithm

- **Goal**: find invariants of loops in procedure `proc`
- For each:
  - `post`: postcondition clause of `proc`
  - `loop`: outer loop in `proc`

compute **all mutations** $M$ of `post` w.r.t. `loop`
  - considering postcondition clauses separately implements term dropping

- **Result**: any formula in $M$ which can be **verified** as invariant of any loop in `proc`
Maximum value of an array

\[
\text{max}(A: \text{ARRAY}[T] ; n: \text{INTEGER}): T
\]

\text{require } A.\text{length} = n \geq 1

\text{local } i: \text{INTEGER}

\text{do}

\text{from } i := 0 ; \text{Result} := A[1];

\text{until } i = n

\text{loop}

\text{i} := i + 1

\text{if } \text{Result} \leq A[i] \text{ then } \text{Result} := A[i] \text{ end}

\text{end}

\text{ensure} \quad (\forall 1 \leq j \leq n \Rightarrow A[j] \leq \text{Result}) \quad \text{and}

(\exists 1 \leq j \leq n \land A[j] = \text{Result})
Maximum value of an array

\[ \text{max}(A: \text{ARRAY}[T]; n: \text{INTEGER}): T \]

\begin{align*}
\text{require } & A.\text{length} = n \geq 1 \\
\text{ensure } & ( \forall 1 \leq j \leq n \Rightarrow A[j] \leq \text{Result} ) \text{ and } \\
& ( \exists 1 \leq j \leq n \wedge A[j] = \text{Result} )
\end{align*}

- **Constant relaxation**: replace “constant” \( n \) by “variable” \( i \)
- **Term dropping**: remove second conjunct

\begin{align*}
\text{Invariant: } & \forall 1 \leq j \leq i \Rightarrow A[j] \leq \text{Result}
\end{align*}
Maximum value of an array (cont'd)

max (A: ARRAY [T]; n: INTEGER): T

require A.length = n ≥ 1

ensure ( ∀ 1 ≤ j ≤ n ⇒ A[j] ≤ Result ) and
( ∃ 1 ≤ j ≤ n ∧ A[j] = Result )

- Term dropping: remove first conjunct

Invariant: ∃ 1 ≤ j ≤ n ∧ A[j] = Result
Maximum value of an array (2\textsuperscript{nd} version)

\[
\text{max}_v^2(A: \text{ARRAY} [T]; n: \text{INTEGER}): T
\]

\text{require } A.length = n \geq 1

\text{local } i: \text{INTEGER}

\text{do}
\hspace{1em} \text{from } i := 1 \text{ ; Result := } A[1];
\hspace{1em} \text{until } i > n
\hspace{1em} \text{loop}
\hspace{2em} \text{if } \text{Result} \leq A[i] \text{ then Result := } A[i] \text{ end}
\hspace{2em} i := i + 1
\hspace{1em} \text{end}

\text{ensure } \forall 1 \leq j \leq n \Rightarrow A[j] \leq \text{Result}
Maximum value of an array (2\textsuperscript{nd} version)

\[ \text{max}_v^2 (A: \text{ARRAY}[T]; n: \text{INTEGER}): T \]
require \( A\.\text{length} = n \geq 1 \)
ensure \( \forall 1 \leq j \leq n \Rightarrow A[j] \leq \text{Result} \)

- **Constant relaxation:** replace “constant” \( n \) by “variable” \( i \)
  \( \forall 1 \leq j \leq i \Rightarrow A[j] \leq \text{Result} \)

- **Variable aging:**
  use expression representing the previous value of \( i \): \( i - 1 \)

Invariant: \( \forall 1 \leq j \leq i - 1 \Rightarrow A[j] \leq \text{Result} \)
Array Partitioning

\[\text{partition}(A: \text{ARRAY}[T]; n: \text{INTEGER}; \text{pivot}: T): \text{INTEGER}\]

require \(A\).length = n \geq 1

local \(l, h\): \text{INTEGER}

do
  from \(l := 1\); \(h := n\) until \(l = h\)
  loop
    from until \(l = h\) or \(A[l] > \text{pivot}\) loop \(l := l + 1\) end
    from until \(l = h\) or \(\text{pivot} > A[h]\) loop \(h := h - 1\) end
    \(A\).swap \((l, h)\)
  end
  if \(\text{pivot} \leq A[l]\) then \(l := l - 1\) end; \(h := l\); \(\text{Result} := h\)
ensure (\(\forall 1 \leq k \leq \text{Result} \Rightarrow A[k] \leq \text{pivot}\)) and
(\(\forall \text{Result} < k \leq n \Rightarrow A[k] \geq \text{pivot}\))
Array Partitioning

```
partition (A: ARRAY [T]; n: INTEGER; pivot: T): INTEGER
  require A.length = n ≥ 1
  ensure (∀ 1≤k≤ Result ⇒ A[k] ≤ pivot) and
            (∀ Result<k≤n ⇒ A[k] ≥ pivot)
```

- **Uncoupling**: replace first occurrence of \textit{Result} by \( l \)
  and second by \( h \)
  \( (∀ 1 ≤ k ≤ l ⇒ A[k] ≤ \text{pivot}) \) and \( (∀ h < k ≤ n ⇒ A[k] ≥ \text{pivot}) \)

- **Variable aging**: use expression representing the previous value of \( l \): \( l - 1 \)

\textbf{Invariant:}
\( (∀ 1 ≤ k ≤ l - 1 ⇒ A[k] ≤ \text{pivot}) \) and \( (∀ h < k ≤ n ⇒ A[k] ≥ \text{pivot}) \)
Array Partitioning

\[
\text{partition}(A: \text{ARRAY}[T]; \; n: \text{INTEGER}; \; \text{pivot}: T): \text{INTEGER}
\]

require \( A\.\text{length} = n \geq 1 \)

ensure (\( \forall 1 \leq k \leq \text{Result} \Rightarrow A[k] \leq \text{pivot} \)) and

(\( \forall \text{Result} < k \leq n \Rightarrow A[k] \geq \text{pivot} \))

- Term dropping: remove first conjunct
  \( \forall \text{Result} < k \leq n \Rightarrow A[k] \geq \text{pivot} \)

- Constant relaxation: replace “constant” Result by “variable” h

Invariant: \( \forall h < k \leq n \Rightarrow A[k] \geq \text{pivot} \)
Implementation: gin-pink

gin-pink: Generation of INvariants by Postcondition Weakening

- written in Eiffel
- command-line tool
  - Boogie in / Boogie out
- works with any high-level language that can be translated to Boogie
- available for download from http://se.inf.ethz.ch/people/furia/software/gin-pink/
Limitations of the approach

Some invariants are not mutations of the postcondition
- “completeness” of the postcondition
- integration with other techniques
- more heuristics

Combinatorial explosion
- user guidance

Dependencies
- especially with nested loops
- user guidance

Limitations of automated reasoning techniques
- they progress quickly
Exact Static Techniques for Invariant Inference:

Constraint-based Approach
In a nutshell:
encode semantics of iteration as constraints on a template invariant

Choose a template invariant expression
  template defines a (infinite) set of assertions

Encode the loop semantics as a set of constraints on the template
  initiation
  consecution

Solve the constraints
  this is usually the complex part

Any solution is an invariant

E.g.:
  2003 -- Henny Sipma et al.; 2004 -- Zohar Manna et al.;
  2007 -- Tom Henzinger et al.
Constraint-based Inv. Inference: Example

- **Template invariant expression:**
  \[ T = c \cdot x + d \cdot n + e \leq 0 \]

- **Constraints encoding loop semantics:**
  - **Initiation:** “T holds for the initial values of x and n”
    \[ T[0/x; n_0/n] \equiv c \cdot 0 + d \cdot n_0 + e \leq 0 \equiv d \cdot n_0 + e \leq 0 \]
**Constraint-based Inv. Inference: Example**

```
dummy_routine (n: NATURAL)
local x: NATURAL do
    from x := 0
    until x ≥ n
    loop x := x + 1 end
end
```

- **Constraints encoding loop semantics:**
  - **Consecution:** “if $T$ holds and one iteration of the loop is executed, $T$ still holds”
    
    $$T [x/x; n/n] \land (\neg(x \geq n) \land x' = x + 1 \land n' = n) \Rightarrow T [x'/x; n'/n]$$

- **Solving** the constraints requires to eliminate occurrences of $x$, $x'$, $n$, $n'$
  - For linear constraints we can use Farkas's Lemma
Farkas's Lemma (1902)

Let $S$ be a system of linear inequalities over $n$ real variables:

$$
S \triangleq \begin{bmatrix}
  a_{11}x_1 + \cdots + a_{1n}x_n + b_1 & \leq & 0 \\
  \vdots & & \vdots \\
  a_{m1}x_1 + \cdots + a_{mn}x_n + b_m & \leq & 0
\end{bmatrix}
$$

and let $\Psi$ be a linear inequality:

$$
\Psi \triangleq c_1x_1 + \cdots + c_nx_n + d \leq 0
$$

Then $S \Rightarrow \Psi$ is valid iff $S$ is unsatisfiable or there exist $m + 1$ real nonnegative coefficients $\lambda_0, \lambda_1, \ldots, \lambda_m$ such that:

$$
c_j = \sum_{i=1}^{m} \lambda_i a_{ij} \quad (1 \leq j \leq m) \quad d = -\lambda_0 + \sum_{i=1}^{m} \lambda_i b_i
$$
Constraint-based Inv. Inference: Example

Use Farkas's lemma to turn the consecution constraint:

\[ T \left[x/x; n/n\right] \land x < n \land x' = x + 1 \land n' = n \]
\[ \Rightarrow T \left[x'/x; n'/n\right] \]

into a constraint over c, d, and e only.

\[
\begin{array}{c|ccc|c}
\lambda_1 & cx & +dn & +e & \leq 0 \\
\lambda_2 & x & -n & +1 & \leq 0 \\
\lambda_3 & -x & +x' & -1 & \leq 0 \\
\lambda_4 & x & -x' & +1 & \leq 0 \\
\lambda_5 & -n & +n' & \leq 0 \\
\lambda_6 & n & -n' & \leq 0 \\
\hline
\end{array}
\]

\[ cx' + dn' + e \leq 0 \]
Constraint-based Inv. Inference: Example

\[
\begin{align*}
\lambda_1 & : cx + dn + e \leq 0 \\
\lambda_2 & : x - n + 1 \leq 0 \\
\lambda_3 & : -x + x' - 1 \leq 0 \\
\lambda_4 & : x - x' + 1 \leq 0 \\
\lambda_5 & : -n + n' \leq 0 \\
\lambda_6 & : n - n' \leq 0 \\
\end{align*}
\]

\[
\Phi \triangleq \exists \lambda_0, \ldots, \lambda_6 \begin{bmatrix}
\lambda_0, \ldots, \lambda_6 \geq 0 \\
\lambda_1 c + \lambda_2 - \lambda_3 + \lambda_4 = 0 \\
\lambda_1 d - \lambda_2 - \lambda_5 + \lambda_6 = 0 \\
\lambda_3 - \lambda_4 = c \\
\lambda_5 - \lambda_6 = d \\
-\lambda_0 + \lambda_1 e + \lambda_2 - \lambda_3 + \lambda_4 = e
\end{bmatrix}
\]
Finally, eliminate existential quantifiers from $\Phi$ to get the constraint:

$$c \leq 0 \lor (c + d = 0 \land e \leq 0)$$

(Quantifier elimination is also quite technical)
**Constraint-based Inv. Inference: Example**

- **Any** solution \([c, d, e]\) to:
  - **Initiation and Consecution:**
    
    \[
    (d \cdot n_0 + e \leq 0) \land (c \leq 0 \lor (c + d = 0 \land e \leq 0))
    \]

    determines an invariant of the loop.

- \([0, -1, 0]\)  \; \rightarrow \; n \geq 0
- \([1, 0, 0]\)  \; \rightarrow \; x \geq 0
- \([1, -1, 0]\)  \; \rightarrow \; x - n \leq 0

```latex
\begin{verbatim}
\texttt{dummy\_routine}(n: \text{NATURAL})
local x: \text{NATURAL} do
  from x := 0
  until x \geq n
  loop x := x + 1 end
end
\end{verbatim}
```
Constraint-based Inv. Inference: Summary

- **Main issues:**
  - choice of invariant templates for which effective decision procedures exist
    - interesting research topic per se, on the brink of undecidability
  - heuristics to extract the "best" invariants from the set of solutions

- **Advantages:**
  - sound & complete (w.r.t. the template)
  - exploit heterogeneous decision procedures together
  - fully automated (possibly except for providing the template)
    - providing the template introduces a "natural" form of user interaction

- **Disadvantages:**
  - suitable mathematical decision theories are usually quite sophisticated
    - hence, hard to extend and customize
  - exact constraint solving is usually quite expensive
  - mostly suitable for "algebraic" invariants
    - requires integration with other techniques to achieve full functional correctness proofs
Dynamic Techniques for Invariant Inference
Dynamic Invariant Inference

In a nutshell:

*testing of candidate invariants*

- Choose a set of *test cases*
- Perform *runtime monitoring* of candidate invariants
- If some test run *violates* a candidate, discard the candidate
- The *surviving* candidates are *guessed invariant*

- Daikon tool, 1999 -- Mike Ernst et al.
- CITADEL: Daikon for Eiffel, 2008 -- Nadia Polikarpova
- AutoInfer for Eiffel (Y. Wei et al.)
Dynamic Invariant Inference: Example

\textbf{dummy\_routine}(n: \text{NATURAL})

\begin{verbatim}
local x: \text{NATURAL} do
    from x := 0
    until x ≥ n
    loop x := x + 1 end
end
\end{verbatim}

- **Test cases:** \{ \( n = k \mid 0 \leq k \leq 1000 \) \}
- **Candidate invariants:**
  - \{ \( x ≥ c \mid -1000 ≤ c ≤ 1000 \) \},
  - \{ \( n ≥ c \mid -1000 ≤ c ≤ 1000 \) \}
  - \{ \( x = c \cdot n + d \mid -500 ≤ c, d ≤ 500 \) \}
  - \{ \( x < n, x ≤ n, x = n, x ≠ n, x ≥ n, x > n \) \}
  - \{ \( x \ n ≥ c \mid -500 ≤ c ≤ 500 \) \}
  - \( \ldots \)
Dynamic Invariant Inference: Example

**dummyRoutine**(n: NATURAL)
local x: NATURAL do
  from x := 0
  until x ≥ n
  loop x := x + 1 end
end

- **Survivors** *(after loop iterations)*:
  - \{ x ≥ -c  |  0 ≤ c ≤ 1000 \}, \{ n ≥ -c  |  0 ≤ c ≤ 1000 \}
  - x ≤ n
  - \{ x + n ≥ c  |  -500 ≤ c ≤ 500 \}
  - ...
Dynamic Invariant Inference: Summary

- **Main issues:**
  - choose suitable test cases
  - handle huge sets of candidate invariants (runtime overhead)
  - estimate soundness/quality of survivor predicates
  - select heuristically the “best” survivor predicates

- **Advantages:**
  - straightforward to implement (at least compared to other techniques)
  - guessing is often rather accurate in practice (possibly with some heuristics)
  - customizable and rather flexible:
    in principle, whatever you can test you can check for invariance

- **Disadvantages:**
  - unsound (educated guessing)
  - without heuristics, large amount of useless, redundant predicates
  - sensitive to choice of test cases
  - some complex candidate invariants are difficult to implement efficiently
Exact Static Techniques for Invariant Inference:

Direct Approach
In a nutshell:

- solve the fixpoint equations underlying the program

- $v(i)$: value of variable $v$ at step $i$ of the computation

- Encode the semantics of loops explicitly and directly as recurrence equations over $v(i)$

- Solve recurrence equations

- Eliminate step parameter $i$ to obtain invariant

- 1973 -- Shmuel Katz & Zohar Manna

- 2005 -- Laura Kovacs et al.
Direct Static Invariant Inference: Example

```
dummy_routine(n: NATURAL)
local x: NATURAL do
  from x := 0
  until x ≥ n
  loop x := x + 1 end
end
```

- $x(i)$, $n(i)$

- Recurrence relations:

$$x(i) = \begin{cases} 
0 & i = 0 \\
(x(i - 1) + 1) & 0 < i ≤ n_0 \\
x(i - 1) & i > n_0
\end{cases}$$

$$n(i) = \begin{cases} 
n_0 ≥ 0 & i = 0 \\
n(i - 1) & i > 0
\end{cases}$$
Direct Static Invariant Inference: Example

$$x(i) = \begin{cases} 
0 & i = 0 \\
x(i-1) + 1 & 0 < i \leq n_0 \\
x(i-1) & i > n_0 
\end{cases}$$

$$n(i) = \begin{cases} 
n_0 \geq 0 & i = 0 \\
n(i-1) & i > 0 
\end{cases}$$

- Solving recurrence relations:
  - $$x(i) = \min(n_0, i) \geq 0$$
  - $$n(i) = n_0$$
- Eliminating step parameter $$i$$:
  - $$x(i) - n(i) = \min(n_0, i) - n_0 \leq 0$$, hence:
  - $$x - n \leq 0$$, hence:
  - $$0 \leq x \leq n$$
Direct Static Invariant Inference: Summary

- **Main issues:**
  - in its bare form, more a set of guidelines than a technique
  - step parameter elimination is tricky

- **Advantages:**
  - since semantics is represented explicitly, obtained invariants are often powerful
  - benefits from the programmer's ingenuity
  - additional information about the program can be “plugged in”

- **Disadvantages:**
  - solving recurrence equations can be very difficult (when possible at all)
  - typically restricted to “algebraic” invariants