Problem Sheet 5: Data Flow Analysis  
Sample Solutions

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Starred exercises (*) are more challenging than the others.

1 Reaching Definitions Analysis

i-ii. The control flow graph and the results of the reaching definitions analysis are given in the diagram below:

iii. We give the use-definition information for x and y in the table below (you could also annotate the diagram above with additional arrows).

∗These sample solutions are adapted from previous iterations of the course when Stephan van Staden was the teaching assistant.
2 Live Variables Analysis

i. Below we identify the blocks of the program:

\[
\begin{align*}
\text{[} & x := y \text{]}^1 \\
\text{[} & x := x - 1 \text{]}^2 \\
\text{[} & x := 4 \text{]}^3 \\
\text{while}[ & y < x]^4 \text{ do} \\
\text{[} & y := y + x \text{]}^5 \\
\text{end} \\
\text{[} & y := 0 \text{]}^6
\end{align*}
\]

ii. The system of equations for a live variable analysis are as follows:

\[
\begin{align*}
LV_{entry}(1) &= (LV_{exit}(1) - \{x\}) \cup \{y\} \\
LV_{entry}(2) &= (LV_{exit}(2) - \{x\}) \cup \{x\} \\
LV_{entry}(3) &= LV_{exit}(3) - \{x\} \\
LV_{entry}(4) &= LV_{exit}(4) \cup \{x, y\} \\
LV_{entry}(5) &= (LV_{exit}(5) - \{y\}) \cup \{x, y\} \\
LV_{entry}(6) &= LV_{exit}(6) - \{y\} \\
LV_{exit}(1) &= LV_{entry}(2) \\
LV_{exit}(2) &= LV_{entry}(3) \\
LV_{exit}(3) &= LV_{entry}(4) \\
LV_{exit}(4) &= LV_{entry}(5) \cup LV_{entry}(6) \\
LV_{exit}(5) &= LV_{entry}(4) \\
LV_{exit}(6) &= \emptyset
\end{align*}
\]

iii. We begin the iteration by initialising every set to \(\emptyset\). Then, we iteratively update the sets by applying the equation system above. (For simplicity, the columns omit sets when a particular iteration does not update the previous value.)
We eliminate blocks $b$ of the form $[x := \ldots]^b$ if $x$ is not an element of $\text{LV}_{\text{exit}}(b)$:

$$
[x := y]^1 \\
[x := 4]^3 \\
\text{while } [y < x]^4 \text{ do} \\
| y := y+x|^5 \\
\text{end}
$$

The program is not yet free of dead variables: $x$ in block 1 is still dead. We strengthen the definition of $\text{LV}_{\text{entry}}$:

$$
\text{LV}_{\text{entry}}(b) = \begin{cases} 
(\text{LV}_{\text{exit}}(b) - \text{kill}_{\text{LV}}(b)) \cup \text{gen}_{\text{LV}}(b) & \text{if } \text{kill}_{\text{LV}}(b) \subseteq \text{LV}_{\text{exit}}(b) \\
\text{LV}_{\text{exit}}(b) & \text{otherwise}
\end{cases}
$$

The rationale is this: if a block assigns to a variable that is not live afterwards, then it must be eliminated, and should not influence the analysis by adding the variables it reads to the live variable set.

Performing a chaotic iteration with this new equation yields the following results:

<table>
<thead>
<tr>
<th>LV Sets</th>
<th>Iterations $\rightarrow$</th>
<th>Final Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>LV$_{\text{entry}}$(1)</td>
<td>$\emptyset$</td>
<td>${y}$</td>
</tr>
<tr>
<td>LV$_{\text{entry}}$(2)</td>
<td>$\emptyset$</td>
<td>${x}$</td>
</tr>
<tr>
<td>LV$_{\text{entry}}$(3)</td>
<td>$\emptyset$</td>
<td>${y}$</td>
</tr>
<tr>
<td>LV$_{\text{entry}}$(4)</td>
<td>$\emptyset$</td>
<td>${x, y}$</td>
</tr>
<tr>
<td>LV$_{\text{entry}}$(5)</td>
<td>$\emptyset$</td>
<td>${x, y}$</td>
</tr>
<tr>
<td>LV$_{\text{entry}}$(6)</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>LV$_{\text{exit}}$(1)</td>
<td>$\emptyset$</td>
<td>${x, y}$</td>
</tr>
<tr>
<td>LV$_{\text{exit}}$(2)</td>
<td>$\emptyset$</td>
<td>${y}$</td>
</tr>
<tr>
<td>LV$_{\text{exit}}$(3)</td>
<td>$\emptyset$</td>
<td>${x, y}$</td>
</tr>
<tr>
<td>LV$_{\text{exit}}$(4)</td>
<td>$\emptyset$</td>
<td>${x, y}$</td>
</tr>
<tr>
<td>LV$_{\text{exit}}$(5)</td>
<td>$\emptyset$</td>
<td>${x, y}$</td>
</tr>
<tr>
<td>LV$_{\text{exit}}$(6)</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

with which we can eliminate all of the dead code in the program:

$$
[x := 4]^3 \\
\text{while } [y < x]^4 \text{ do} \\
| y := y+x|^5 \\
\text{end}
$$