Software Verification

Assertion Inference

Carlo A. Furia
Proving Programs Automatically

The Program Verification problem:

- **Given**: a program $P$ and a specification $S = [\text{Pre}, \text{Post}]$
- **Determine**: if every execution of $P$, for any value of input parameters, satisfies $S$
- **Equivalently**: establish whether $\{\text{Pre}\} \ P \ {\text{Post}}$ is (totally) correct

- A **general and fully automated solution** to the Program Verification problem is **unachievable** because the problem is **undecidable**

- One of the consequences of this inescapable limitation is the **impossibility of computing intermediate assertions fully automatically**

  (It is not an obvious consequence: formally, a reduction between undecidable problems)
Proving Programs Automatically

The Program Verification problem:

- **Given**: a program $P$ and a specification $S = \text{[Pre, Post]}$
- **Determine**: if every execution of $P$, for any value of input parameters, satisfies $S$
- **Equivalently**: establish whether $\{\text{Pre}\} P \{\text{Post}\}$ is (totally) correct

One way to put it, practically:

Proving the correctness of a computer program requires knowledge about the program that is not readily available in the program text

--- Chang & Leino

In this lecture, we survey techniques to automatically infer assertions in interesting special cases
The Assertion Inference Paradox

Correctness is consistency of implementation to specification

The paradox:

if the specification is inferred from the implementation, what do we prove?
The Assertion Inference Paradox

The **paradox**: if the specification is inferred from the implementation, what do we prove?

Possible **retorts**:
- The paradox only arises for correctness proofs; there are **other applications** (e.g. reverse-engineering legacy software)
- The result may be presented to a programmer for **assessment**
- Inferred specification may be **inconsistent**, thus denoting a problem
The Assertion Inference Paradox

The paradox:
if the specification is inferred from the implementation, what do we prove?

The paradox does not arise if we only infer intermediate assertions and not specifications (pre and postconditions)

- Intermediate assertions are a technical means to an end (proving correctness)
  - tools infer loop invariants
- The specification is a formal description of what the implementation should do
  - programmers write specifications
Let us consider a general (and somewhat informal) definition of invariant:

**Def. Invariant:** assertion whose truth is preserved by the execution of (parts of) a program.

\[ x: \text{INTEGER} \]
\[ \text{from } x := 1 \text{ until } \ldots \text{ loop } x := -x \text{ end} \]

Some invariants:
- \(-1 \leq x \leq 1\)
- \(x = -1 \lor x = 0 \lor x = 1\)
- \(x \geq -10\)
Kinds of Invariants

Def. Invariant: assertion whose truth is preserved by the execution of (parts of) a program.

We can identify different types of invariants, according to what parts of the program preserve the invariant:

- **Location invariant at** \( x \): assertion that holds whenever the computation reaches location \( x \)
- **Program invariant**: predicate that holds in any reachable state of the computation
- **Class invariant**: predicate that holds between (external) feature invocations
- **Loop invariant**: predicate that holds after every iteration of a loop body

\[
\begin{align*}
\{P\} & \ A \ \{I\} \\
\{I \land \neg \ c\} & \ B \ \{I\} \\
\hline \\
\{P\} & \ \text{from} \ A \ \text{until} \ c \\
\text{loop} \ B \ \text{end} & \ \{I \land c\}
\end{align*}
\]
Kinds of Invariants

1: \[ x: \text{INTEGER} \]
2: 
3: from \( x := 1 \)
4: until ...
5: loop \( x := -x \) end

- Location invariant at 2:
- Loop invariant:
- Program invariant:
Kinds of Invariants

- **Location invariant at 2:**
  \[ x = 0 \]

- **Loop invariant:**
  \[ x = -1 \lor x = 1 \]

- **Program invariant:**
  \[ x \geq -10 \]
Focus on Loop Invariants

If we have loop invariants we can get (basically) everything else at little cost

- while getting loop invariants requires invention

In the following discussion we focus on loop invariants (and call them simply “invariants”)

This focus is also consistent with the Assertion Inference Paradox
Focus on Loop Invariants

The various kinds of invariants are closely related by the inference rules of Hoare logic.

- If \( L_x \) is a location invariant at \( x \) then:
  \[ @x \Rightarrow L_x \]
  is a program invariant.

- If \( P \) is a program invariant then it is also a location invariant at every location \( x \).

- If \( I \) is a loop invariant of:
  \[ x: \text{from ... until } c \text{ loop ... end} \]
  then \( I \land c \) is a location invariant at \( x+1 \).

- If \( L \) is a location invariant at \( x+1 \):
  \[ x: a := b + 3 \]
  then \( L[b + 3 / a] \) is a location invariant at \( x \).

- Etc...
Techniques for Invariant Inference

Classification of invariant inference techniques:

- **Dynamic techniques**
- **Static techniques**
  - statistical techniques
  - exact techniques

(Roughly) direction of increasing sophistication:
Exact Static Techniques for Invariant Inference
Static Invariant Inference: classification

Static exact techniques for invariant inference are further classified in categories:

- **Direct**
- **Assertion-based**
  - postcondition mutation
- **Based on abstract interpretation**
  - forward analysis (bottom-up)
  - backward analysis (top-down)
- **Constraint-based**
  - usually, template-based
Exact Static Techniques for Invariant Inference:

Postcondition-mutation Approach
The Role of User-provided Contracts

Techniques for invariant inference rarely take advantage of other annotations in the program text, such as contracts provided by the user.

- Not every annotation can (or should, cf. Assertion Inference Paradox) be inferred automatically!

However, there is a close connection between a loop's invariant and its postcondition.
The Role of User-provided Contracts

However, there is a close connection between a loop's invariant and its postcondition

The invariant is a weakened form of the postcondition

- It is a larger collection of program states

Example: \[ \text{from } x := 0 \text{ until } x = n \text{ loop } x := x + 1 \text{ end} \]

- Post: \( x = n \) (for some \( n > 0 \))
- Invariant: \( 0 \leq x \leq n \)
In a nutshell:

Static verification of candidate invariants obtained by mutating postconditions

- Assume the availability of postconditions
- Mutate postconditions according to various heuristics
  - the heuristics mirror common patterns that link postconditions to invariants
  - each mutated postcondition is a candidate invariant
- Verify which candidates are indeed invariants
  - With an automatic program prover such as Boogie
- Retain all verified invariants

2009 - CAF & BM

Implementation: gin-pink which finds invariants in Boogie programs
Loop invariant inference

Input

Pre
Post
Routine

Candidate Invariants

mutate

checking invariance

Loop

Loop Invariants

proving correctness
(possibly using additional info)

Output

\{Pre\} Routine \{Post\}
Postcondition Mutation Heuristics

Constant relaxation

- replace “constant” by “variable”
  - cannot/may be changed by any of the loop bodies

Uncoupling

- replace subexpression appearing twice by two subexpressions
  - for example: subexpression = variable id

Term dropping

- remove a conjunct

Variable aging

- replace subexpression by another expression representing its previous value
Invariant Inference: the Algorithm

**Goal:** find invariants of loops in procedure `proc`

For each:
- **post**: postcondition clause of `proc`
- **loop**: outer loop in `proc`

compute all mutations $M$ of `post` w.r.t. `loop` considering postcondition clauses separately implements term dropping

**Result:** any formula in $M$ which can be verified as invariant of any loop in `proc`
Maximum value of an array

\[ \text{max}(A: \text{ARRAY}[T]; n: \text{INTEGER}): T \]

\text{require} \quad A.length = n \geq 1

\text{local} \ i: \text{INTEGER}

\text{do}

\text{from} \ i := 0; \ \text{Result} := A[1];

\text{until} \ i = n

\text{loop}

\quad i := i + 1

\quad \text{if} \ \text{Result} \leq A[i] \ \text{then} \ \text{Result} := A[i] \ \text{end}

\text{end}

\text{ensure} \quad (\forall 1 \leq j \leq n \Rightarrow A[j] \leq \text{Result}) \quad \text{and} \quad (\exists 1 \leq j \leq n \land A[j] = \text{Result})
Maximum value of an array

\[
\text{max} (A: \text{ARRAY} [T] ; n: \text{INTEGER}): T \\
\text{require } A\.length = n \geq 1 \\
\text{ensure } ( \forall 1 \leq j \leq n \Rightarrow A[j] \leq \text{Result} ) \text{ and } \\
( \exists 1 \leq j \leq n \land A[j] = \text{Result} )
\]

- **Constant relaxation**: replace “constant” \( n \) by “variable” \( i \)
- **Term dropping**: remove second conjunct

**Invariant**: \( \forall 1 \leq j \leq i \Rightarrow A[j] \leq \text{Result} \)
Maximum value of an array (cont'd)

```plaintext
max (A: ARRAY [T]; n: INTEGER): T
    require A.length = n ≥ 1
    ensure ( ∀ 1 ≤ j ≤ n ⇒ A[j] ≤ Result ) and
           ( ∃ 1 ≤ j ≤ n ∧ A[j] = Result )
```

- Term dropping: remove first conjunct
  
  Invariant:  ∃ 1 ≤ j ≤ n ∧ A[j] = Result
Maximum value of an array (2\textsuperscript{nd} version)

\begin{align*}
\text{max\_v2} (A: \text{ARRAY} [T]; n: \text{INTEGER}): & T \\
\text{require } & A.\text{length} = n \geq 1 \\
\text{local } & i: \text{INTEGER} \\
\text{do } & \\
\text{from } & i := 1 ; \text{ Result := } A[1]; \\
\text{until } & i > n \\
\text{loop } & \\
\text{if } & \text{ Result } \leq A[i] \text{ then } \text{ Result := } A[i] \text{ end} \\
\text{ i := } & i + 1 \\
\text{end } & \\
\text{ensure } & \forall 1 \leq j \leq n \Rightarrow A[j] \leq \text{Result} \\
\end{align*}
Maximum value of an array (2\textsuperscript{nd} version)

\[
\texttt{max\_v2}(A: \text{ARRAY}[T]; n: \text{INTEGER}): T
\]
require \(A\).\text{length} = n \geq 1
ensure \(\forall 1 \leq j \leq n \Rightarrow A[j] \leq \text{Result}\)

- **Constant relaxation:** replace “constant” \(n\) by “variable” \(i\)
  \(\forall 1 \leq j \leq i \Rightarrow A[j] \leq \text{Result}\)

- **Variable aging:**
  use expression representing the previous value of \(i\): \(i - 1\)
  Invariant: \(\forall 1 \leq j \leq i - 1 \Rightarrow A[j] \leq \text{Result}\)
Array Partitioning

**partition** \((A: \text{ARRAY} [T]; n: \text{INTEGER}; \text{pivot}: T): \text{INTEGER}\)

require \(A.length = n \geq 1\)

local \(l, h: \text{INTEGER}\)

do
    from \(l := 1 ; h := n\) until \(l = h\)
    loop
        from until \(l = h\) or \(A[l] > \text{pivot}\) loop \(l := l + 1\) end
        from until \(l = h\) or \(\text{pivot} > A[h]\) loop \(h := h - 1\) end
        \(A.swap(l, h)\)
    end
    if \(\text{pivot} \leq A[l]\) then \(l := l - 1\) end; \(h := l; \text{Result} := h\)
end

ensure \((\forall 1 \leq k \leq \text{Result} \Rightarrow A[k] \leq \text{pivot})\) and

\((\forall \text{Result} < k \leq n \Rightarrow A[k] \geq \text{pivot})\)
Array Partitioning

\[ \text{partition}(A: \text{ARRAY}[T]; \ n: \text{INTEGER}; \ pivot: T): \text{INTEGER} \]

require \( A.length = n \geq 1 \)

ensure \((\forall 1 \leq k \leq \text{Result} \Rightarrow A[k] \leq \text{pivot})\) and 
\((\forall \text{Result} < k \leq n \Rightarrow A[k] \geq \text{pivot})\)

- **Uncoupling**: replace first occurrence of \( \text{Result} \) by \( l \) and second by \( h \)
  \((\forall 1 \leq k \leq l \Rightarrow A[k] \leq \text{pivot})\) and \((\forall h < k \leq n \Rightarrow A[k] \geq \text{pivot})\)

- **Variable aging**: use expression representing the previous value of \( l \): \( l - 1 \)

**Invariant**: 
\((\forall 1 \leq k \leq l - 1 \Rightarrow A[k] \leq \text{pivot})\) and \((\forall h < k \leq n \Rightarrow A[k] \geq \text{pivot})\)
Array Partitioning

\[\text{partition}(A: \text{ARRAY}[T]; n: \text{INTEGER}; \text{pivot}: T): \text{INTEGER}\]

require \(A.\text{length} = n \geq 1\)

ensure (\(\forall 1 \leq k \leq \text{Result} \Rightarrow A[k] \leq \text{pivot}\)) and
\((\forall \text{Result} < k \leq n \Rightarrow A[k] \geq \text{pivot})\)

- **Term dropping**: remove first conjunct
  \(\forall \text{Result} < k \leq n \Rightarrow A[k] \geq \text{pivot}\)

- **Constant relaxation**: replace “constant” Result by “variable” \(h\)

  Invariant: \(\forall h < k \leq n \Rightarrow A[k] \geq \text{pivot}\)
Limitations of the approach

Some invariants are not mutations of the postcondition

- “completeness” of the postcondition
- integration with other techniques
- more heuristics

Combinatorial explosion

- user guidance

Dependencies

- especially with nested loops
- user guidance

Limitations of automated reasoning techniques

- they progress quickly
Exact Static Techniques for Invariant Inference: Constraint-based Approach
Constraint-based Invariant Inference

- **In a nutshell:**
  encode semantics of iteration as constraints on a template invariant

  - Choose a template invariant expression
    - template defines an (infinite) set of assertions
  - Encode the loop semantics as a set of constraints on the template
    - initiation
    - consecution
  - **Solve** the constraints
    - this is usually the complex part
  - Any solution is an invariant

Constraint-based Inv. Inference: Example

**dummy_routine** (n: NATURAL)
local x: NATURAL do
  from x := 0
  until x ≥ n
  loop x := x + 1 end
end

- **Template invariant expression:**
  \[ T = c \cdot x + d \cdot n + e \leq 0 \]

- **Constraints encoding loop semantics:**
  - **Initiation:** “T holds for the initial values of x and n”
    \[ T[0/x; n_0/n] \equiv c \cdot 0 + d \cdot n_0 + e \leq 0 \equiv d \cdot n_0 + e \leq 0 \]
Constraint-based Inv. Inference: Example

**dummy_routine**(n: NATURAL)
local x: NATURAL do
  from x := 0
  until x ≥ n
  loop x := x + 1 end
end

- Constraints encoding loop semantics:

  **Consecution:** “if T holds and one iteration of the loop is executed, T still holds”
  \[ T [x/x; n/n] \land ( \neg(x \geq n) \land x' = x + 1 \land n' = n ) \Rightarrow T [x'/x; n'/n] \]

- Solving the constraints requires to eliminate occurrences of x, x', n, n'
  - For linear constraints we can use Farkas's Lemma
Farkas's Lemma (1902)

Let $S$ be a system of linear inequalities over $n$ real variables:

$$S \triangleq \begin{bmatrix}
  a_{11}x_1 + \cdots + a_{1n}x_n + b_1 & \leq & 0 \\
  \vdots & \vdots & \vdots \\
  a_{m1}x_1 + \cdots + a_{mn}x_n + b_m & \leq & 0
\end{bmatrix}$$

and let $\Psi$ be a linear inequality:

$$\psi \triangleq c_1x_1 + \cdots + c_nx_n + d \leq 0$$

Then $S \Rightarrow \Psi$ is valid iff $S$ is unsatisfiable or there exist $m+1$ real nonnegative coefficients $\lambda_0, \lambda_1, \ldots, \lambda_m$ such that:

$$c_j = \sum_{i=1}^{m} \lambda_i a_{ij} \quad (1 \leq j \leq m) \quad d = -\lambda_0 + \sum_{i=1}^{m} \lambda_i b_i$$
Constraint-based Inv. Inference: Example

Use Farkas's lemma to turn the consecution constraint:

\[ T[x/x; n/n] \land x < n \land x' = x + 1 \land n' = n \\Rightarrow T[x'/x; n'/n] \]

into a constraint over \( c, d, \) and \( e \) only.

\[
\begin{array}{c|ccc}
\lambda_1 & cx & +dn & +e \leq 0 \\
\lambda_2 & x & -n & +1 \leq 0 \\
\lambda_3 & -x & +x' & -1 \leq 0 \\
\lambda_4 & x & -x' & +1 \leq 0 \\
\lambda_5 & -n & +n' & \leq 0 \\
\lambda_6 & n & -n' & \leq 0 \\
\hline
\text{.................................} \\
cx' & +dn' & +e \leq 0
\end{array}
\]
### Constraint-based Inv. Inference: Example

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>$cx$</th>
<th>$+dn$</th>
<th>$+e$</th>
<th>$\leq 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_2$</td>
<td>$x$</td>
<td>$-n$</td>
<td></td>
<td>$+1$</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>$-x$</td>
<td></td>
<td>$+x'$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>$x$</td>
<td></td>
<td>$-x'$</td>
<td></td>
</tr>
<tr>
<td>$\lambda_5$</td>
<td></td>
<td>$-n$</td>
<td>$+n'$</td>
<td>$\leq 0$</td>
</tr>
<tr>
<td>$\lambda_6$</td>
<td></td>
<td>$n$</td>
<td>$-n'$</td>
<td>$\leq 0$</td>
</tr>
</tbody>
</table>

\[ cx' + dn' + e \leq 0 \]

\[ \Phi \triangleq \exists \lambda_0, \ldots, \lambda_6 \]

\[ \begin{bmatrix}
\lambda_0, \ldots, \lambda_6 \geq 0 \\
\lambda_1 c + \lambda_2 - \lambda_3 + \lambda_4 = 0 \\
\lambda_1 d - \lambda_2 - \lambda_5 + \lambda_6 = 0 \\
\lambda_3 - \lambda_4 = c \\
\lambda_5 - \lambda_6 = d \\
-\lambda_0 + \lambda_1 e + \lambda_2 - \lambda_3 + \lambda_4 = e
\end{bmatrix} \]
Constraint-based Inv. Inference: Example

Finally, eliminate existential quantifiers from $\Phi$ to get the constraint:

$$c \leq 0 \lor (c + d = 0 \land e \leq 0)$$

(Quantifier elimination is also quite technical)
Constraint-based Inv. Inference: Example

\textit{dummy\_routine} (n: NATURAL)  
local x: NATURAL do  
    from x := 0  
    until x \geq n  
    loop x := x + 1 end  
end

- Any solution \([c, d, e]\) to:
  - Initiation and Consecution:
    \(( d \cdot n_0 + e \leq 0 ) \land ( c \leq 0 \lor ( c + d = 0 \land e \leq 0 ) )\)

  determines an invariant of the loop.

- \([0, -1, 0]\) \quad \longrightarrow \quad n \geq 0
- \([1, 0, 0]\) \quad \longrightarrow \quad x \geq 0
- \([1, -1, 0]\) \quad \longrightarrow \quad x - n \leq 0
Constraint-based Inv. Inference: Summary

- **Main issues:**
  - choice of invariant templates for which effective decision procedures exist
    - interesting research topic per se, on the brink of undecidability
  - heuristics to extract the “best” invariants from the set of solutions

- **Advantages:**
  - sound & complete (w.r.t. the template)
  - exploit heterogeneous decision procedures together
  - fully automated (possibly except for providing the template)
    - providing the template introduces a “natural” form of user interaction

- **Disadvantages:**
  - suitable mathematical decision theories are usually quite sophisticated
    - hence, hard to extend and customize
  - exact constraint solving is usually quite expensive
  - mostly suitable for “algebraic” invariants
    - requires integration with other techniques to achieve full functional correctness proofs
Dynamic Techniques for Invariant Inference
Dynamic Invariant Inference

- In a nutshell:
  
  testing of candidate invariants
  
  - Choose a set of test cases
  - Perform runtime monitoring of candidate invariants
  - If some test run violates a candidate, discard the candidate
  - The surviving candidates are guessed invariant

- Daikon tool, 1999 -- Mike Ernst et al.
- CITADEL: Daikon for Eiffel, 2008 -- Nadia Polikarpova
- AutoInfer for Eiffel (Yi “Jason” Wei et al.)
Dynamic Invariant Inference: Example

\[\text{dummy_routine}(n: \text{NATURAL})\]
\[
\text{local } x: \text{NATURAL do}
\]
  \[
  \text{from } x := 0
  \]
  \[
  \text{until } x \geq n
  \]
  \[
  \text{loop } x := x + 1 \text{ end}
  \]
\end{end}

- **Test cases:** \{ \( n = k \) | \( 0 \leq k \leq 1000 \) \}

- **Candidate invariants:**
  - \{ \( x \geq c \) | \(-1000 \leq c \leq 1000 \) \},
    \{ \( n \geq c \) | \(-1000 \leq c \leq 1000 \) \}
  - \{ \( x = c \cdot n + d \) | \(-500 \leq c, d \leq 500 \) \}
  - \{ \( x < n, x \leq n, x = n, x \neq n, x \geq n, x > n \) \}
  - \{ \( x \leq n \geq c \) | \(-500 \leq c \leq 500 \) \}
  - \( \ldots \)
Dynamic Invariant Inference: Example

\begin{verbatim}
\textbf{dummy\_routine} (n: NATURAL)
local x: NATURAL do
  from x := 0
  until x ≥ n
  loop x := x + 1 end
end
\end{verbatim}

- \textbf{Survivors (after loop iterations)}:
  - \{ x ≥ -c \mid 0 ≤ c ≤ 1000 \},
    \{ n ≥ -c \mid 0 ≤ c ≤ 1000 \}
  - x ≤ n
  - \{ x + n ≥ c \mid -500 ≤ c ≤ 500 \}
  - ...
Dynamic Invariant Inference: Summary

- **Main issues:**
  - choose suitable test cases
  - handle huge sets of candidate invariants (runtime overhead)
  - estimate soundness/quality of survivor predicates
  - select heuristically the “best” survivor predicates

- **Advantages:**
  - straightforward to implement (at least compared to other techniques)
  - guessing is often rather accurate in practice (possibly with some heuristics)
  - customizable and rather flexible:
    in principle, whatever you can test you can check for invariance

- **Disadvantages:**
  - unsound (educated guessing)
  - without heuristics, large amount of useless, redundant predicates
  - sensitive to choice of test cases
  - some complex candidate invariants are difficult to implement efficiently
Exact Static Techniques for Invariant Inference:

Direct Approach
Direct Static Invariant Inference

• **In a nutshell:**
  solve the fixpoint equations underlying the program
  
  • \( v(i) \): value of variable \( v \) at step \( i \) of the computation
  • Encode the semantics of loops explicitly and directly as recurrence equations over \( v(i) \)
  • Solve recurrence equations
  • Eliminate step parameter \( i \) to obtain invariant

• 1973 -- Shmuel Katz & Zohar Manna
• 2005 -- Laura Kovacs et al.
Direct Static Invariant Inference: Example

\[ \text{dummy\_routine}(n: \text{NATURAL}) \]
local \( x: \text{NATURAL} \) do
\[ \text{from } x := 0 \]
\[ \text{until } x \geq n \]
loop \( x := x + 1 \) end
end

- \( x(i), n(i) \)

- **Recurrence relations**:

\[
x(i) = \begin{cases} 
0 & i = 0 \\
x(i - 1) + 1 & 0 < i \leq n_0 \\
x(i - 1) & i > n_0
\end{cases}
\]

\[
n(i) = \begin{cases} 
n_0 \geq 0 & i = 0 \\
n(i - 1) & i > 0
\end{cases}
\]
Direct Static Invariant Inference: Example

\[
x(i) = \begin{cases} 
0 & i = 0 \\
x(i - 1) + 1 & 0 < i \leq n_0 \\
x(i - 1) & i > n_0 
\end{cases} \\
\]

\[
n(i) = \begin{cases} 
n_0 \geq 0 & i = 0 \\
n(i - 1) & i > 0 
\end{cases} 
\]

- **Solving recurrence relations:**
  - \(x(i) = \min(n_0, i) \geq 0\)
  - \(n(i) = n_0\)

- **Eliminating step parameter \(i\):**
  - \(-x(i) - n(i) = \min(n_0, i) - n_0 \leq 0, \text{ hence:}\)
  - \(-x - n \leq 0, \text{ hence:}\)
  - \(0 \leq x \leq n\)
Direct Static Invariant Inference: Summary

- **Main issues:**
  - in its bare form, more a set of guidelines than a technique
  - step parameter elimination is tricky

- **Advantages:**
  - since semantics is represented explicitly, obtained invariants are often powerful
  - benefits from the programmer's ingenuity
  - additional information about the program can be “plugged in”

- **Disadvantages:**
  - solving recurrence equations can be very difficult (when possible at all)
  - typically restricted to “algebraic” invariants