Software Verification

Assertion Inference

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Proving Programs Automatically

The Program Verification problem:

- Given: a program P and a specification S = [Pre, Post]
- Determine: if every execution of P, for any value of input parameters, satisfies S
- Equivalently: establish whether {Pre} P {Post} is (totally) correct

- A general and fully automated solution to the Program Verification problem is unachievable because the problem is undecidable
- One of the consequences of this inescapable limitation is the impossibility of computing intermediate assertions fully automatically

(It is not an obvious consequence: formally, a reduction between undecidable problems)

Proving Programs Automatically

The Program Verification problem:

- Given: a program P and a specification S = [Pre, Post]
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- Equivalently: establish whether {Pre} P {Post} is (totally) correct

One way to put it, practically:

> Proving the correctness of a computer program requires knowledge about the program that is not readily available in the program text
>
> -- Chang & Leino

In this lecture, we survey techniques to automatically infer assertions in interesting special cases

The Assertion Inference Paradox

Correctness is consistency of implementation to specification

The paradox:

> if the specification is inferred from the implementation, what do we prove?

The Assertion Inference Paradox

The paradox:

if the specification is inferred from the implementation,
what do we prove?

Possible retorts:

- The paradox only arises for correctness proofs; there are other applications (e.g. reverse-engineering legacy software)
- The result may be presented to a programmer for assessment
- Inferred specification may be inconsistent, thus denoting a problem

The Assertion Inference Paradox

The paradox:

> if the specification is inferred from the implementation, what do we prove?

The paradox does not arise if we only infer intermediate assertions and not specifications (pre and postconditions)

- Intermediate assertions are a technical means to an end (proving correctness)
  - tools infer loop invariants
- The specification is a formal description of what the implementation should do
  - programmers write specifications

Invariants

Let us consider a general (and somewhat informal) definition of invariant:

Def. Invariant: assertion whose truth is preserved by the execution of (parts of) a program.

```
x: INTEGER
from x := 1  until ...  loop x := - x end
```

Some invariants:

- $-1 \leq x \leq 1$
- $x = -1 \lor x = 0 \lor x = 1$
- $x \geq -10$

Kinds of Invariants

Def. Invariant: assertion whose truth is preserved by the execution of (parts of) a program.

We can identify different types of invariants, according to what parts of the program preserve the invariant:

- Location invariant at x: assertion that holds whenever the computation reaches location x
- Program invariant: predicate that holds in any reachable state of the computation
- Class invariant: predicate that holds between (external) feature invocations
- Loop invariant: predicate that holds after every iteration of a loop body

$$\frac{\begin{array}{c}\{P\}\ A\ \{I\} \\ \{I \land \neg\, c\}\ B\ \{I\}\end{array}}{\begin{array}{c}\{P\} \\ \textbf{from}\ \ A\ \textbf{until}\ \ c \\ \textbf{loop}\ B\ \textbf{end} \\ \{I \land c\}\end{array}}$$

Kinds of Invariants

```
1:          x: INTEGER
2:
3:          from x := 1
4:          until ...
5:          loop x := - x end
```

- Location invariant at 2:

- Loop invariant:

- Program invariant:

Kinds of Invariants

```
1:          x: INTEGER
2:
3:          from x := 1
4:          until ...
5:          loop x := - x end
```

- Location invariant at 2:

  $x = 0$

- Loop invariant:

  $x = -1 \lor x = 1$

- Program invariant:

  $x \geq -10$

Focus on Loop Invariants

If we have loop invariants we can get (basically) everything else at little cost

- while getting loop invariants requires invention

In the following discussion we focus on loop invariants (and call them simply "invariants")

This focus is also consistent with the Assertion Inference Paradox

Focus on Loop Invariants

The various kinds of invariants are closely related by the inference rules of Hoare logic

- If Lx is a location invariant at x then:

  @x ⇒ Lx

  is a program invariant

- If P is a program invariant then it is also a location invariant at every location x

- If I is a loop invariant of:

  x: from … until c loop … end

  then I ∧ c is a location invariant at x+1

$$\frac{\begin{array}{c}\{P\}\ A\ \{I\} \\ \{I \wedge \neg\, c\}\ B\ \{I\}\end{array}}{\begin{array}{c}\{P\} \\ \textbf{from}\ \ A\ \textbf{until}\ \ c \\ \textbf{loop}\ B\ \textbf{end} \\ \{I \wedge c\}\end{array}}$$

- If L is a location invariant at x+1:

  x: a := b + 3

  then L [b + 3 / a] is a location invariant at x

$$\{P\ [e\ /\ x]\}\ x := e\ \{P\}$$

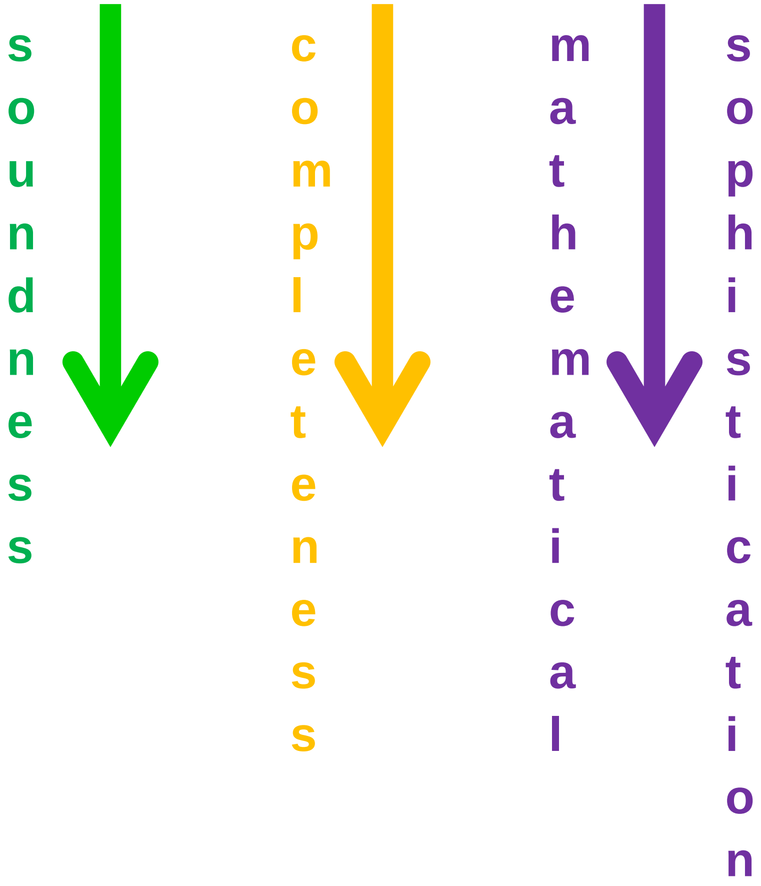
- Etc…

Techniques for Invariant Inference

Classification of invariant inference techniques:

- Dynamic techniques
- Static techniques
  - statistical techniques
  - exact techniques

(Roughly) direction of increasing:



soundness

completeness

mathematical

sophistication

Exact Static Techniques for Invariant Inference

Static Invariant Inference: classification

Static exact techniques for invariant inference are further classified in categories:

- Direct
- Assertion-based
    - postcondition mutation
- Based on abstract interpretation
    - forward analysis (bottom-up)
    - backward analysys (top-down)
- Constraint-based
    - usually, template-based

Exact Static Techniques for Invariant Inference:

Postcondition-mutation Approach

The Role of User-provided Contracts

Techniques for invariant inference rarely take advantage of other annotations in the program text, such as contracts provided by the user

- Not every annotation can (or should, cf. Assertion Inference Paradox) be inferred automatically!

However, there is a close connection between a loop's invariant and its postcondition

The Role of User-provided Contracts

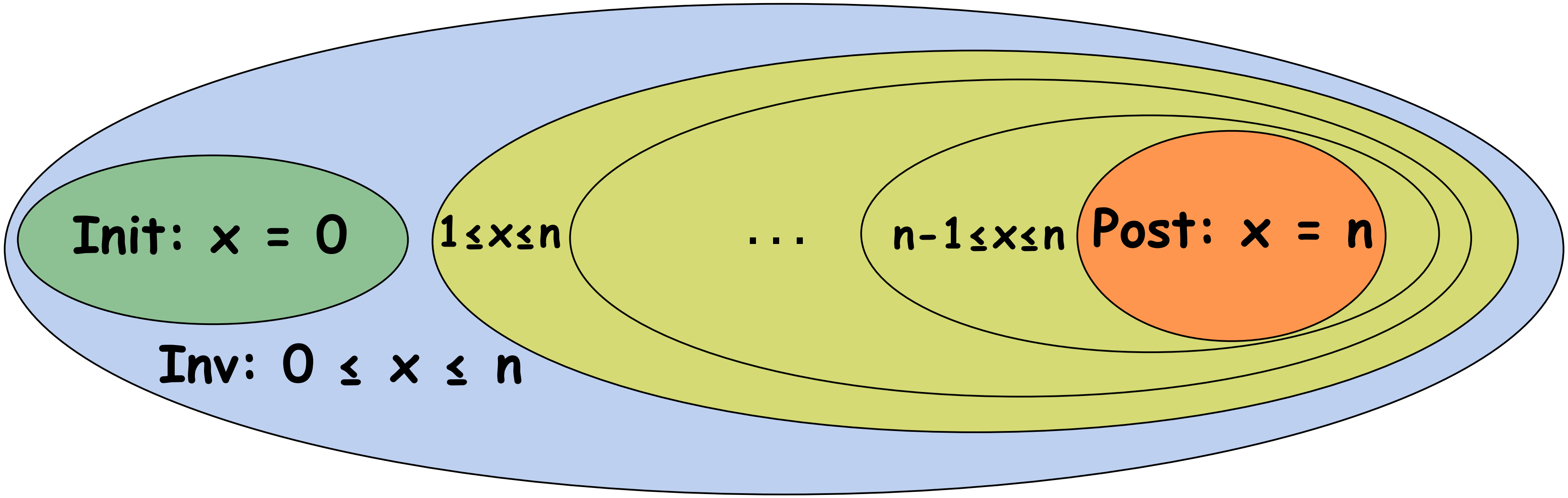
However, there is a close connection between a loop's invariant and its postcondition

The invariant is a weakened form of the postcondition

- It is a larger collection of program states

Example: from x := 0 until x = n loop x := x + 1 end

- Post: x = n (for some n > 0)
- Invariant: 0 ≤ x ≤ n



Invariants by Postcondition Mutation

- In a nutshell:

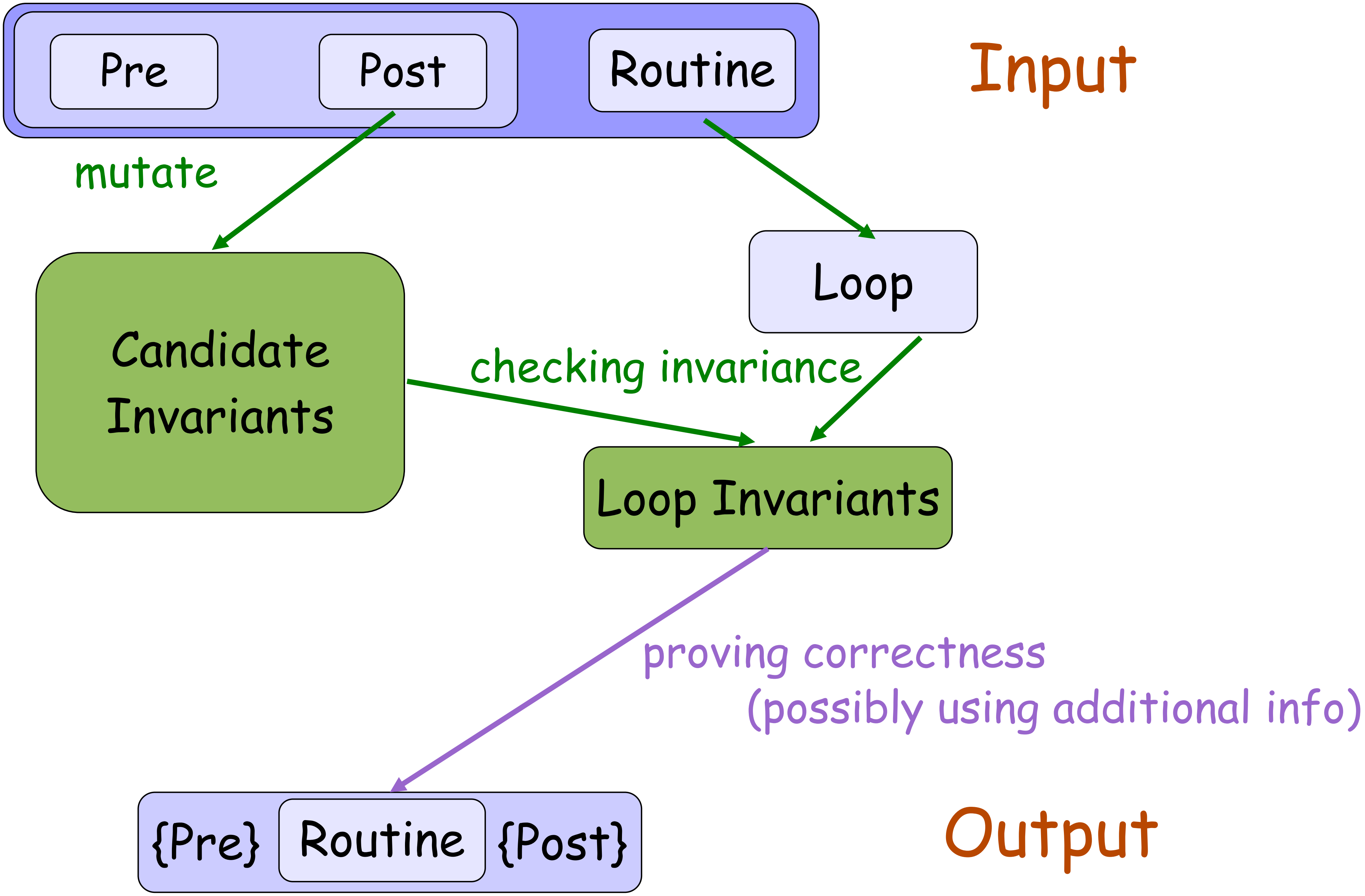
  Static verification of candidate invariants
  obtained by mutating postconditions

  - Assume the availability of postconditions
  - Mutate postconditions according to various heuristics
    - the heuristics mirror common patterns that link postconditions to invariants
    - each mutated postcondition is a candidate invariant
  - Verify which candidates are indeed invariants
    - With an automatic program prover such as Boogie
  - Retain all verified invariants

- 2009 – CAF & BM

- Implementation: *gin-pink* which finds invariants in Boogie programs

Loop invariant inference



Pre

Post

Routine

Input

mutate

Loop

Candidate Invariants

checking invariance

Loop Invariants

proving correctness (possibly using additional info)

{Pre} Routine {Post}

Output

Postcondition Mutation Heuristics

Constant relaxation

- replace “constant” by “variable”
  - cannot/may be changed by any of the loop bodies

Uncoupling

- replace subexpression appearing twice by two subexpressions
  - for example: subexpression = variable id

Term dropping

- remove a conjunct

Variable aging

- replace subexpression by another expression representing its previous value

Invariant Inference: the Algorithm

Goal: find invariants of loops in procedure proc

For each:

- post: postcondition clause of proc
- loop: outer loop in proc

compute all mutations M of post w.r.t. loop

- considering postcondition clauses separately implements term dropping

Result: any formula in M which can be verified as invariant of any loop in proc

Maximum value of an array

```
max (A: ARRAY [T] ; n: INTEGER): T
   require A.length = n ≥ 1
   local i: INTEGER
   do
      from i := 0 ;  Result := A[1];
      until i = n
      loop
         i := i + 1
         if  Result ≤ A[i]  then  Result := A[i]  end
      end
   ensure        ( ∀ 1 ≤ j ≤ n ⇒ A[j] ≤ Result )   and
                         ( ∃ 1 ≤ j ≤ n ∧ A[j] = Result )
```

Maximum value of an array

```
max (A: ARRAY [T] ; n: INTEGER): T
   require A.length = n ≥ 1
   ensure       ( ∀ 1 ≤ j ≤ n ⇒ A[j] ≤ Result )  and
                        ( ∃ 1 ≤ j ≤ n ∧ A[j] = Result )
```

- Constant relaxation: replace "constant" n by "variable" i
- Term dropping: remove second conjunct

Invariant: $\forall\ 1 \le j \le i \Rightarrow A[j] \le Result$

Maximum value of an array (cont'd)

```
max (A: ARRAY [T] ; n: INTEGER): T
    require A.length = n ≥ 1
    ensure ( ∀ 1 ≤ j ≤ n ⇒ A[j] ≤ Result )  and
                              ( ∃ 1 ≤ j ≤ n ∧ A[j] = Result )
```

- Term dropping: remove first conjunct

Invariant: $\exists\ 1 \leq j \leq n \wedge A[j] = Result$

Maximum value of an array (2nd version)

```
max_v2 (A: ARRAY [T] ; n: INTEGER): T
    require A.length = n ≥ 1
    local i: INTEGER
    do
        from i := 1 ;  Result := A[1];
        until i > n
        loop
            if  Result ≤ A[i]  then  Result := A[i]  end
            i := i + 1
        end
    ensure ∀ 1 ≤ j ≤ n ⇒ A[j] ≤ Result
```

Maximum value of an array (2^{nd} version)

```
max_v2 (A: ARRAY [T] ; n: INTEGER): T
  require A.length = n ≥ 1
  ensure ∀ 1 ≤ j ≤ n ⇒ A[j] ≤ Result
```

- Constant relaxation: replace “constant” n by “variable” i

  $\forall\, 1 \le j \le i \Rightarrow A[j] \le Result$

- Variable aging:

  use expression representing the previous value of i: i - 1

Invariant: $\forall\, 1 \le j \le i - 1 \Rightarrow A[j] \le Result$

Array Partitioning

```
partition (A: ARRAY [T]; n: INTEGER; pivot: T): INTEGER
   require A.length = n ≥ 1
   local l, h: INTEGER
   do
     from l := 1 ; h := n        until l = h
     loop
        from until l = h or A[l] > pivot loop l := l + 1 end
        from until l = h or pivot > A[h] loop h := h - 1 end
        A.swap (l, h)
     end
     if pivot ≤ A[l] then l := l - 1 end ;  h := l ; Result := h
   ensure  (∀ 1≤k≤ Result ⇒ A[k] ≤ pivot) and
           (∀ Result<k≤n ⇒ A[k] ≥ pivot)
```

Array Partitioning

```
partition (A: ARRAY [T]; n: INTEGER; pivot: T): INTEGER
   require A.length = n ≥ 1
   ensure   (∀ 1≤k≤ Result ⇒ A[k] ≤ pivot) and
            (∀ Result<k≤n ⇒ A[k] ≥ pivot)
```

- Uncoupling: replace first occurrence of Result by l and second by h

  $(\forall\ 1 \le k \le l \Rightarrow A[k] \le pivot)$ and $(\forall\ h < k \le n \Rightarrow A[k] \ge pivot)$

- Variable aging: use expression representing the previous value of l: l - 1

Invariant:

$$(\forall\ 1 \le k \le l - 1 \Rightarrow A[k] \le pivot) \text{ and } (\forall\ h < k \le n \Rightarrow A[k] \ge pivot)$$

Array Partitioning

```
partition (A: ARRAY [T]; n: INTEGER; pivot: T): INTEGER
   require A.length = n ≥ 1
   ensure  (∀ 1≤k≤ Result ⇒ A[k] ≤ pivot) and
           (∀ Result<k≤n ⇒ A[k] ≥ pivot)
```

- Term dropping: remove first conjunct

$$\forall\ Result < k \le n \Rightarrow A[k] \ge pivot$$

- Constant relaxation: replace “constant” Result by “variable” h

Invariant: $\forall\ h < k \le n \Rightarrow A[k] \ge pivot$

Limitations of the approach

Some invariants are not mutations of the postcondition

- “completeness” of the postcondition
- integration with other techniques
- more heuristics

Combinatorial explosion

- user guidance

Dependencies

- especially with nested loops
- user guidance

Limitations of automated reasoning techniques

- they progress quickly

Exact Static Techniques for Invariant Inference:

Constraint-based Approach

Constraint-based Invariant Inference

- In a nutshell:

  encode semantics of iteration as constraints on a template invariant

  - Choose a template invariant expression
    - template defines a (infinte) set of assertions
  - Encode the loop semantics as a set of constraints on the template
    - initation
    - consecution
  - Solve the constraints
    - this is usually the complex part
  - Any solution is an invariant

- E.g.: 2003 -- Henny Sipma et al.; 2004 -- Zohar Manna et al. ;
  2007 -- Tom Henzinger et al.

Constraint-based Inv. Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Template invariant expression:

$$T \;=\; c \cdot x + d \cdot n + e \leq 0$$

- Constraints encoding loop semantics:
  - Initiation: “T holds for the initial values of x and n”

$$T\,[0/x;\, n_0/n] \;\equiv\; c \cdot 0 + d \cdot n_0 + e \leq 0 \equiv d \cdot n_0 + e \leq 0$$

Constraint-based Inv. Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Constraints encoding loop semantics:

  Consecution: "if T holds and one iteration of the loop is executed, T still holds"

  $T\,[x/x;\, n/n] \land (\, \neg(x \geq n) \land x' = x + 1 \land n' = n \,) \Rightarrow T\,[x'/x;\, n'/n]$

- Solving the constraints requires to eliminate occurrences of x, x', n, n'
  - For linear constraints we can use Farkas's Lemma

Farkas's Lemma (1902)

Let S be a system of linear inequalities over *n* real variables:

$$S \triangleq \left[\begin{array}{ccc} a_{11}x_1 + \cdots + a_{1n}x_n + b_1 & \leqslant & 0 \\ \vdots & \vdots & \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n + b_m & \leqslant & 0 \end{array}\right]$$

and let Ψ be a linear inequality:

$$\psi \triangleq c_1x_1 + \cdots + c_nx_n + d \leqslant 0$$

Then S ⇒ Ψ is valid iff S is unsatisfiable or there exist *m* +1 real nonnegative coefficients $\Lambda_0, \Lambda_1, \ldots, \Lambda_m$ such that:

$$c_j = \sum_{i=1}^{m} \lambda_i a_{ij} \quad (1 \leqslant j \leqslant m) \qquad d = -\lambda_0 + \sum_{i=1}^{m} \lambda_i b_i$$

Constraint-based Inv. Inference: Example

Use Farkas's lemma to turn the consecution constraint:

$$T[x/x;\ n/n] \land x < n \land x' = x + 1 \land n' = n$$
$$\Rightarrow T[x'/x;\ n'/n]$$

into a constraint over c, d, and e only.

$$\begin{array}{c|cccccl}
\lambda_1 & cx & +dn & & & +e & \leqslant 0 \\
\lambda_2 & x & -n & & & +1 & \leqslant 0 \\
\lambda_3 & -x & & +x' & & -1 & \leqslant 0 \\
\lambda_4 & x & & -x' & & +1 & \leqslant 0 \\
\lambda_5 & & -n & & +n' & & \leqslant 0 \\
\lambda_6 & & n & & -n' & & \leqslant 0 \\
\hline
 & & & cx' & +dn' & +e & \leqslant 0
\end{array}$$

Constraint-based Inv. Inference: Example

$$\begin{array}{c|cccccc}
\lambda_1 & cx & +dn & & & +e & \leqslant 0 \\
\lambda_2 & x & -n & & & +1 & \leqslant 0 \\
\lambda_3 & -x & & +x' & & -1 & \leqslant 0 \\
\lambda_4 & x & & -x' & & +1 & \leqslant 0 \\
\lambda_5 & & -n & & +n' & & \leqslant 0 \\
\lambda_6 & & n & & -n' & & \leqslant 0 \\
\hline
 & & & cx' & +dn' & +e & \leqslant 0
\end{array}$$

$$\Phi \quad \triangleq \quad \exists\lambda_0,\ldots,\lambda_6 \left[\begin{array}{c}
\lambda_0,\ldots,\lambda_6 \geqslant 0 \\
\lambda_1 c + \lambda_2 - \lambda_3 + \lambda_4 = 0 \\
\lambda_1 d - \lambda_2 - \lambda_5 + \lambda_6 = 0 \\
\lambda_3 - \lambda_4 = c \\
\lambda_5 - \lambda_6 = d \\
-\lambda_0 + \lambda_1 e + \lambda_2 - \lambda_3 + \lambda_4 = e
\end{array}\right]$$

Constraint-based Inv. Inference: Example

$$\Phi \quad \triangleq \quad \exists \lambda_0, \ldots, \lambda_6 \left[\begin{array}{c} \lambda_0, \ldots, \lambda_6 \geqslant 0 \\ \lambda_1 c + \lambda_2 - \lambda_3 + \lambda_4 = 0 \\ \lambda_1 d - \lambda_2 - \lambda_5 + \lambda_6 = 0 \\ \lambda_3 - \lambda_4 = c \\ \lambda_5 - \lambda_6 = d \\ -\lambda_0 + \lambda_1 e + \lambda_2 - \lambda_3 + \lambda_4 = e \end{array} \right]$$

Finally, eliminate existential quantifiers from Φ to get the constraint:

$$c \leq 0 \lor (c + d = 0 \land e \leq 0)$$

(Quantifier elimination is also quite technical)

Constraint-based Inv. Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Any solution [c, d, e] to:
  - Initiation and Consecution:

$$(d \cdot n_0 + e \leq 0) \wedge (c \leq 0 \ \vee \ (c + d = 0 \wedge e \leq 0))$$

determines an invariant of the loop.

– [0, -1, 0]	--->	$n \geq 0$
– [1, 0, 0]	--->	$x \geq 0$
– [1, -1, 0]	--->	$x - n \leq 0$

Constraint-based Inv. Inference: Summary

- Main issues:
  - choice of invariant templates for which effective decision procedures exist
    - interesting research topic per se, on the brink of undecidability
  - heuristics to extract the “best” invariants from the set of solutions
- Advantages:
  - sound & complete (w.r.t. the template)
  - exploit heterogeneous decision procedures together
  - fully automated (possibly except for providing the template)
    - providing the template introduces a “natural” form of user interaction
- Disadvantages:
  - suitable mathematical decision theories are usually quite sophisticated
    - hence, hard to extend and customize
  - exact constraint solving is usually quite expensive
  - mostly suitable for “algebraic” invariants
    - requires integration with other techniques to achieve full functional correctness proofs

Dynamic Techniques for Invariant Inference

Dynamic Invariant Inference

- In a nutshell:

  testing of candidate invariants

  - Choose a set of test cases
  - Perform runtime monitoring of candidate invariants
  - If some test run violates a candidate, discard the candidate
  - The surviving candidates are guessed invariant

- Daikon tool, 1999 -- Mike Ernst et al.
- CITADEL: Daikon for Eiffel, 2008 -- Nadia Polikarpova
- AutoInfer for Eiffel (Yi "Jason" Wei et al.)

Dynamic Invariant Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Test cases: $\{ n = k \mid 0 \le k \le 1000 \}$
- Candidate invariants:
  - $\{ x \ge c \mid -1000 \le c \le 1000 \}$,
$\{ n \ge c \mid -1000 \le c \le 1000 \}$
  - $\{ x = c \cdot n + d \mid -500 \le c, d \le 500 \}$
  - $\{ x < n, x \le n, x = n, x \ne n, x \ge n, x > n \}$
  - $\{ x \quad n \ge c \mid -500 \le c \le 500 \}$
  - …

Dynamic Invariant Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Survivors (after loop iterations) :
  - $\{ x \geq -c \mid 0 \leq c \leq 1000 \}$,
    $\{ n \geq -c \mid 0 \leq c \leq 1000 \}$
  - $x \leq n$
  - $\{ x + n \geq c \mid -500 \leq c \leq 500 \}$
  - ...

Dynamic Invariant Inference: Summary

- Main issues:
  - choose suitable test cases
  - handle huge sets of candidate invariants (runtime overhead)
  - estimate soundness/quality of survivor predicates
  - select heuristically the “best” survivor predicates
- Advantages:
  - straightforward to implement (at least compared to other techniques)
  - guessing is often rather accurate in practice (possibly with some heuristics)
  - customizable and rather flexible:
    in principle, whatever you can test you can check for invariance
- Disadvantages:
  - unsound (educated guessing)
  - without heuristics, large amount of useless, redundant predicates
  - sensitive to choice of test cases
  - some complex candidate invariants are difficult to implement efficiently

Exact Static Techniques for Invariant Inference:

Direct Approach

Direct Static Invariant Inference

- In a nutshell:

  solve the fixpoint equations underlying the program

  - v(i): value of variable v at step i of the computation
  - Encode the semantics of loops explicitly and directly as recurrence equations over v(i)
  - Solve recurrence equations
  - Eliminate step parameter i to obtain invariant

- 1973 -- Shmuel Katz & Zohar Manna
- 2005 -- Laura Kovacs et al.

Direct Static Invariant Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- x(i), n(i)
- Recurrence relations:

$$x(i) = \begin{cases} 0 & i = 0 \\ x(i-1)+1 & 0 < i \leqslant n_0 \\ x(i-1) & i > n_0 \end{cases} \qquad n(i) = \begin{cases} n_0 \geqslant 0 & i = 0 \\ n(i-1) & i > 0 \end{cases}$$

Direct Static Invariant Inference: Example

$$x(i) = \begin{cases} 0 & i = 0 \\ x(i-1) + 1 & 0 < i \leqslant n_0 \\ x(i-1) & i > n_0 \end{cases} \qquad n(i) = \begin{cases} n_0 \geqslant 0 & i = 0 \\ n(i-1) & i > 0 \end{cases}$$

- Solving recurrence relations:
  - $x(i) = \min(n_0, i) \geq 0$
  - $n(i) = n_0$
- Eliminating step parameter i:
  - $x(i) - n(i) = \min(n_0, i) - n_0 \leq 0$, hence:
  - $x - n \leq 0$, hence:
  - $0 \leq x \leq n$

Direct Static Invariant Inference: Summary

- Main issues:
  - in its bare form, more a set of guidelines than a technique
  - step parameter elimination is tricky
- Advantages:
  - since semantics is represented explicitly, obtained invariants are often powerful
  - benefits from the programmer's ingenuity
  - additional information about the program can be "plugged in"
- Disadvantages:
  - solving recurrence equations can be very difficult (when possible at all)
  - typically restricted to "algebraic" invariants