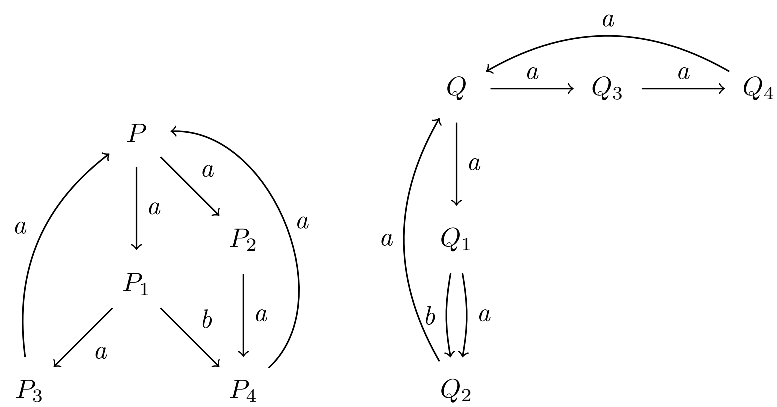
Assignment 12: CCS advanced concepts

ETH Zurich

1 Strong Bisimulation

Consider the following labelled transition system:



Show that $P \sim Q$ by finding a strong bisimulation $\mathcal{R}$ such that $P \,\mathcal{R}\, Q$.

1.1 Solution

A strong bisimulation $\mathcal{R}$ is given by the following relation:

$$\mathcal{R} = \{(P, Q), (P_1, Q_1), (P_3, Q_2), (P_4, Q_2), (P_2, Q_3), (P_4, Q_4)\}$$

2 Weak Bisimulation

Suppose we have the following definitions of processes

$$\begin{aligned} \mathrm{S} &\overset{\mathsf{def}}{=} a.\overline{b}.\mathrm{S} \\ \mathrm{T} &\overset{\mathsf{def}}{=} \overline{a}.e.b.\mathrm{T} \\ \mathrm{ST} &\overset{\mathsf{def}}{=} (\mathrm{S} \,|\, \mathrm{T}) \setminus \{a, b\} \end{aligned}$$

Further we have

$$\begin{aligned} \mathrm{U} &\overset{\mathsf{def}}{=} e.x.y.\mathrm{U} \\ \mathrm{V} &\overset{\mathsf{def}}{=} \overline{x}.\overline{y}.\mathrm{V} \\ \mathrm{UV} &\overset{\mathsf{def}}{=} (\mathrm{U} \,|\, \mathrm{V}) \setminus \{x, y\} \end{aligned}$$

Your task is to

1. Represent ST and UV as LTSs.

2. Show that ST and UV are weakly bisimilar.

3. Suppose we further have $\mathrm{UV}' \stackrel{\mathsf{def}}{=} (\mathrm{U} \mid \mathrm{V}) \setminus \{y\}$. Show that ST and UV′ are not weakly bisimilar.

2.1 Solution

1.

$$\begin{tikzcd} \mathrm{UV} \arrow[d, "e"] \\ (x.y.\mathrm{U} \mid \overline{x}.\overline{y}.\mathrm{V}) \setminus \{x, y\} \text{ or UV2} \arrow[d, "\tau"] \\ (y.\mathrm{U} \mid \overline{y}.\mathrm{V}) \setminus \{x, y\} \text{ or UV3} \arrow[uu, bend left, "\tau"] \end{tikzcd} \qquad \begin{tikzcd} \mathrm{ST} \arrow[d, "\tau"] \\ (\overline{b}.\mathrm{S} \mid e.b.\mathrm{T}) \setminus \{a, b\} \text{ or ST2} \arrow[d, "e"] \\ (\overline{b}.\mathrm{S} \mid b.\mathrm{T}) \setminus \{a, b\} \text{ or ST3} \arrow[uu, bend left, "\tau"] \end{tikzcd}$$

2. The weak bisimulation here is $\{ST, ST2, ST3\} \times \{UV, UV2, UV3\}$. An alternative weak bisimulation relation is $\{(UV, ST), (UV, ST2), (UV2, ST3), (UV3, ST3)\}$.

3. This is no longer a weak bisimulation. Due to the exposure of x, UV' can now make transitions that are impossible in ST.

3 In a nutshell

3.1 Background

Consider the labeled transition system describing the behavior of a process P:

$$\begin{tikzcd} \mathrm{P} \arrow[r, shift left, "b"] & \mathrm{P}_1 \arrow[l, shift left, "\bar{b}"] \arrow[r, shift left, "b"] & \mathrm{P}_2 \arrow[l, shift left, "\bar{b}"] \end{tikzcd}$$

Furthermore, consider the CCS process Q defined by the following equations:

$$\begin{aligned} \mathrm{Q} &\stackrel{\mathsf{def}}{=} (\mathrm{Q}_1 \mid \mathrm{Q}_2) \setminus \{a\} \\ \mathrm{Q}_1 &\stackrel{\mathsf{def}}{=} a.\bar{b}.\mathrm{Q}_1 \\ \mathrm{Q}_2 &\stackrel{\mathsf{def}}{=} b.\bar{a}.\mathrm{Q}_2 \end{aligned}$$

3.2 Tasks

1. Draw a labeled transition system that describes the behavior of process Q.

2. (a) Are the processes P and Q strongly bisimilar?

   (b) Are the processes P and Q weakly bisimilar?

   Justify your answers: if yes, give a strong (weak) bisimulation $\mathcal{R}$ such that $\mathrm{P}\,\mathcal{R}\,\mathrm{Q}$; if no, argue why not.

3.3 Master solution

1.

$$
\begin{tikzcd}
\mathrm{Q} \arrow[r, "b"] & \mathrm{Q}' \arrow[d, bend left, "\tau"] & \mathrm{Q}'' \arrow[l, "\bar{b}"'] \\
& \mathrm{Q}''' \arrow[ul, "\bar{b}"] \arrow[u, bend left, "\tau"] \arrow[ur, "b"'] &
\end{tikzcd}
$$

$$
\text{where}\quad
\begin{aligned}
\mathrm{Q}' &\stackrel{\mathsf{def}}{=} (\mathrm{Q}_1 \mid \bar{a}.\mathrm{Q}_2) \setminus \{a\} \\
\mathrm{Q}'' &\stackrel{\mathsf{def}}{=} (\bar{b}.\mathrm{Q}_1 \mid \bar{a}.\mathrm{Q}_2) \setminus \{a\} \\
\mathrm{Q}''' &\stackrel{\mathsf{def}}{=} (\bar{b}.\mathrm{Q}_1 \mid \mathrm{Q}_2) \setminus \{a\}
\end{aligned}
$$

2. (a) The processes P and Q are not strongly bisimilar: if $(P, Q) \in \mathcal{R}$ then must also be $(P_1, Q') \in \mathcal{R}$; however, P_1 has an outgoing b transition, which cannot be matched by Q'.

   (b) The processes P and Q are weakly bisimilar: $\mathcal{R} = \{(P, Q), (P_1, Q'), (P_2, Q''), (P_1, Q''')\}$.