Assignment 12: CCS advanced concepts

ETH Zurich

1 Strong Bisimulation

Consider the following labelled transition system:

Show that $P \sim Q$ by finding a strong bisimulation $R$ such that $P R Q$.

1.1 Solution

A strong bisimulation $R$ is given by the following relation:

$$R = \{(P,Q), (P_1,Q_1), (P_2,Q_2), (P_3,Q_3), (P_4,Q_4)\}$$

2 Weak Bisimulation

Suppose we have the following definitions of processes

$$
S \overset{\text{def}}{=} a.\overline{a}.S \\
T \overset{\text{def}}{=} \pi.x.e.b.T \\
ST \overset{\text{def}}{=} (S \mid T) \setminus \{a, b\}
$$

Further we have

$$
U \overset{\text{def}}{=} e.x.y.U \\
V \overset{\text{def}}{=} \pi.y.V \\
UV \overset{\text{def}}{=} (U \mid V) \setminus \{x, y\}
$$

Your task is to

1. Represent ST and UV as LTSs.
2. Show that ST and UV are weakly bisimilar.

3. Suppose we further have \( UV' \overset{\text{def}}{=} (U \mid V) \setminus \{y\} \). Show that ST and UV' are not weakly bisimilar.

### 2.1 Solution

1. 

2. The weak bisimulation here is \( \{ST,ST_2,ST_3\} \times \{UV,UV_2,UV_3\} \). An alternative weak bisimulation relation is \( \{(UV,ST), (UV,ST_2), (UV_2,ST_3), (UV_3,ST_3)\} \).

3. This is no longer a weak bisimulation. Due to the exposure of \( x \), \( UV' \) can now make transitions that are impossible in \( ST \).

### 3 In a nutshell

#### 3.1 Background

Consider the labeled transition system describing the behavior of a process P:

\[
P \xleftrightarrow{b} b \quad P_1 \xleftrightarrow{b} b \quad P_2
\]

Furthermore, consider the CCS process Q defined by the following equations:

\[
Q \overset{\text{def}}{=} (Q_1 \mid Q_2) \setminus \{a\}
\]

\[
Q_1 \overset{\text{def}}{=} a.b.Q_1
\]

\[
Q_2 \overset{\text{def}}{=} b.a.Q_2
\]

#### 3.2 Tasks

1. Draw a labeled transition system that describes the behavior of process Q.

2. (a) Are the processes P and Q strongly bisimilar?

   (b) Are the processes P and Q weakly bisimilar?

Justify your answers: if yes, give a strong (weak) bisimulation \( R \) such that \( P R Q \); if no, argue why not.
3.3 Master solution

1. \[
\begin{align*}
Q \xrightarrow{b} Q' & \xleftarrow{b} Q'' \\
Q'' \xrightarrow{\tau} Q' & \xleftarrow{\tau} Q''
\end{align*}
\]

\[
Q' \overset{\text{def}}{=} (Q_1 \mid \bar{a}.Q_2) \setminus \{a\}
\]

\[
Q'' \overset{\text{def}}{=} (\bar{b}.Q_1 \mid \bar{a}.Q_2) \setminus \{a\}
\]

\[
Q''' \overset{\text{def}}{=} (\bar{b}.Q_1 \mid Q_2) \setminus \{a\}
\]

2. (a) The processes P and Q are not strongly bisimilar: if \((P, Q) \in R\) then must also be \((P_1, Q') \in R\); however, \(P_1\) has an outgoing \(b\) transition, which cannot be matched by \(Q'\).

(b) The processes P and Q are weakly bisimilar: \(R = \{(P, Q), (P_1, Q'), (P_2, Q''), (P_1, Q''')\}\).