ETH Zürich
Chair of Software Engineering

Concepts of Concurrent Computation

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Lecture 3: Synchronization Algorithms

Today's lecture

In this lecture you will learn about:

- the mutual exclusion problem, a common framework for evaluating solutions to the problem of exclusive resource access
- solutions to the mutual exclusion problem (Peterson's algorithm, the Bakery algorithm) and their properties
- ways of proving properties for concurrent programs

The mutual exclusion problem

Mutual exclusion

- As discussed in the last lecture, race conditions can corrupt the result of a concurrent computation if processes are not properly synchronized
- We want to develop techniques for ensuring mutual exclusion
- *Mutual exclusion*: a form of synchronization to avoid the simultaneous use of a shared resource
- To identify the program parts that need attention, we introduce the notion of a critical section
- *Critical section*: part of a program that accesses a shared resource.

The mutual exclusion problem (1)

- We assume to have *n* processes of the following form:

```
while true loop
    entry protocol
    critical section
    exit protocol
    non-critical section
end
```

- Design the entry and exit protocols to ensure:
  - *Mutual exclusion*: At any time, at most one process may be in its critical section
  - *Freedom from deadlock*: If two or more processes are trying to enter their critical sections, one of them will eventually succeed
  - *Freedom from starvation*: If a process is trying to enter its critical section, it will eventually succeed

The mutual exclusion problem (2)

```
while true loop
    entry protocol
    critical section
    exit protocol
    non-critical section
end
```

- Further important conditions:
  - Processes can communicate with each other only via atomic read and write operations
  - If a process enters its critical section, it will eventually exit from it
  - A process may loop forever or terminate while being in its non-critical section
  - The memory locations accessed by the protocols may not be accessed outside of them

Locks

- Synchronization mechanisms based on the ideas of entry- and exit-protocols are called *locks*
- They can typically be implemented as a pair of functions:

```
lock
    do
        entry protocol
    end

unlock
    do
        exit protocol
    end
```

Busy waiting

- We will use the following statement in pseudo code

    await b

which is equivalent to

    while not b **loop end**

- This type of waiting is called *busy waiting* or "*spinning*"
- Busy waiting is inefficient on multitasking systems
- Busy waiting makes sense if waiting times are typically so short that a context switch would be more expensive
- Therefore spin locks (locks using busy waiting) are often used in operating system kernels

Towards a solution

- The mutual exclusion problem is quite tricky: in the 1960's many incorrect solutions were published
- We will work along a series of failing attempts until establishing a solution
- We will start with trying to find a solution for only two processes

Solution attempt I

- **First idea:** use two variables enter1 and enter2; if enter*i* is true, it means that process P_i intends to enter the critical section

enter1 := **false** enter2 := **false**	
P1	P2

while true loop
    await not enter2
    enter1 := **true**
    critical section
    enter1 := **false**
    non-critical section
end
	while true loop
	    await not enter1
	    enter2 := **true**
	    critical section
	    enter2 := **false**
	    non-critical section
	end

Solution attempt I is incorrect

- The solution attempt fails to ensure mutual exclusion
- The two processes can end up in their critical sections at the same time, as demonstrated by the following execution sequence

P2	1	**await not** enter1
P1	1	**await not** enter2
P1	2	enter1 := **true**
P2	2	enter2 := **true**
P2	3	critical section
P1	3	critical section

Solution attempt II

- When analyzing the failure, we see that we set the variable enter*i* only after the await statement, which is guarding the critical section
- **Second idea:** switch these statements around

enter1 := **false** enter2 := **false**			
P1		P2	
	while true loop		**while true loop**
1	enter1 := **true**	1	enter2 := **true**
2	**await not** enter2	2	**await not** enter1
3	critical section	3	critical section
4	enter1 := **false**	4	enter2 := **false**
5	non-critical section	5	non-critical section
	end		**end**

Solution attempt II is incorrect

- The solution provides mutual exclusion
- However, the processes can deadlock:

P1	1	enter1 := **true**
P2	1	enter2 := **true**
P2	2	**await not** enter1
P1	2	**await not** enter2

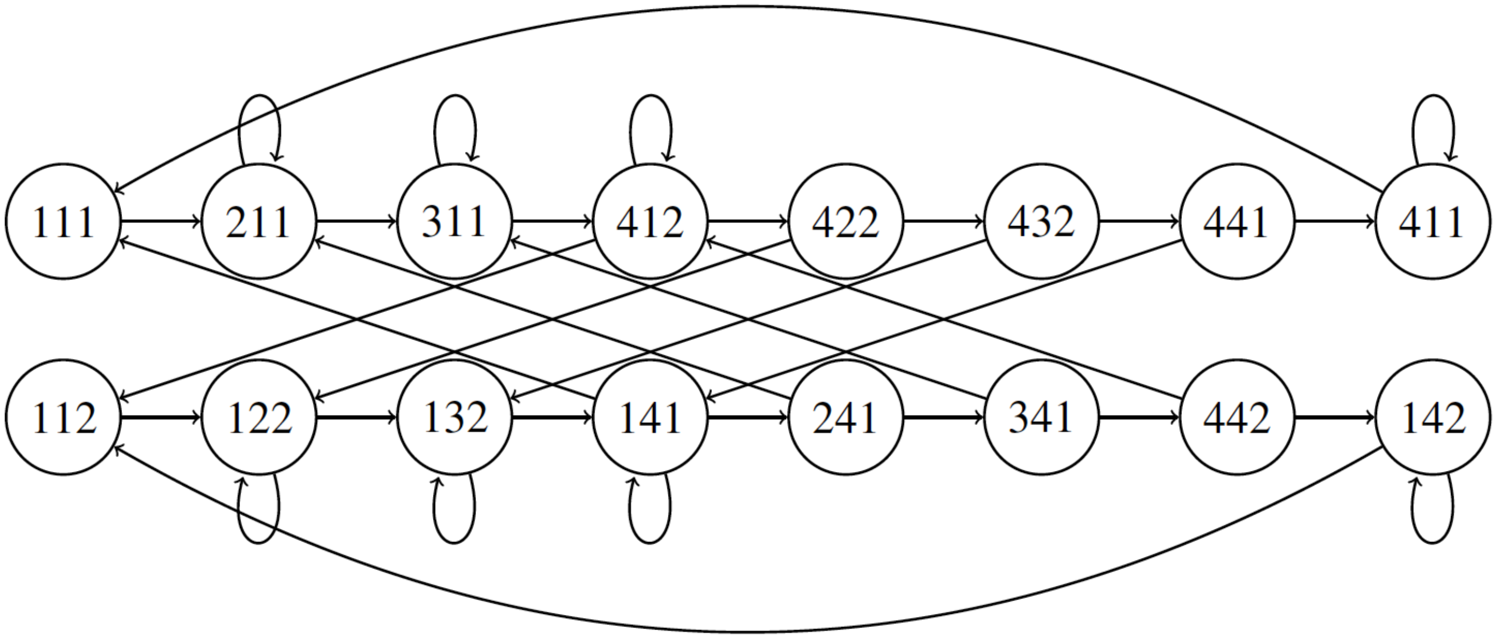
Solution attempt III

- **Third idea:** let's try something new, namely a single variable turn that has value *i* if it's P_i's turn to enter the critical section

<table>
<tr><td colspan="4">turn := 1 or turn := 2</td></tr>
<tr><td colspan="2">P1</td><td colspan="2">P2</td></tr>
<tr><td>
1
2
3
4
</td><td>while true loop
    await turn = 1
    critical section
    turn := 2
    non-critical section
end</td><td>
1
2
3
4
</td><td>while true loop
    await turn = 2
    critical section
    turn := 1
    non-critical section
end</td></tr>
</table>

Proving correctness of solution attempt III

- Solution attempt III looks good to us, let's try to prove it correct
- Draw the related transition system; states are labeled with triples (i, j, k): program pointer values P1▹i and P2▹j, and value of the variable turn = k.



Proving correctness of solution attempt III

- *Solution attempt III satisfies mutual exclusion*

Proof. Mutual exclusion expressed as LTL formula:

$$\mathbf{G}\ \neg(P1 \triangleright 2 \wedge P2 \triangleright 2)$$

Easy to see that this formula holds, as there are no states of the form (2, 2, k).

- *Solution attempt III is deadlock-free*

Proof. Deadlock-freedom expressed as LTL formula:

$$\mathbf{G}\ ((P1 \triangleright 1 \wedge P2 \triangleright 1) \to \mathbf{F}\ (P1 \triangleright 2 \vee P2 \triangleright 2))$$

We have to examine the states (1, 1, 1) and (1, 1, 2); in both cases, one of the processes is enabled to enter its critical section.

Another setback

- Let's check starvation-freedom
- Expressed as LTL formula: for i = 1, 2

$$\mathbf{G}\ (P_i \triangleright 1 \rightarrow \mathbf{F}\ (P_i \triangleright 2))$$

- Recall: processes may terminate in non-critical section
- A problematic case is (1, 4, 2): variable turn = 2, P1 trying to enter critical section (although not its turn), P2 in non-critical section
- If P2 terminates, turn will never be set to 1: P1 will starve

Peterson's algorithm

Peterson's algorithm (for two processes)

- Peterson's algorithm combines the ideas of solution attempts II and III
- If both processes have set their enter-flag to true, then the value of turn decides who may enter the critical section

```
enter1 := false
enter2 := false
turn := 1 or turn := 2
```

P1

```
while true loop
    enter1 := true
    turn := 2
    await not enter2 or turn = 1
    critical section
    enter1 := false
    non-critical section
end
```

P2

```
while true loop
    enter2 := true
    turn := 1
    await not enter1 or turn = 2
    critical section
    enter2 := false
    non-critical section
end
```

Peterson's algorithm: mutual exclusion

- *Peterson's algorithm satisfies mutual exclusion*

Proof.

- Assume that both P1 and P2 are in their critical section and that P1 entered before P2
- When P1 entered the critical section we have enter1 = true, and P2 must thus have seen turn = 2 upon entering its critical section
- P2 could not have executed line 2 after P1 entered, as this sets turn = 1 and would have excluded P2, as P1 does not change turn while being in the critical section
- However, P2 could not have executed line 2 before P1 entered either because then P1 would have seen enter2 = true and turn = 1, although P2 should have seen turn = 2
- Contradiction

Peterson's algorithm: starvation-freedom

- *Peterson's algorithm is starvation-free*

Proof.

- Assume P1 is forced to wait in the entry protocol forever
- P2 can eventually do only one of three actions:
    1. Be in its non-critical section: then enter2 is false, thus allowing P1 to enter.
    2. Wait forever in its entry protocol: impossible because turn cannot be both 1 and 2
    3. Repeatedly cycle through its code: then P2 will set turn to 1 at some point and never change it back

Peterson's algorithm for *n* processes

- Up until now, we have only seen a solution to the mutual exclusion problem for two processes; the problem is however posed for n processes
- Peterson's algorithm has a direct generalization

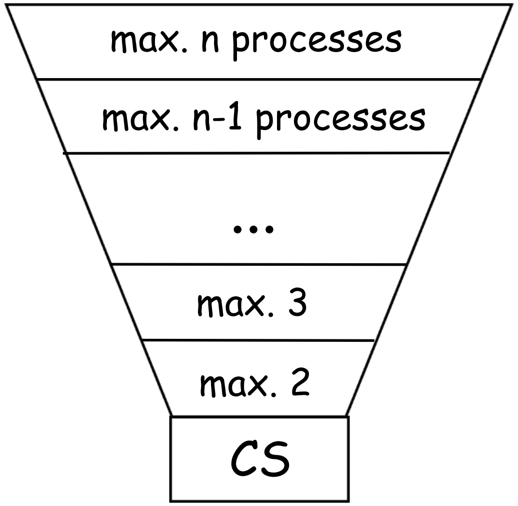
```
enter[1] := 0; ...; enter[n] := 0
turn[1] := 0; ...; turn[n – 1] := 0

P_i

for j = 1 to n – 1 do
    enter[i] := j
    turn[j] := i
    await (for all k != i : enter[k] < j) or turn[j] != i
end
critical section
enter[i] := 0
non-critical section
```

Peterson's algorithm for *n* processes

- Every process has to go through n – 1 stages to reach the critical section: variable j indicates the stage
- enter[i]: stage the process P_i is currently in
- turn[j]: which process entered stage j last
- Waiting: P_i waits if there are still processes at higher stages, or if there are processes at the same stage unless P_i is no longer the last process to have entered this stage
- Idea for mutual exclusion proof:
at most n – j processes can have passed stage j =>
at most n – (n - 1) = 1 processes can be in the critical section



The Bakery algorithm

Fairness again

- Freedom from starvation still allows that processes may enter their critical sections before a certain, already waiting process is allowed access
- We study an algorithm that has very strong fairness guarantees

Bounded waiting

- The following definitions help analyze the fairness with respect to process waiting in mutual exclusion algorithms
- *Bounded waiting*: If a process is trying to enter its critical section, then there is a bound on the number of times any other process can enter its critical section before the given process does so.
- *r-bounded waiting*: If a process tries to enter its critical section then it will be able to enter before any other process is able to enter its critical section $r + 1$ times.
- This means: bounded waiting = there exists an r such that the waiting is r-bounded
- *First-come-first-served*: 0-bounded waiting

Relating the definitions

- starvation-freedom $\Rightarrow$ deadlock-freedom
- starvation-freedom $\not\Rightarrow$ bounded waiting
- bounded waiting $\not\Rightarrow$ starvation-freedom
- bounded waiting + deadlock-freedom
  $\Rightarrow$ starvation-freedom

deadlock-freedom If two or more processes are trying to enter their critical sections, one of them will eventually succeed.

starvation-freedom If a process is trying to enter its critical section, it will eventually succeed.

bounded waiting If a process is trying to enter its critical section, then there is a bound on the number of times any other process can enter its critical section before the given process does so.

Peterson's algorithm: no bounded waiting

- Assume a scenario with three competing processes

P1	2	enter[1] := 1	
P2	2	enter[2] := 1	
P2	3	turn[1] := 2	
P3	2	enter[3] := 1	
P3	3	turn[1] := 3	turn[1] != 2: P2 can proceed
P2	...	enters + leaves critical section	
P2	2	enter[2] := 1	
P2	3	turn[1] := 2	turn[1] != 3: P3 can proceed
P3	...	enters + leaves critical section	
	...		P3 can unblock P2 etc.

- P2 and P3 can overtake P1 unboundedly often
- Still P1 is not starved as it eventually (fairness) executes turn[1] := 1 and can proceed into the critical section

The Bakery algorithm: first attempt

- **Idea:** ticket systems for customers, at any turn the customer with the lowest number will be served
- number[i]: ticket number drawn by a process P_i
- Waiting: until P_i has the lowest number currently drawn

number[1] := 0; ...; number[n] := 0
P_i

```
number[i] := 1 + max(number[1], ..., number[n])
for all j != i do
    await number[j] = 0 or number[i] < number[j]
end
critical section
number[i] := 0
non-critical section
```

- Where is the problem?

Problem with the first attempt

- Line 1 may not be executed atomically
- Hence two processes may get the same ticket number
- Then a deadlock can happen in line 3, as none of the processes' ticket numbers is less than the other

A suggestion for a fix

- Replace the comparison number[i] < number[j] by (number[i], i) < (number[j], j)
- The "less than" relation is defined in this case as

$$(a, b) < (c, d) \quad \text{if} \quad (a < c) \text{ or } ((a = c) \text{ and } (b < d))$$

- **Idea:** if two ticket numbers turn out to be the same, the process with the lower identifier gets precedence

The fix doesn't work

- Unfortunately, with the fix we no longer have mutual exclusion:
  - P1 and P2 both compute the current maximum as 0
  - P2 assigns itself ticket number 1 (number[2] := 1) and proceeds into critical section
  - P1 assigns itself ticket number 1 (number[1] := 1) and proceeds into critical section, because

    (number[1], 1) < (number[2], 2)

The bakery algorithm

- Finally, we indicate with a flag if a process is currently calculating its ticket number

```
number[1] := 0; ...; number[n] := 0
choosing[1] := false, ..., choosing[n] := false

P_i

choosing[i] := true
number[i] := 1 + max(number[1], ..., number[n])                    doorway
choosing[i] := false
for all j != i do
    await choosing[j] = false                                      bakery
    await number[j] = 0 or (number[i], i) < (number[j], j)
end
critical section
number[i] := 0
non-critical section
```

Two lemmas

Lemma 1. *If processes P_i and P_k are in the bakery and P_i entered the bakery before P_k entered the doorway, then number[i] < number[k].*

Lemma 2. *If process P_i is in its critical section and process P_k is in the bakery then (number[i], i) < (number[k], k).*

For P_i choosing[k] = false when reading it in line 5

If we have the situation of Lemma 1, we are finished.

If P_k had left the doorway before P_i read number[k], it was reading its current value.

Since process P_i went on into the critical section, it must have found (number[i], i) < (number[k], k).

Correctness of the bakery algorithm

- *The Bakery algorithm satisfies mutual exclusion.*

Proof. Follows from Lemma 2.

- *The Bakery algorithm is deadlock-free.*

Proof. Some waiting process P_i has the minimum value of (number[i], i) among all the processes in the bakery. This process must eventually complete the for loop and enter the critical section.

- *The Bakery algorithm is first-come-first-served.*

Proof. Follows from Lemmas 1 and 2.

Unbounded ticket numbers

- *Drawback of the Bakery algorithm*: values of the ticket numbers can grow unboundedly
    - Assume P1 gets ticket number 1 and proceeds to its critical section.
    - Then process P2 gets ticket number 2, lets P1 exit from its critical section and enters its own critical section.
    - As P1 tries to re-enter its critical section it draws ticket number 3.
    - In this manner two processes could alternatingly draw ticket numbers until the maximum size of an integer on the system is reached.

Space bounds for synchronization algorithms

- Size and number of shared memory locations is an important measure to compare synchronization algorithms
- For Peterson's algorithm, we count 2n – 1 registers (bounded by n), and in the case of the Bakery algorithm 2n registers (unbounded in size)
- Large overhead: can we do better?
- One can prove in general a lower bound: mutual exclusion problem for n processes satisfying mutual exclusion and global progress needs to use n shared one-bit registers
- The bound is tight (Lamport's one bit algorithm)

Non-atomic memory access

- The mutual exclusion problem makes the assumption that memory accesses are executed atomically
- This might not be a valid assumption on multiprocessor systems, leading to inconsistencies
- The Bakery algorithm can help here as well: each memory location is only written by a single process, hence conflicting write operations cannot occur

Other atomic primitives (1)

- Having only atomic read and write to implement locks makes efficient implementation difficult
- Where available, locks can be built from more complex atomic primitives

```
test-and-set (x, value)
    do
        temp := *x
        *x := value
        result := temp
    end
```

- Note that x in this pseudo-code is treated as a reference

Other atomic primitives (2)

- Using more powerful primitives, concise solutions to the mutual exclusion problem can be obtained:

b := false	
P_i	
1	**await not** test-and-set(b, true)
2	critical section
3	b := false
4	non-critical section

Other atomic primitives (3)

```
fetch-and-add (x, value)
   do
      temp := *x
      *x := *x + value
      result := temp
   end
compare-and-swap (x, old, new)
   do
      if *x = old then
         *x := new; result := true
      else
         result := false
      end
   end
```