Concepts of Concurrent Computation
Spring 2014
Lecture 11: Petri Nets

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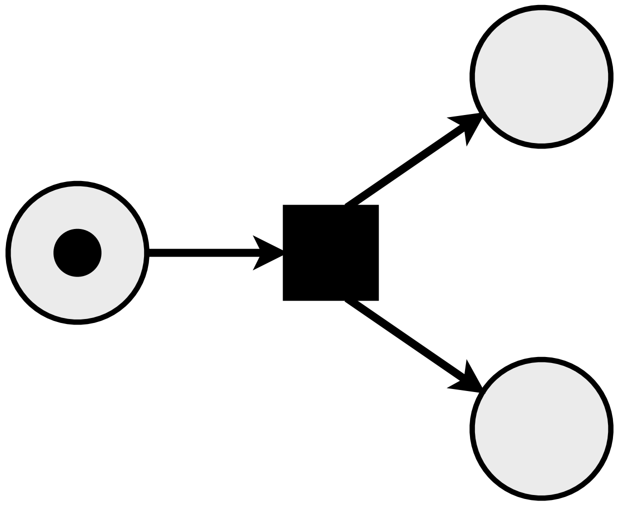
Petri nets

- Petri nets are mathematical models for describing systems with concurrency and resource sharing
- they facilitate many automatic analyses of interest for concurrent systems
- rich, intuitive graphical notation for choice, concurrent execution, interaction with the environment, ...



Petri nets - the origins

- proposed by Carl Adam Petri in his famous thesis *Kommunikation mit Automaten* (1962)

- aimed for a system architecture that could be expanded indefinitely
  => no central components
  => in particular, no central, synchronising clock
  => actions with *locally confined* causes/effects

- original presentation omitted the graphical representation

Next on the agenda

1. modelling concepts: *cookies for everyone!*
2. synchronisation problems as Petri nets
3. Petri net analyses
4. true concurrency semantics; unfoldings

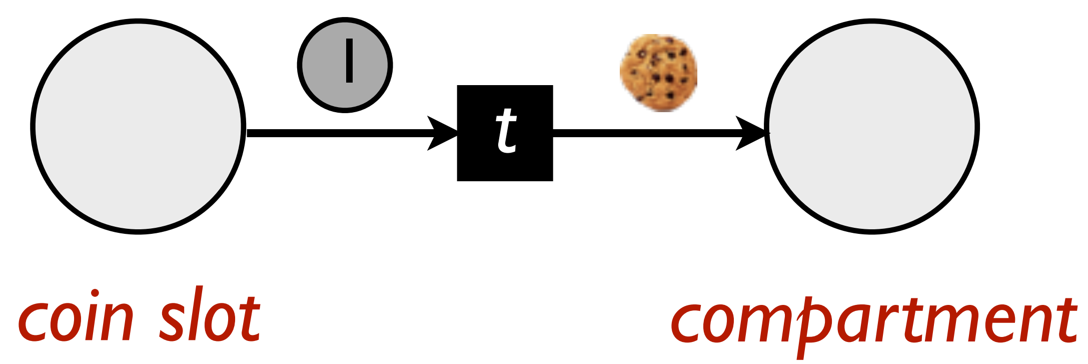
Let's design a cookie vending machine

coin slot

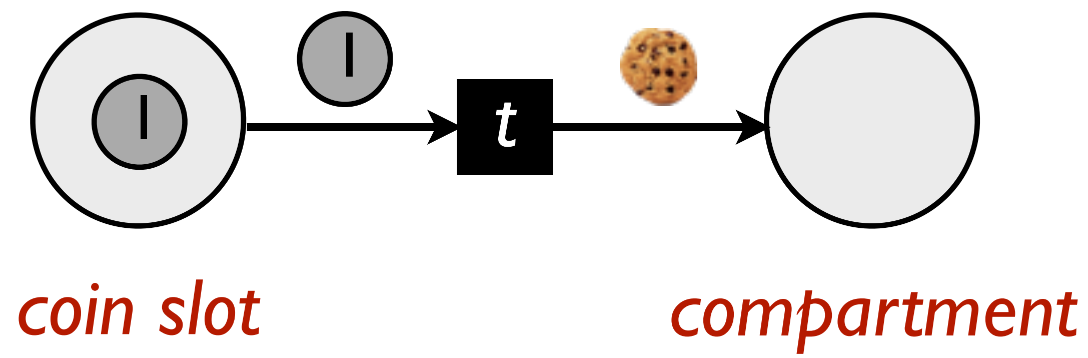
compartment

Let’s design a cookie vending machine

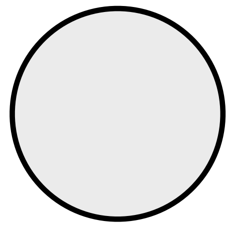
Let's design a cookie vending machine



Let's design a cookie vending machine

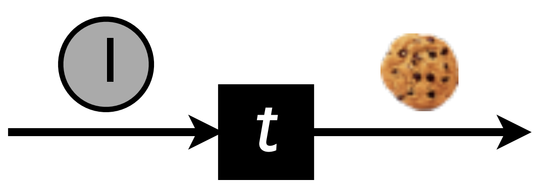


Terminology

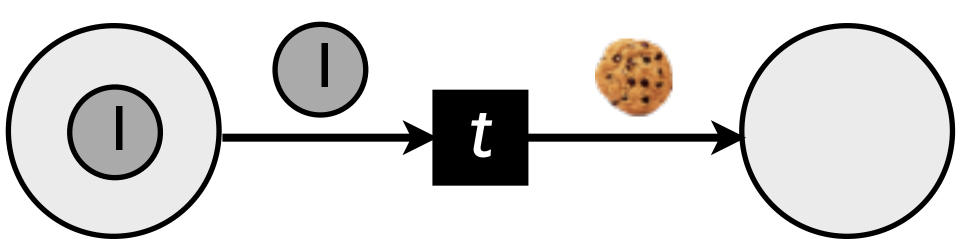


place

tokens

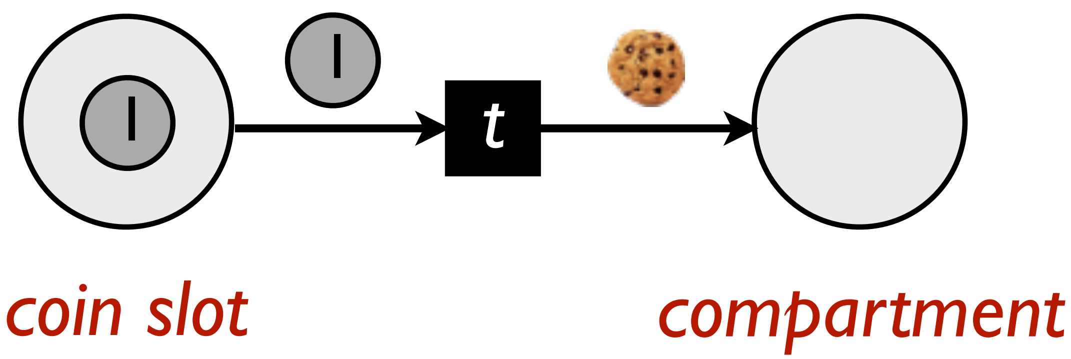


transition (with precondition I *)*



marking (distribution of tokens)

Let's design a cookie vending machine



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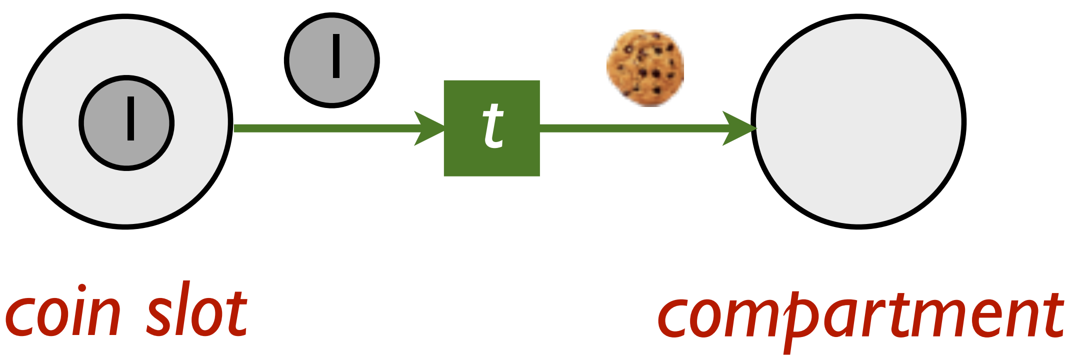

transition t is <u>enabled</u>
it can <u>occur</u> and change the <u>marking</u>

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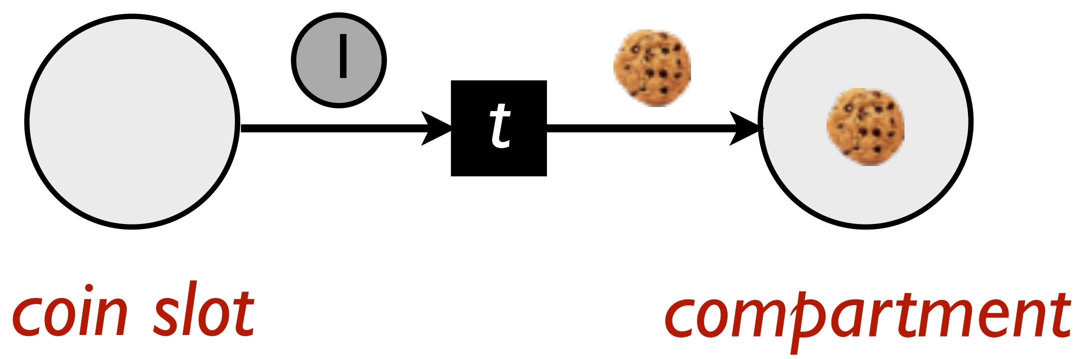

transition t is <u>enabled</u>
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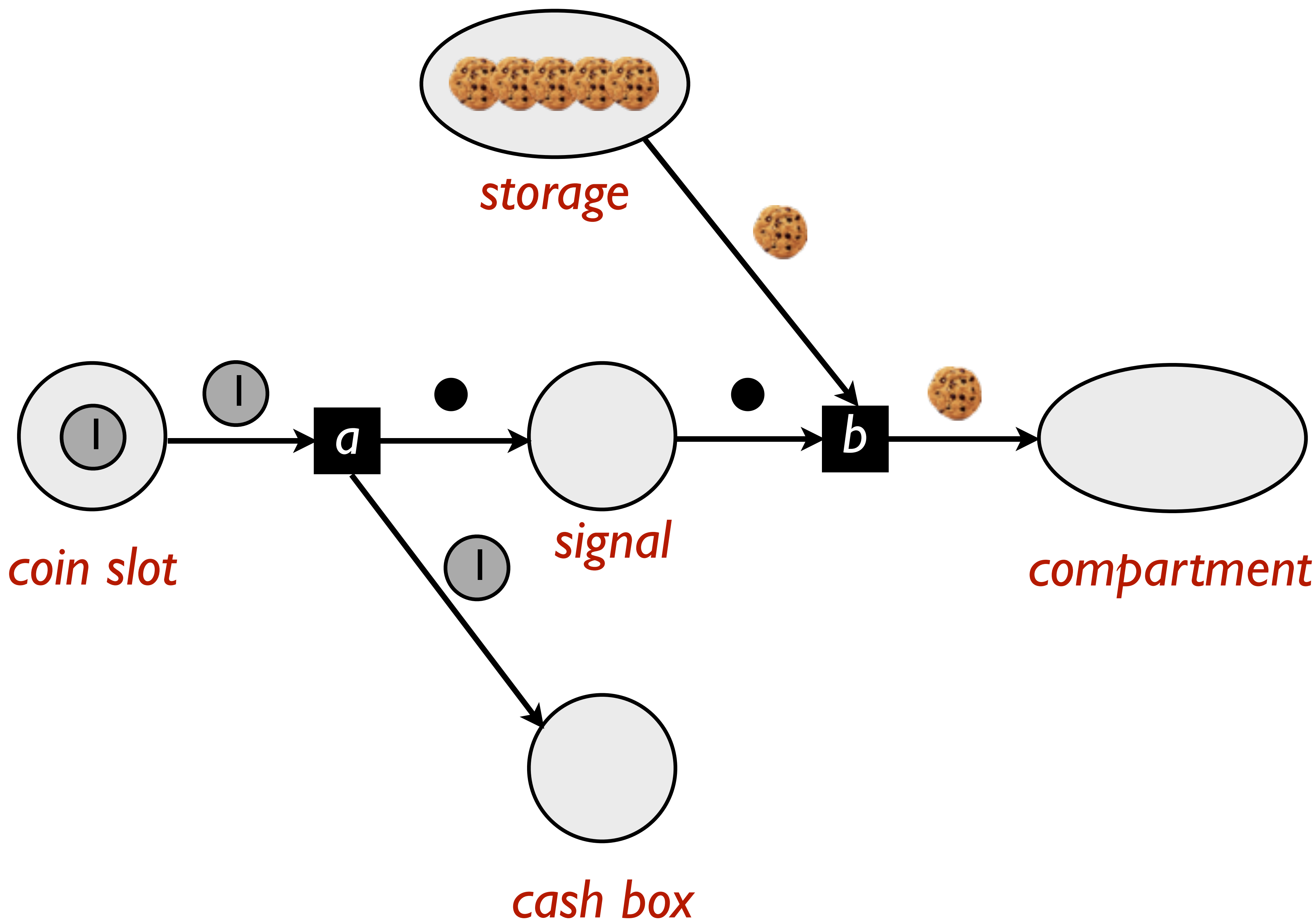
Let's look inside

cash box?
finitely many cookies?

Let's look inside



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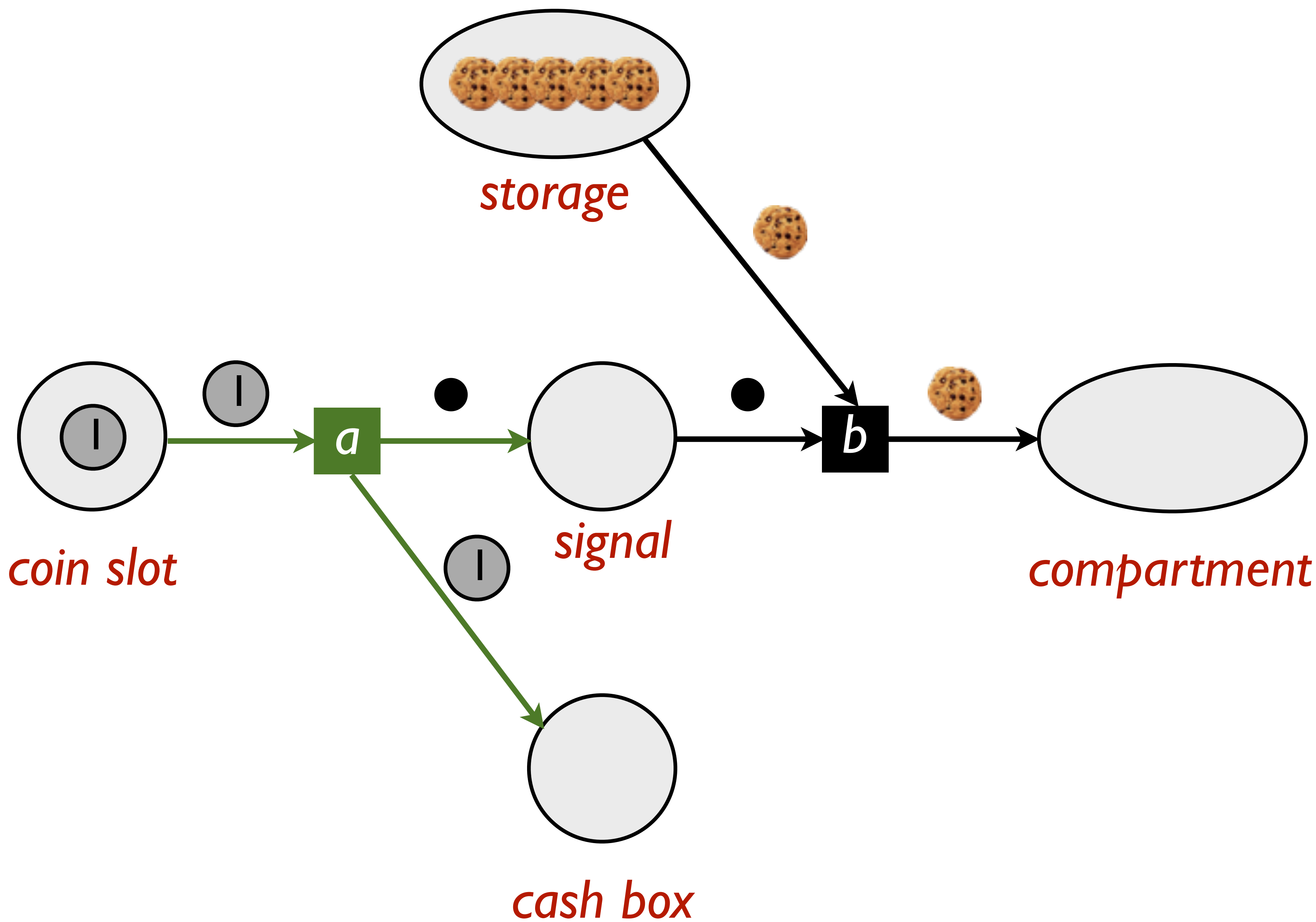

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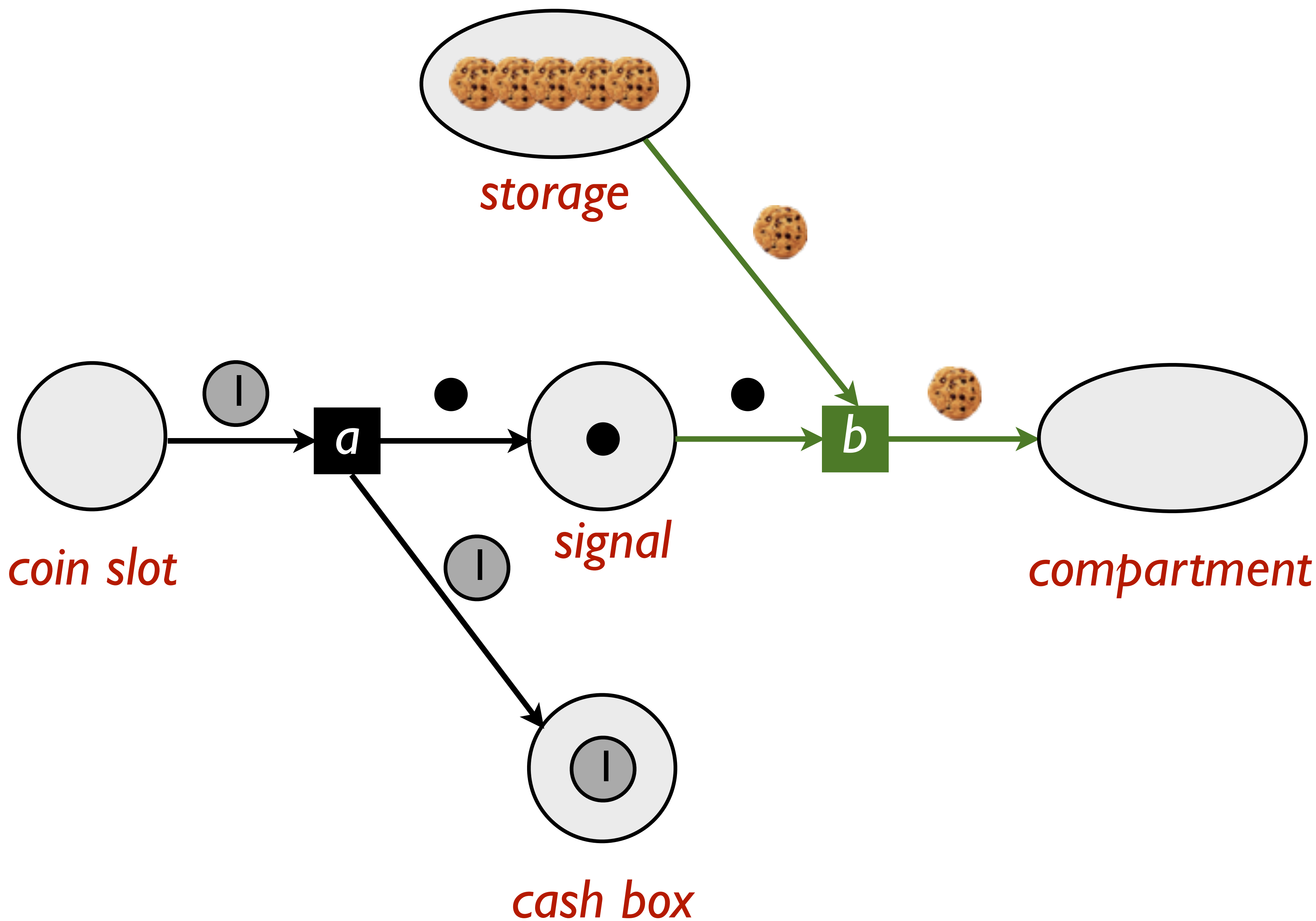

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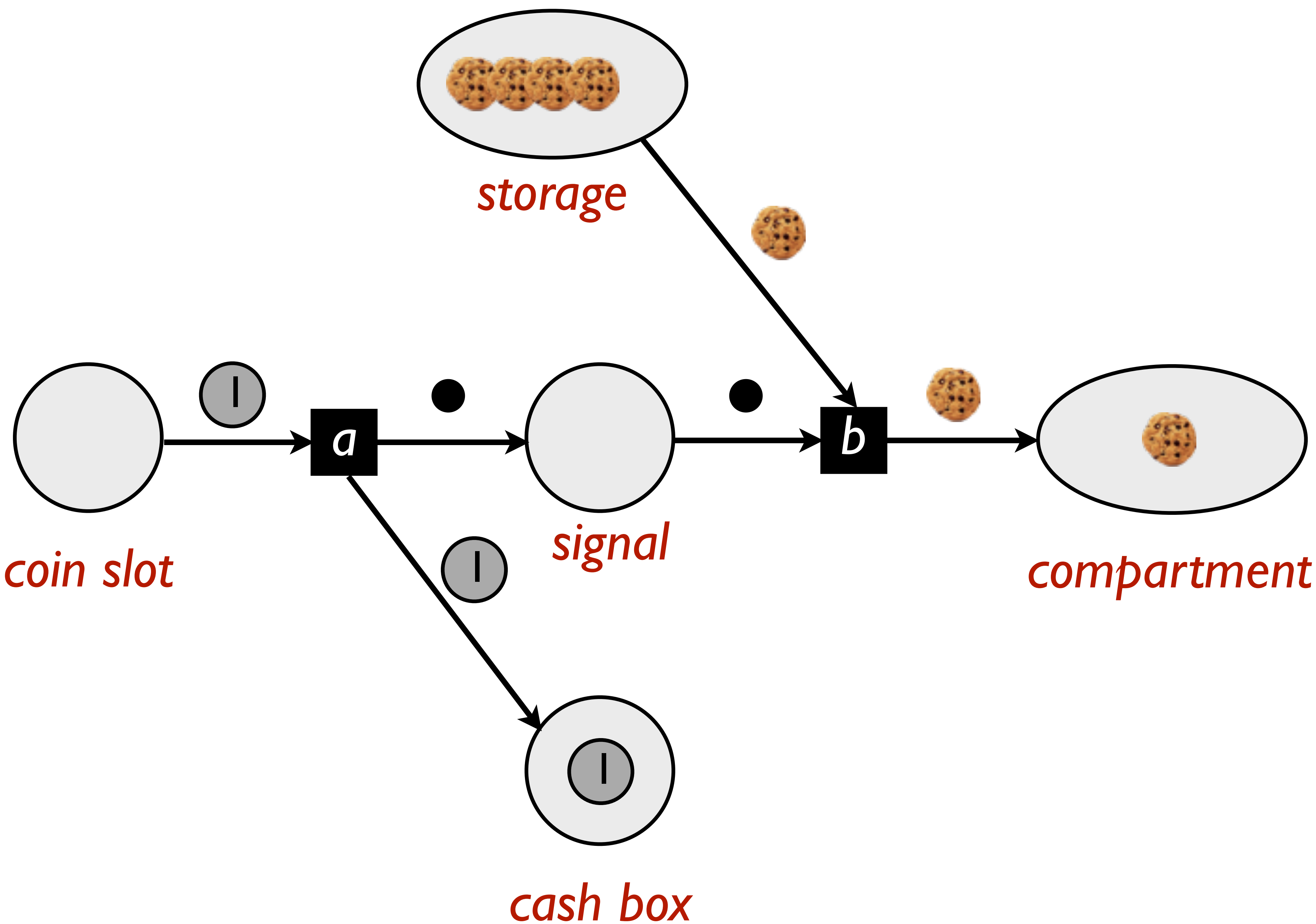
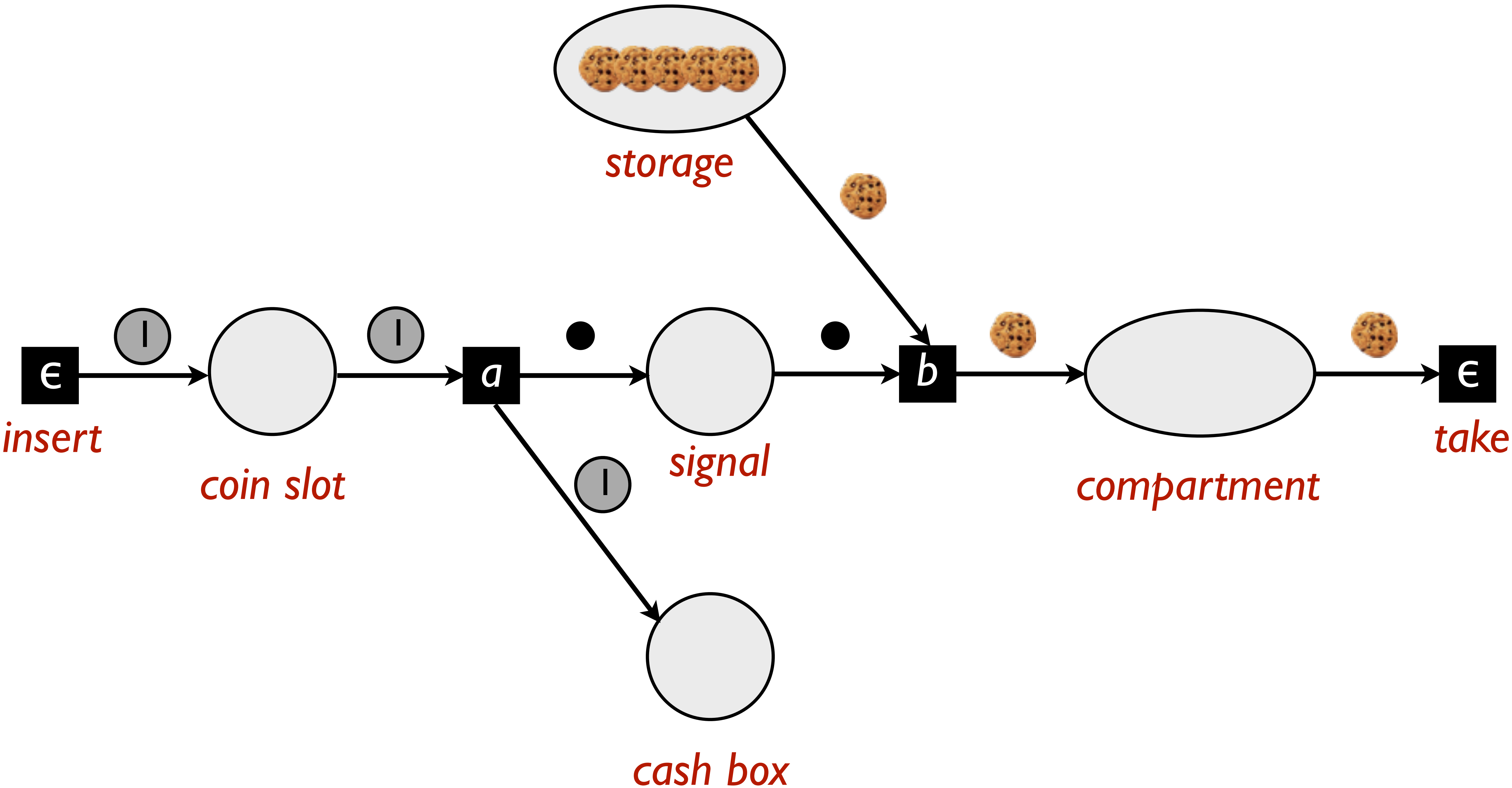
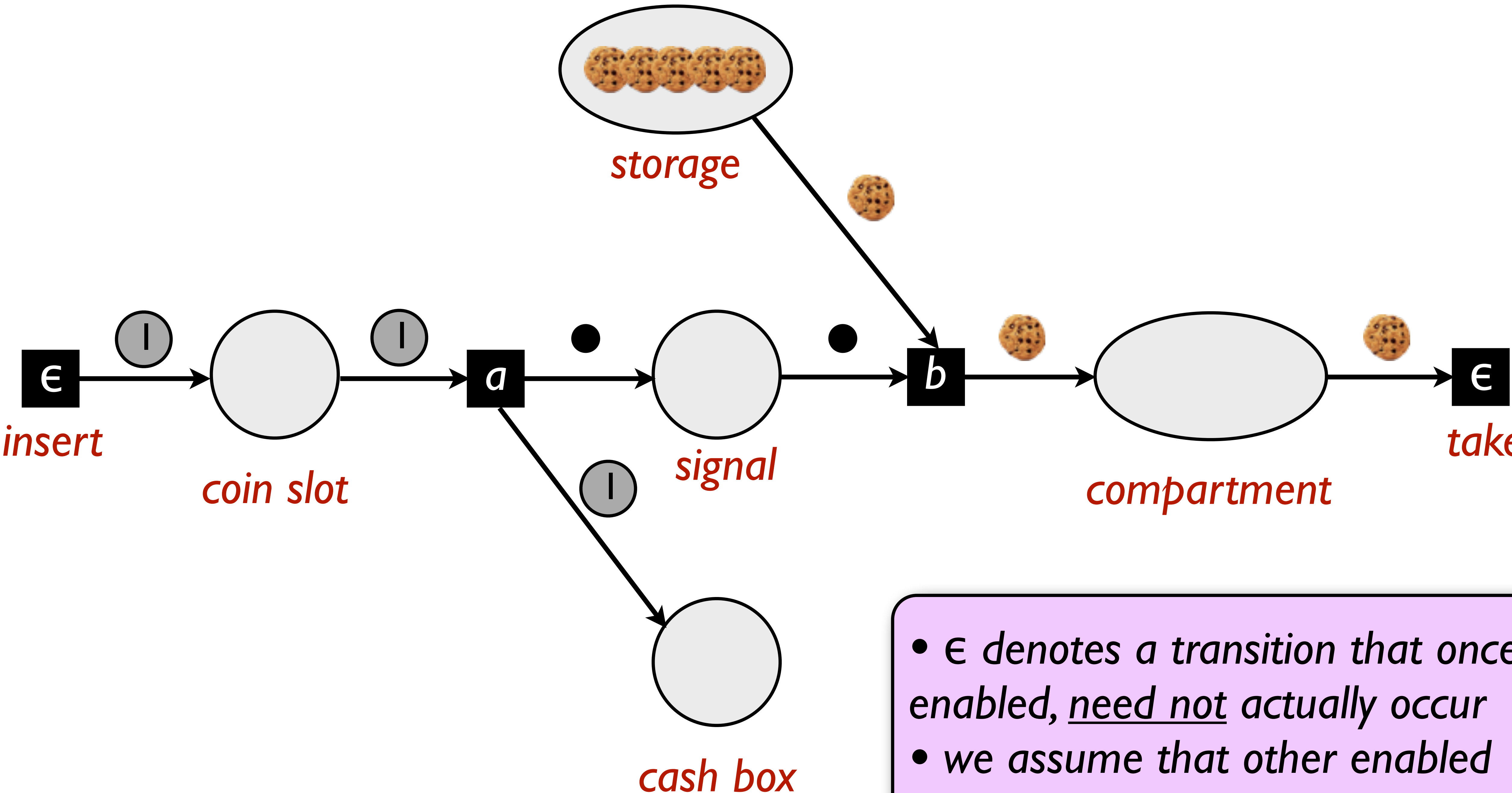


Let's open it up to the world

Let's open it up to the world

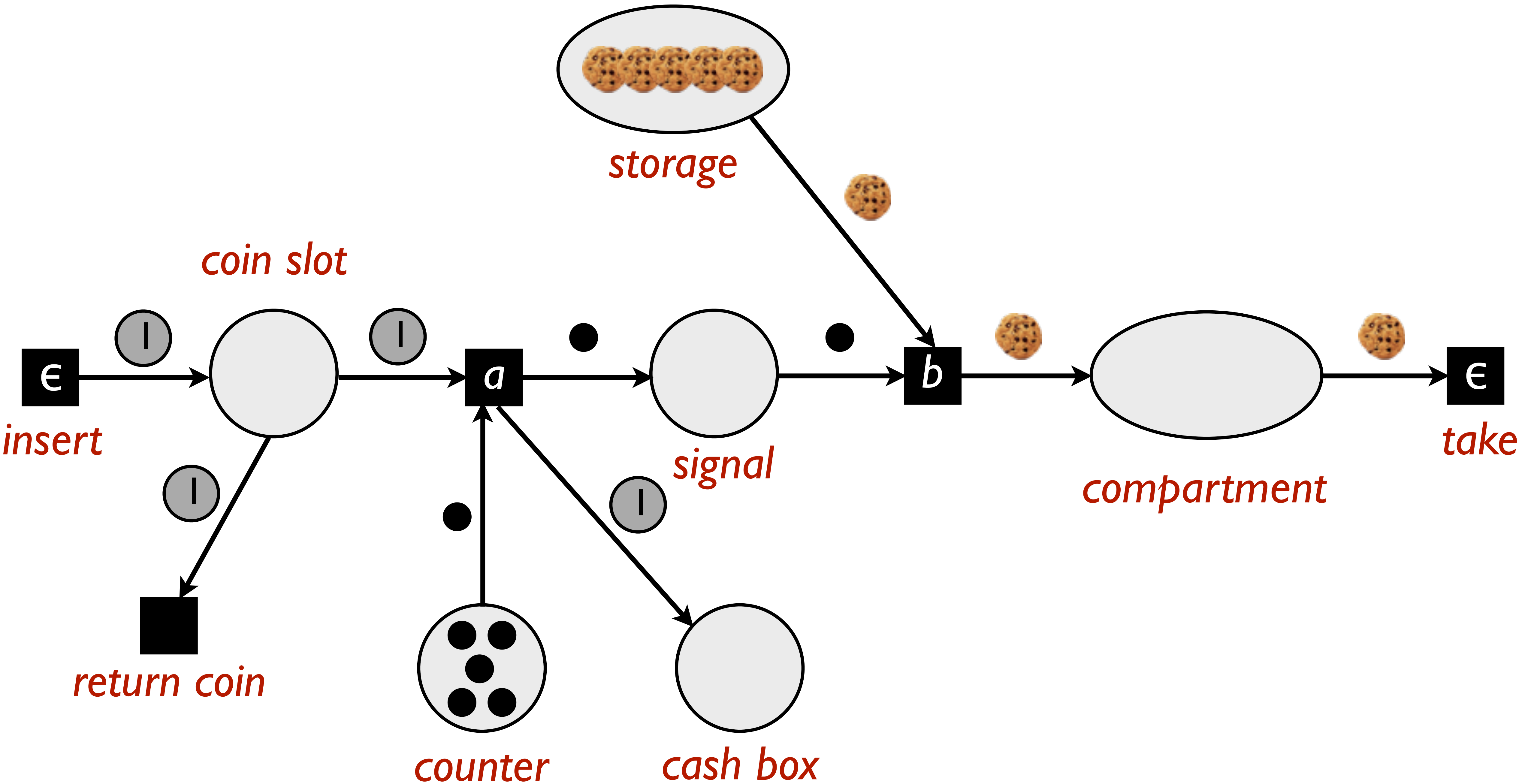


Let's open it up to the world



- ϵ denotes a transition that once enabled, <u>need not</u> actually occur
- we assume that other enabled transitions occur eventually

The ultimate cookie machine (design)



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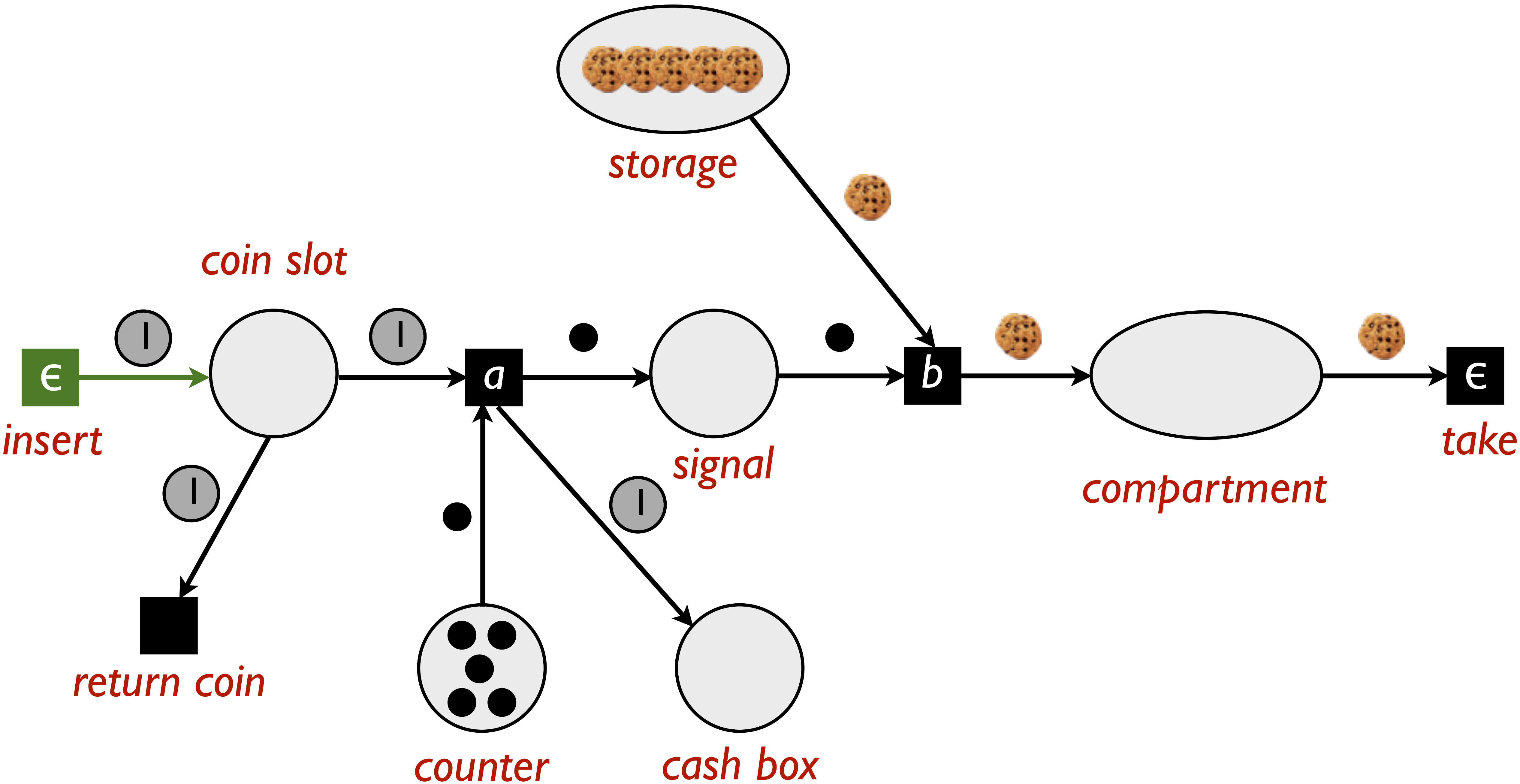

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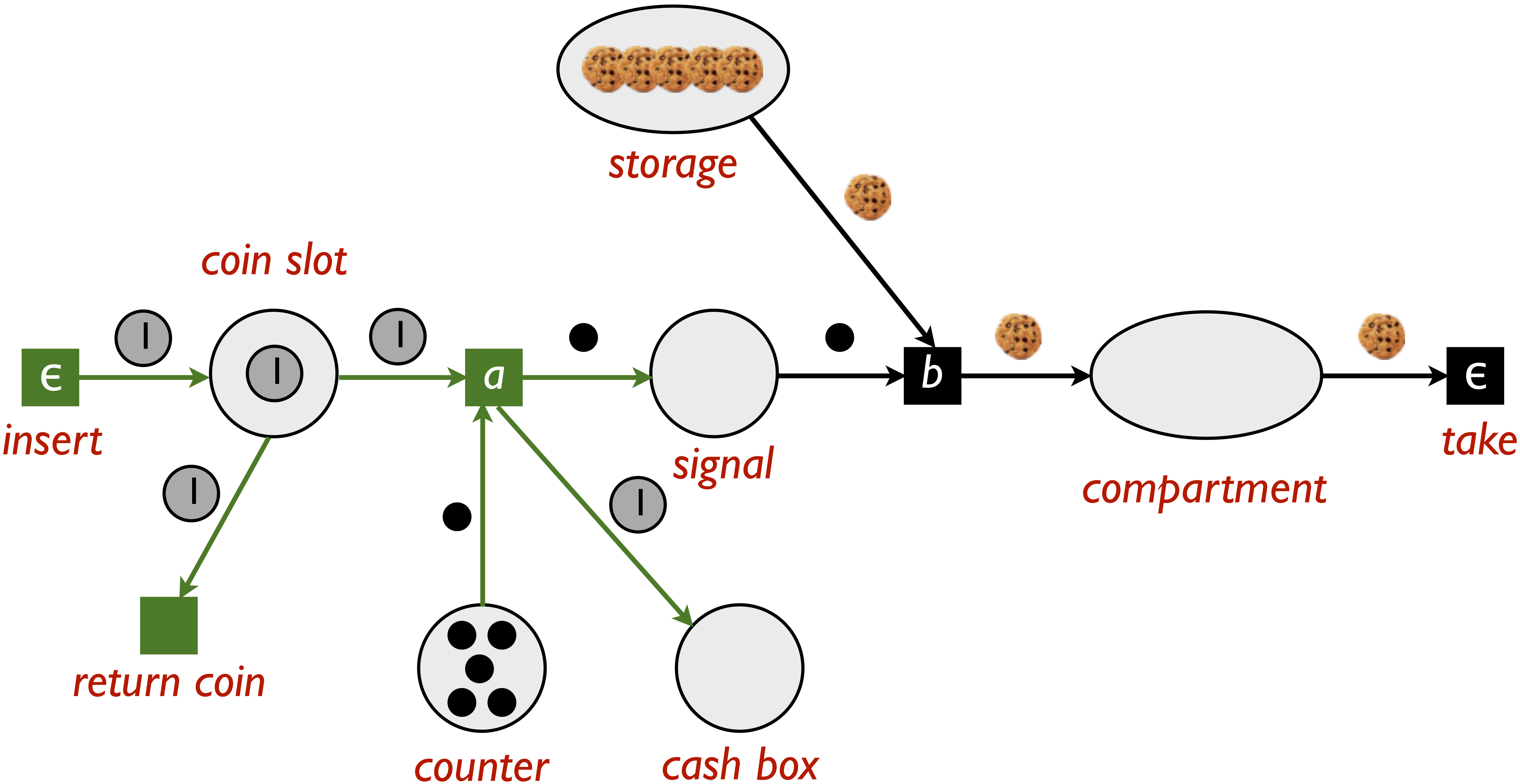

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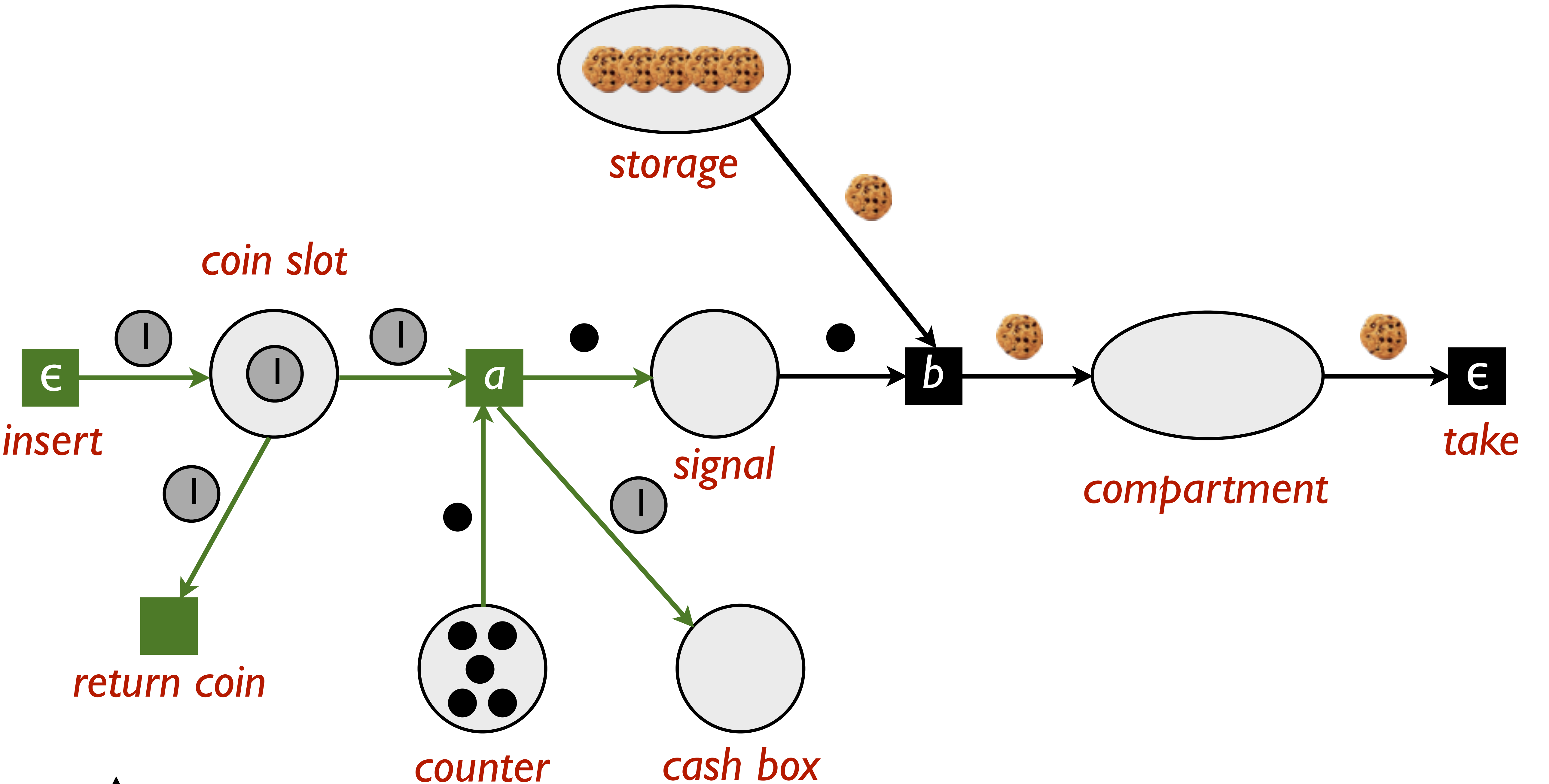
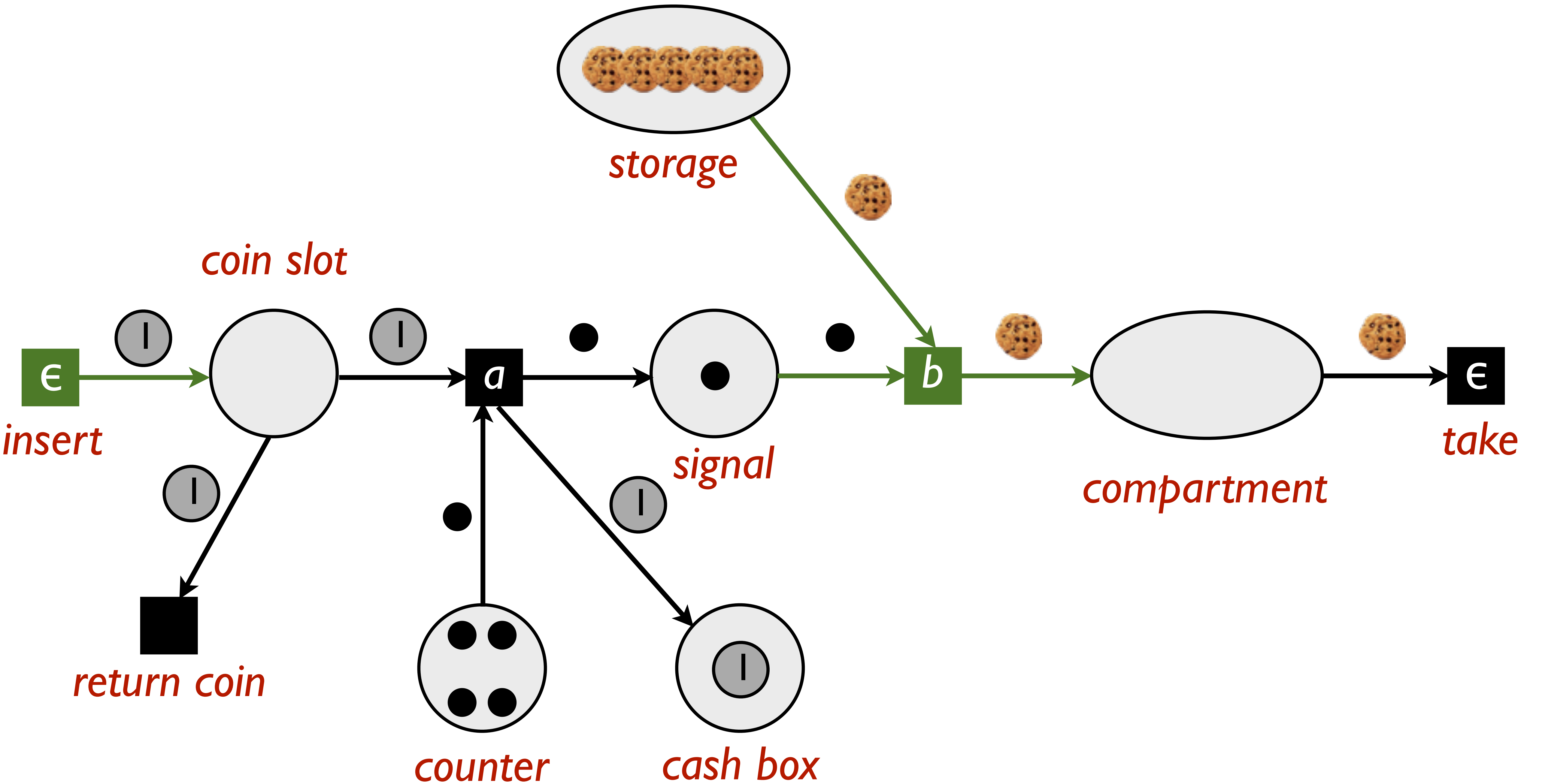
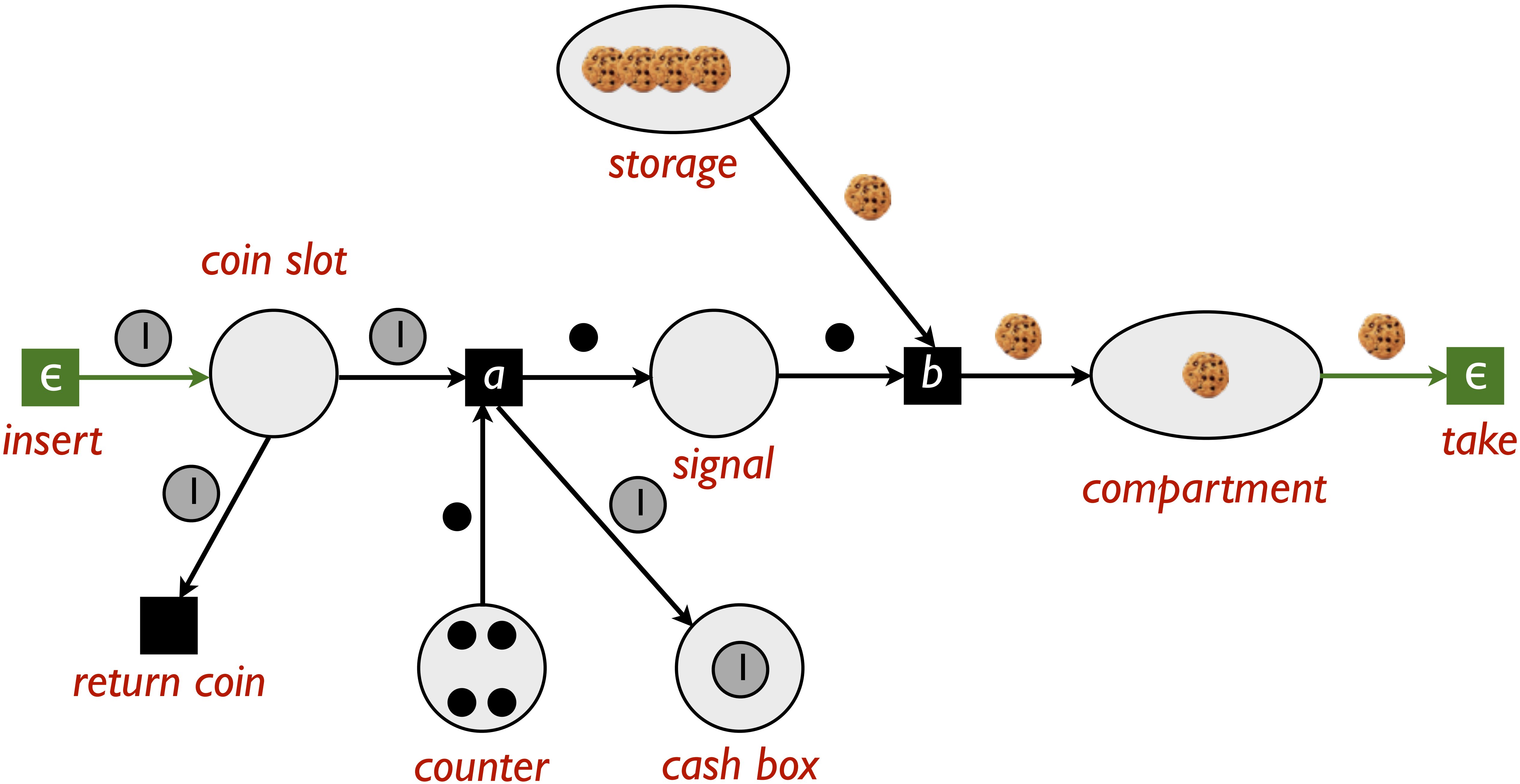


conflict! nondeterminism!

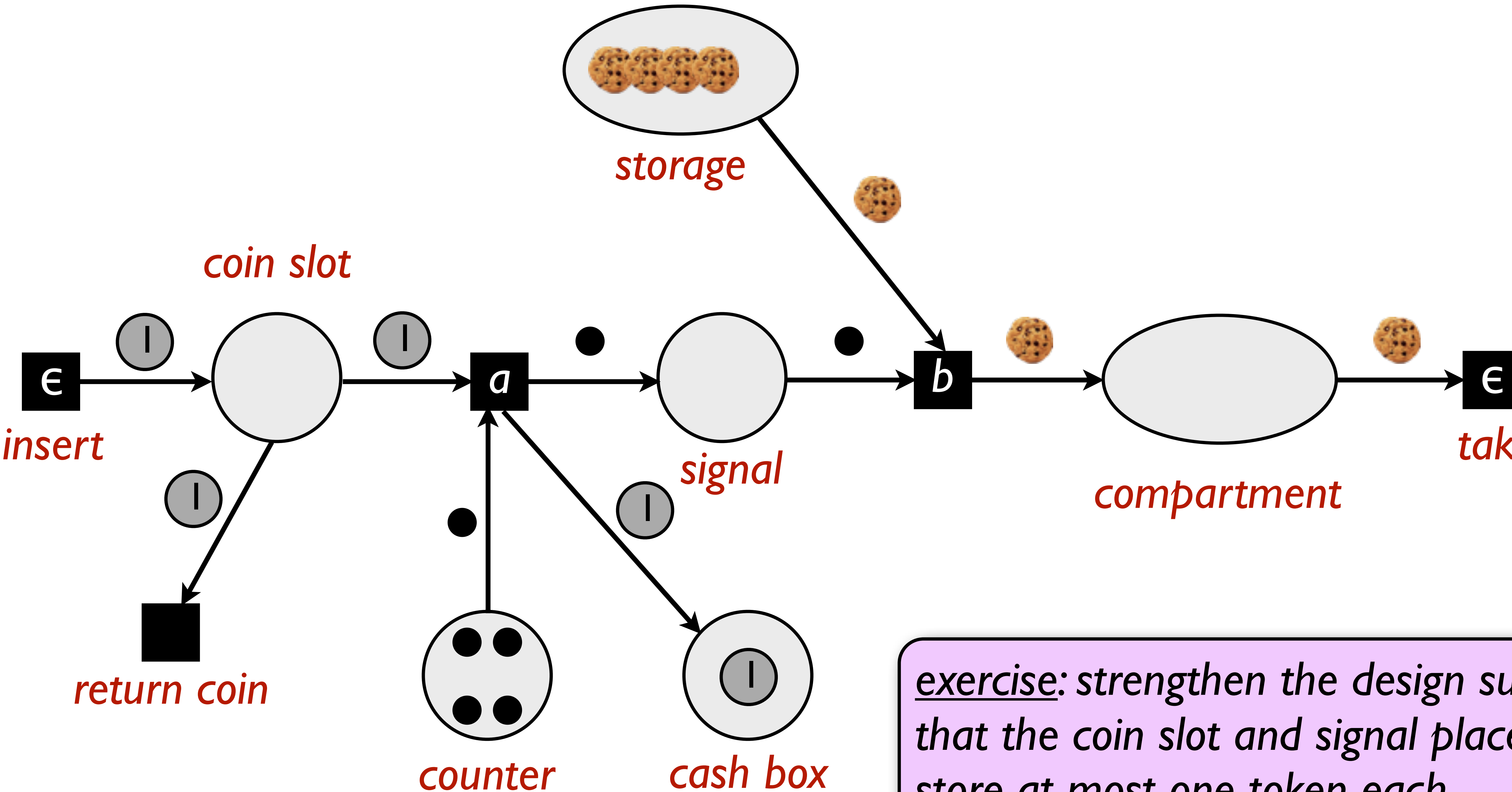
The ultimate cookie machine (design)



The ultimate cookie machine (design)



The ultimate cookie machine (design)



<u>exercise</u>: strengthen the design such that the coin slot and signal places store at most one token each

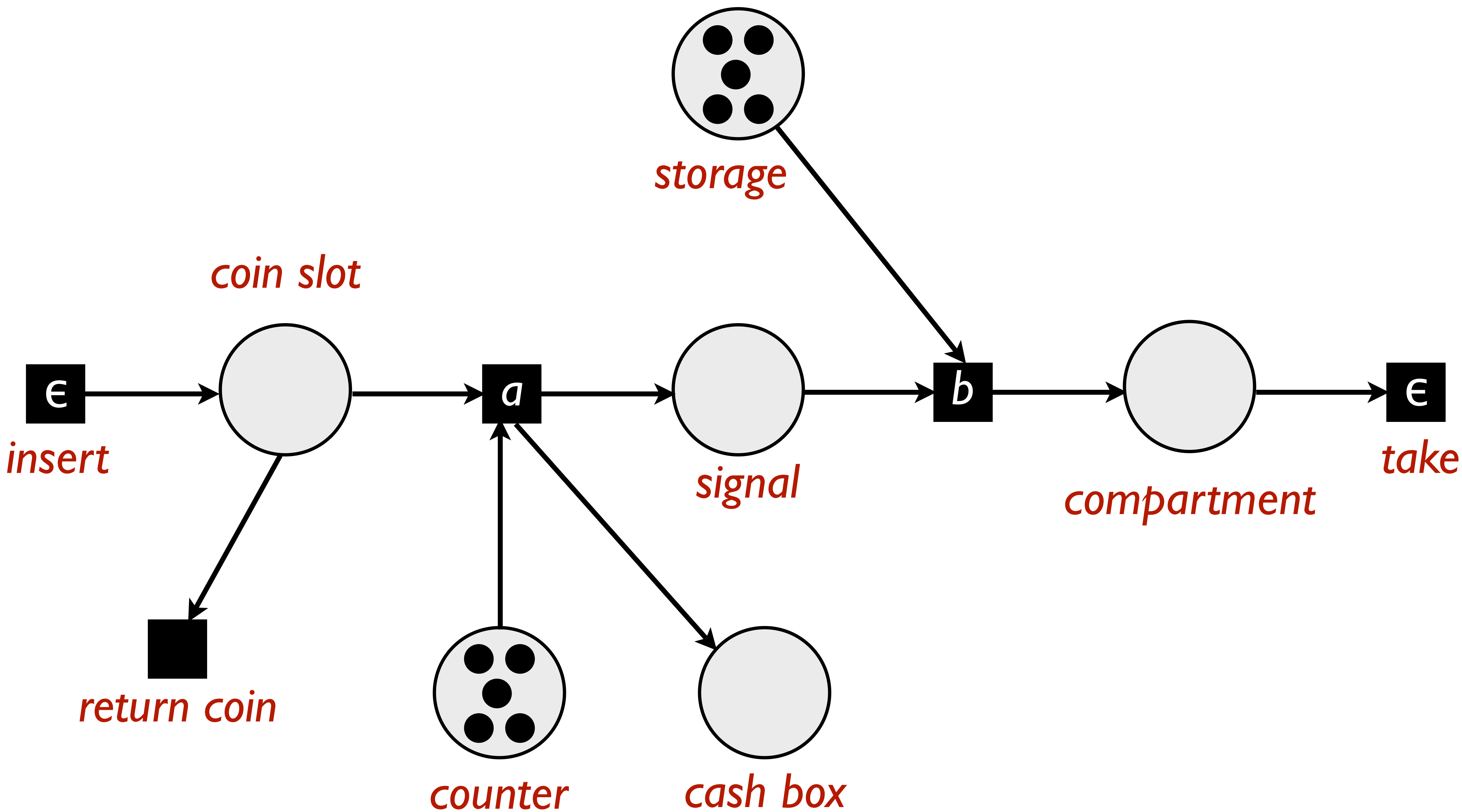
Elementary Petri nets

- if we are interested in only control flow, we can use a special case - elementary Petri nets - where <u>all</u> tokens are simply black dots

- assume all edges to be labelled by: “●“

- henceforth, we assume all Petri nets to be elementary

Elementary cookie vending machine



Petri nets: definition

- an (elementary) Petri net consists of a net structure:

  $$N = (P, T, F)$$

  with finite sets P and T of places and transitions, F an edge relation $F \subseteq (P \times T) \cup (T \times P)$ and an initial marking $M_0: P \rightarrow \boldsymbol{N}$

- transitions marked with ϵ are cold

- markings have the form $M: P \rightarrow \boldsymbol{N}$; each place p holds $M(p)$ tokens

Petri nets: definition

- the preset of a transition *t* is the set of places *p* connected by edges from *p* to *t* (postset defined analogously)

- a transition is enabled if $M(p) \geq 1$ for all places *p* in the preset

- an enabled transition can occur, removing a token from each place in the preset and adding one to each place in the postset

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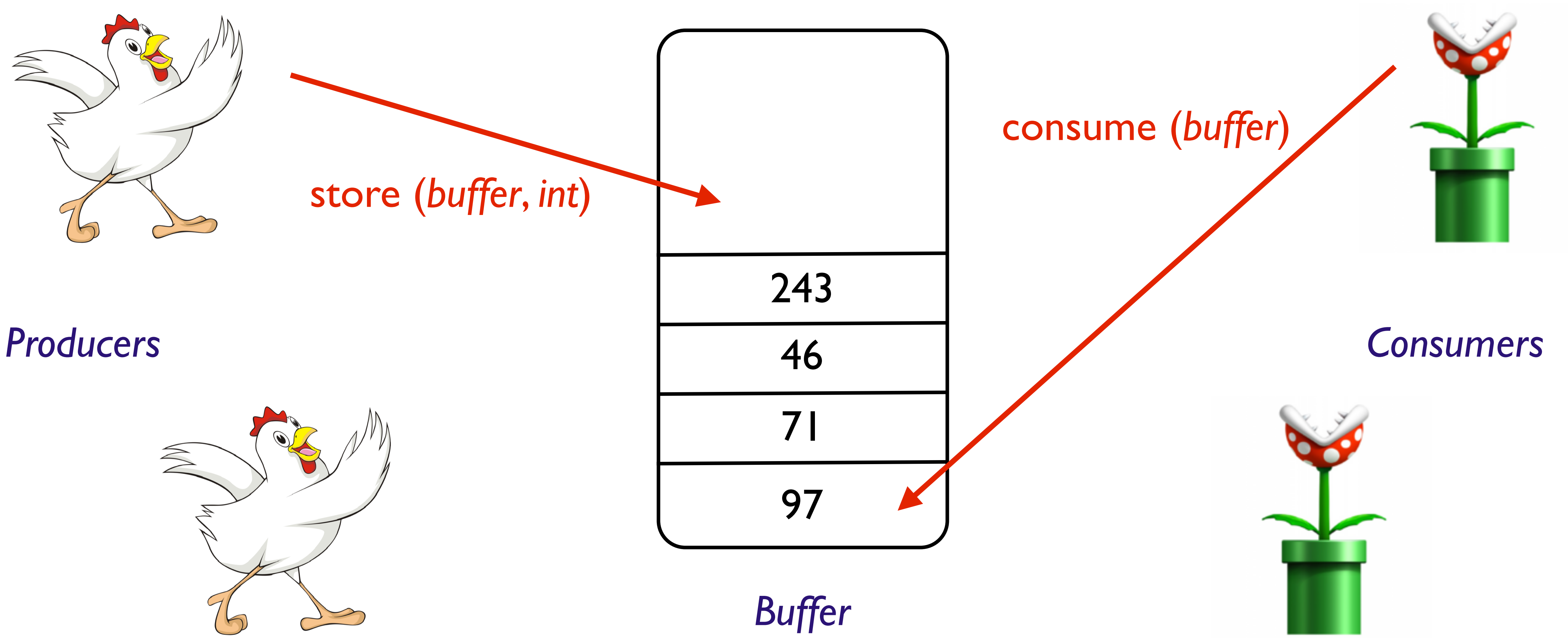
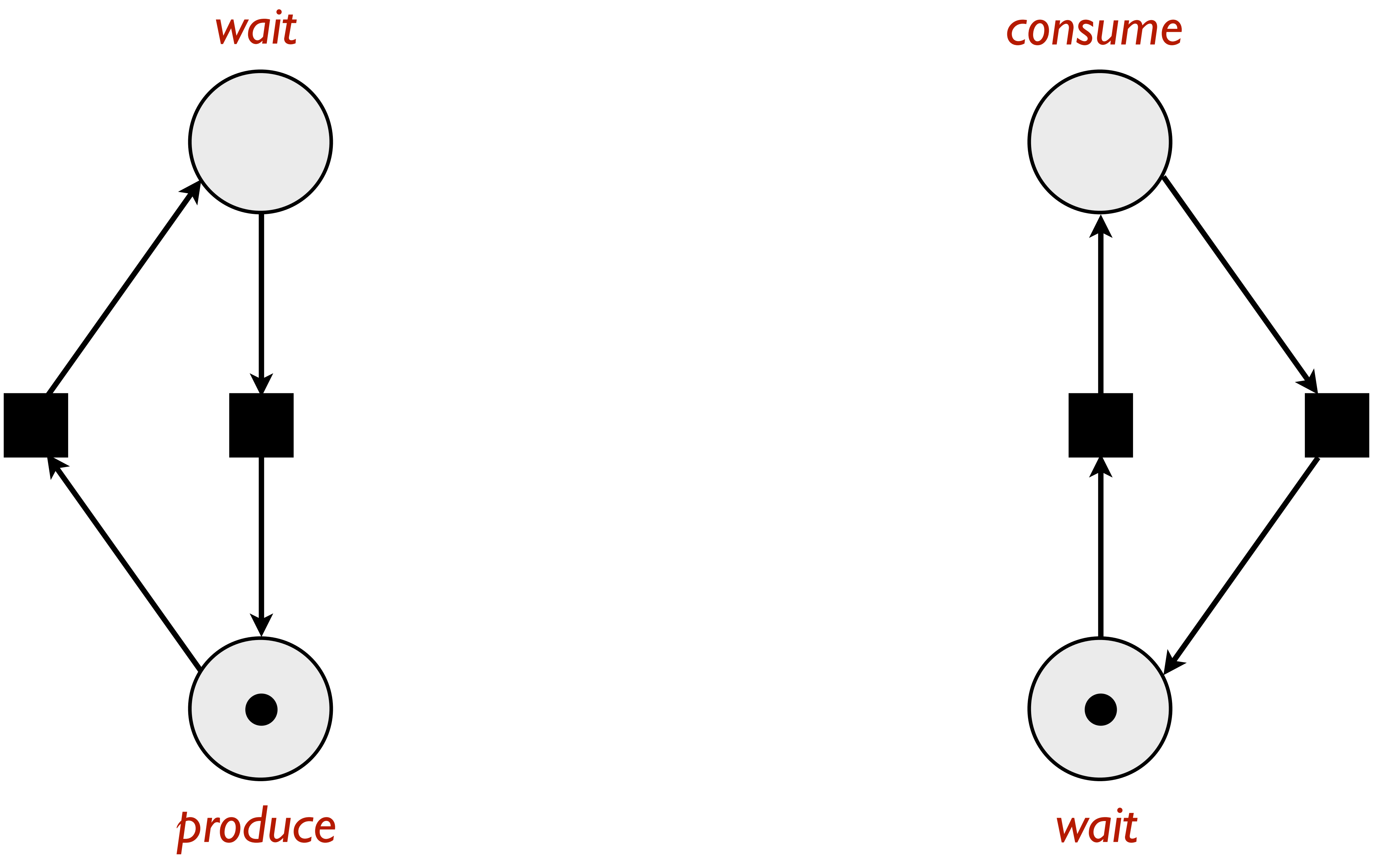
4. true concurrency semantics; unfoldings

Producer-consumer problem



Producer-consumer problem



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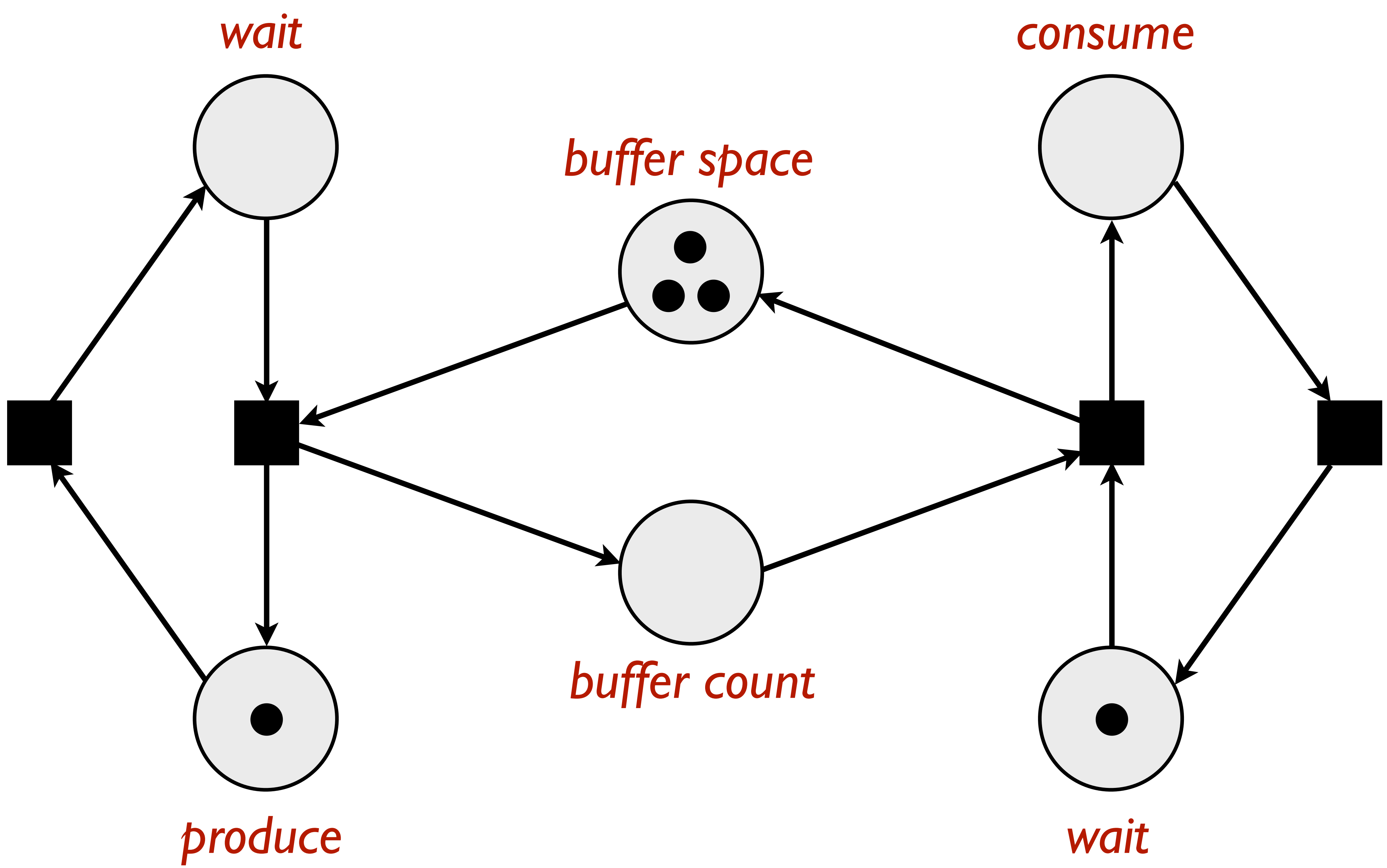

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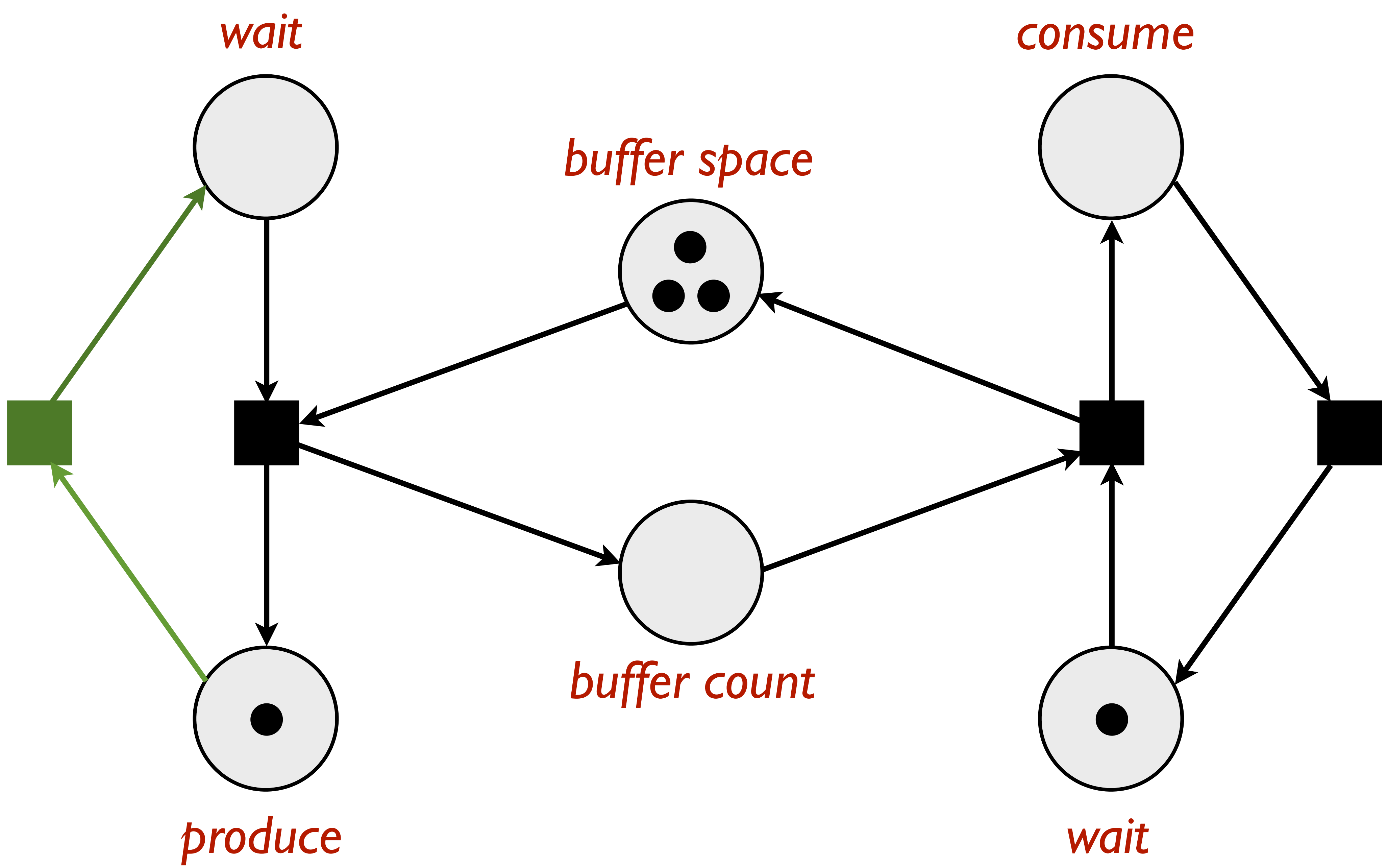

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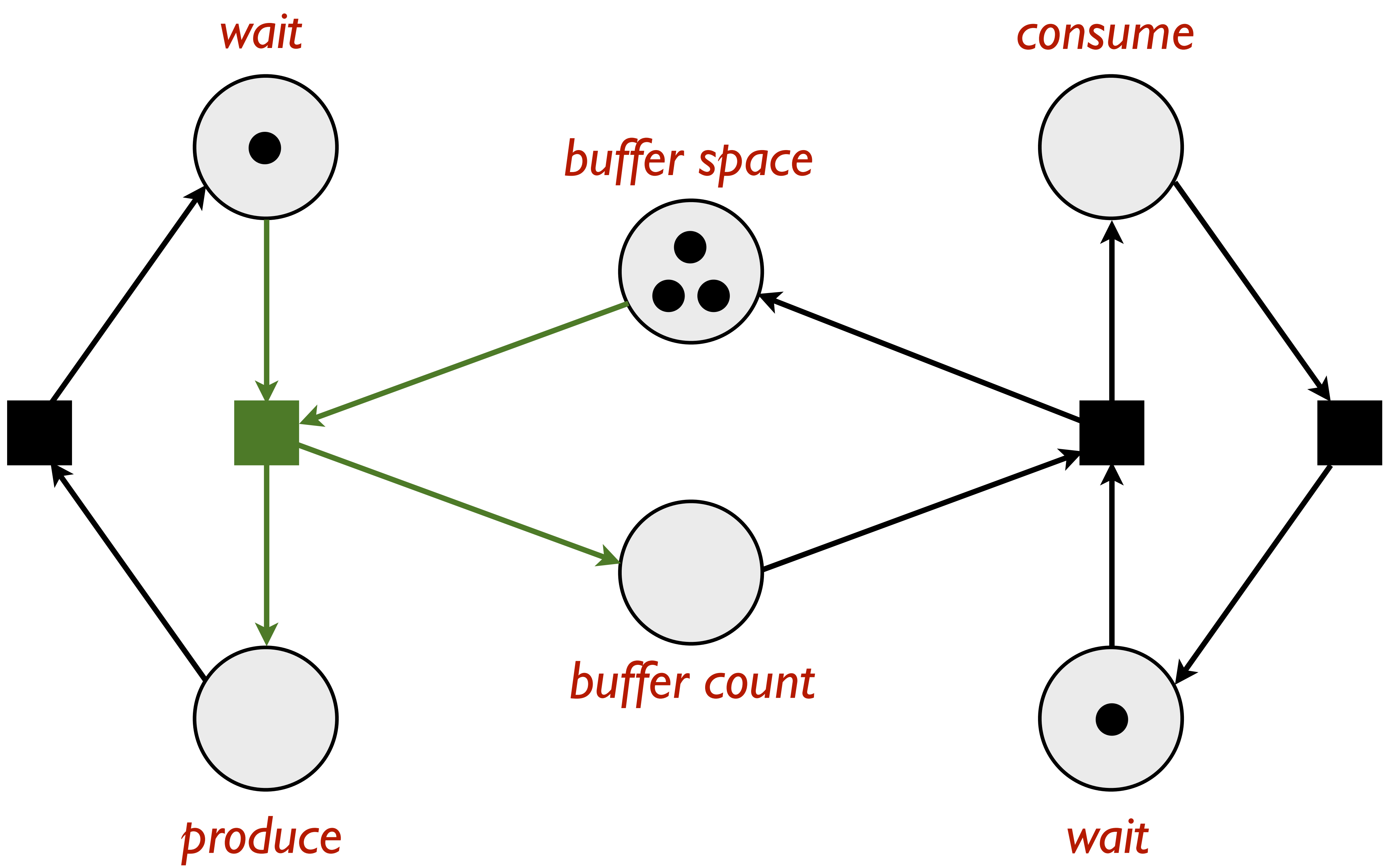

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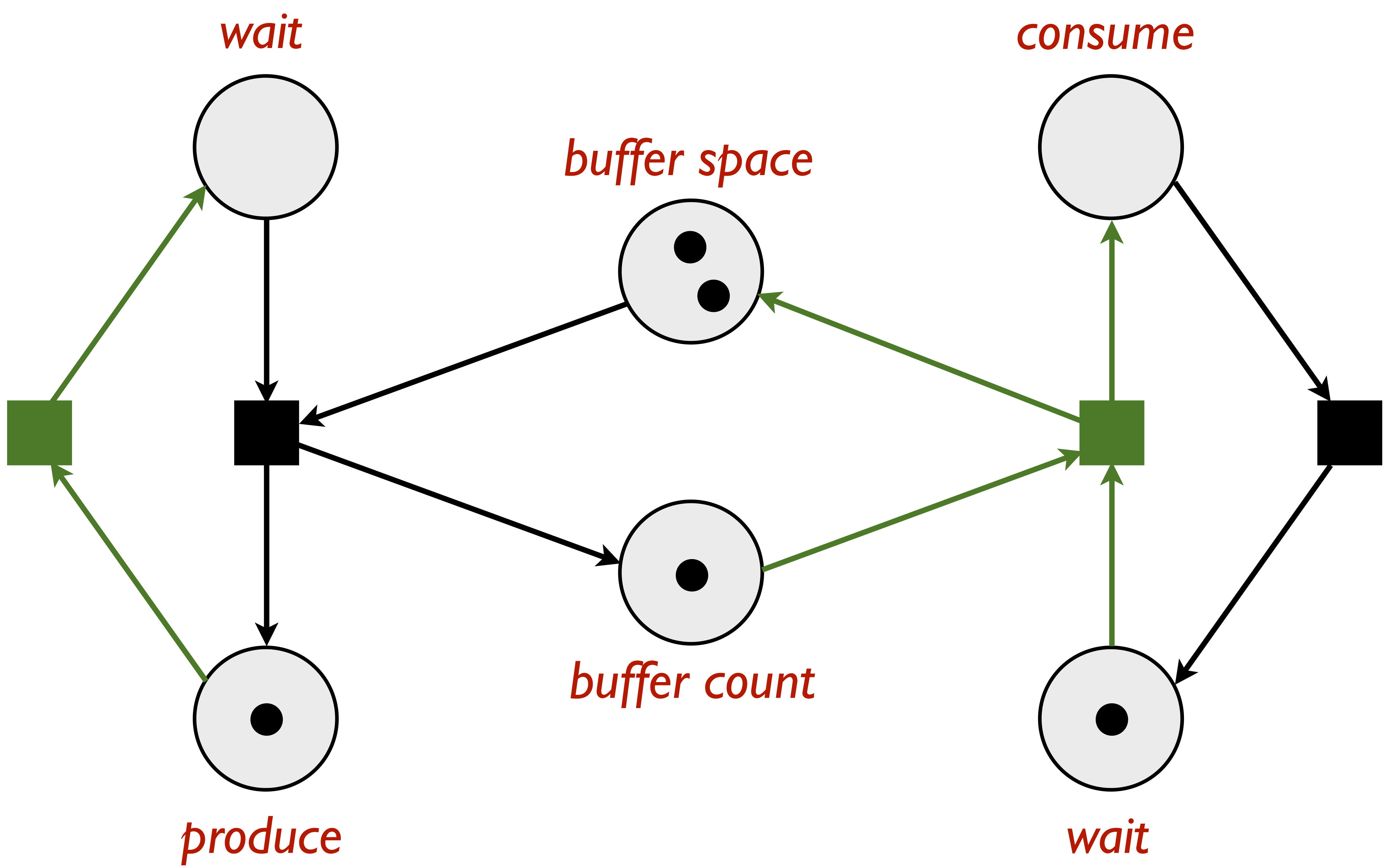

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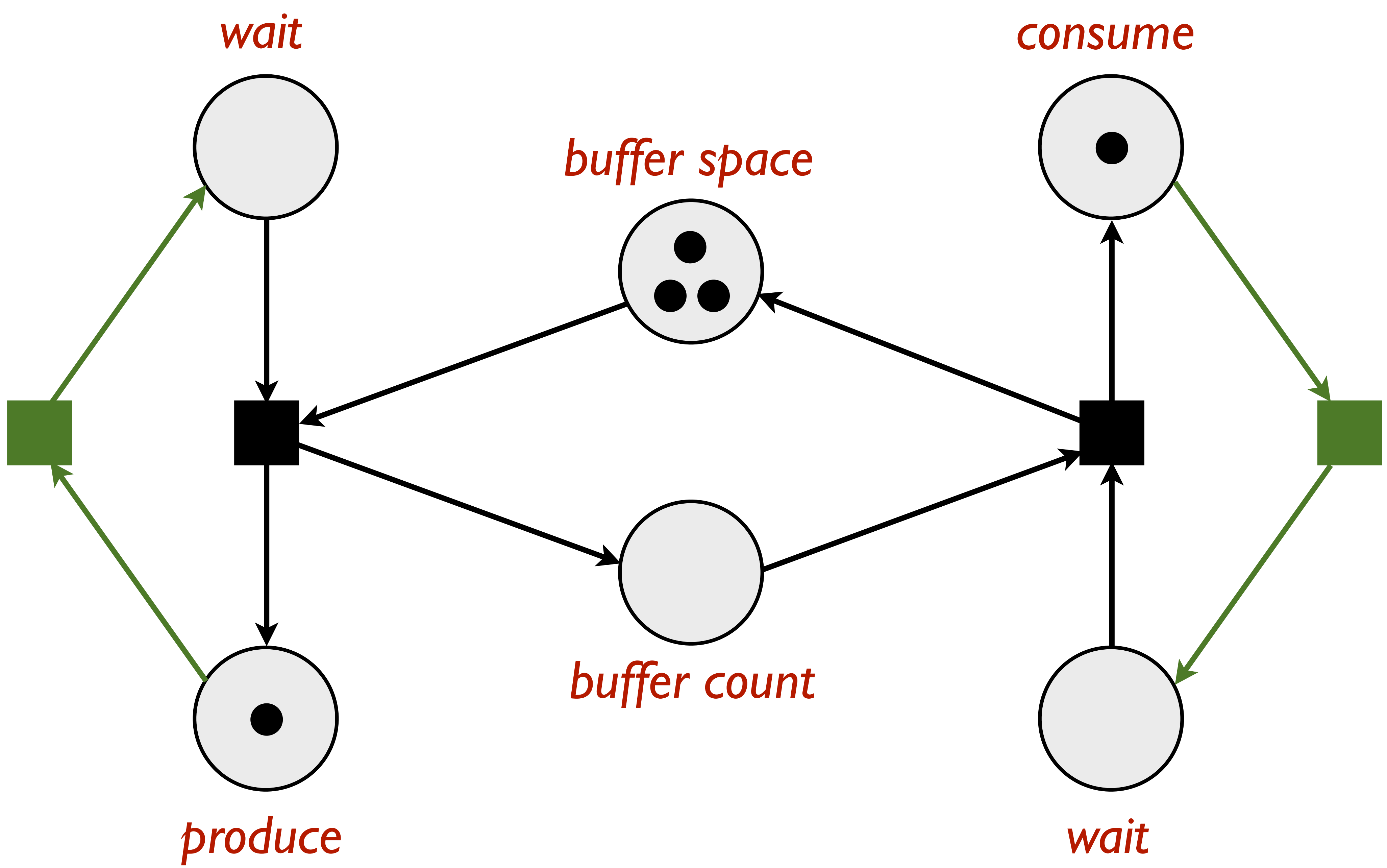

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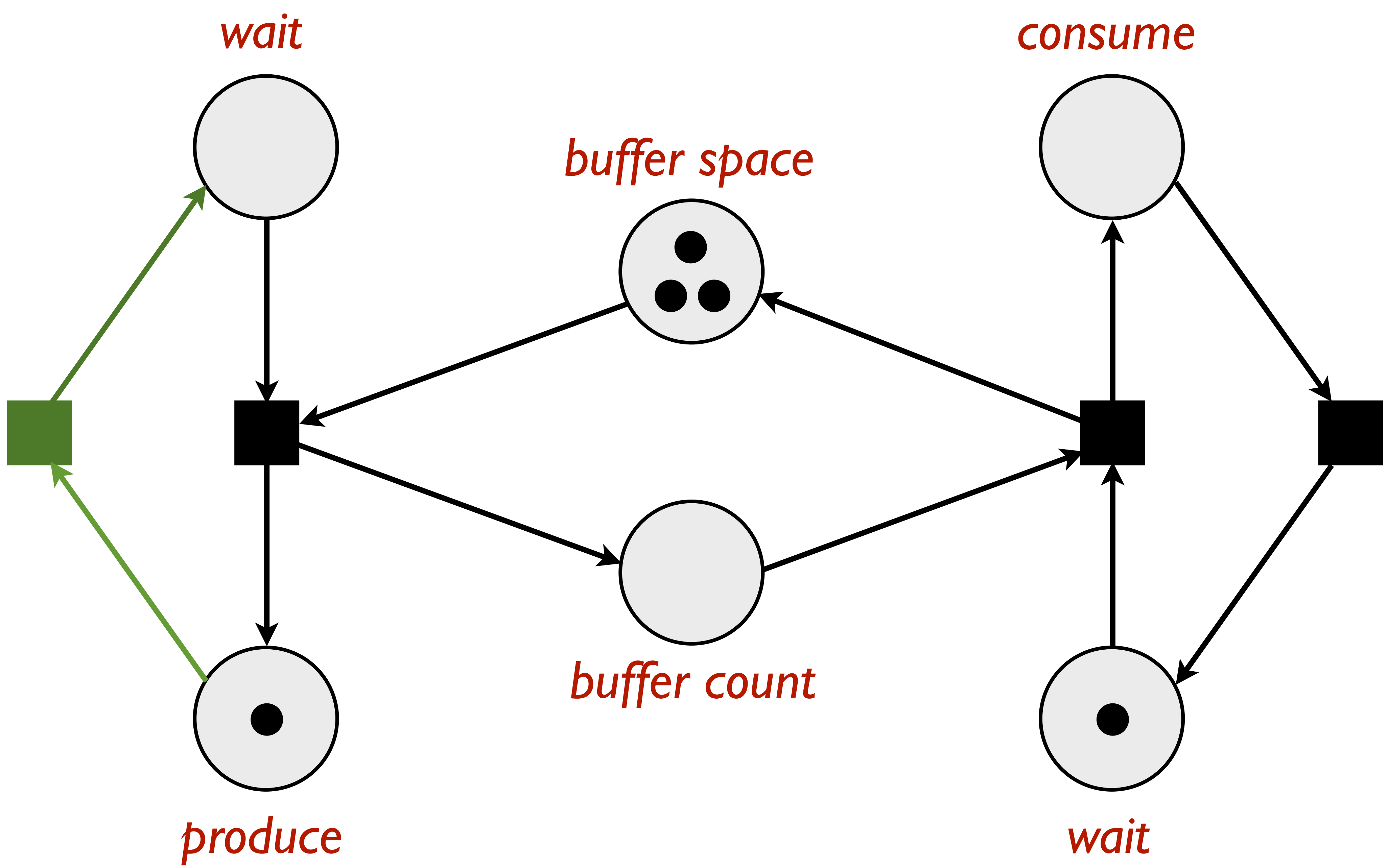
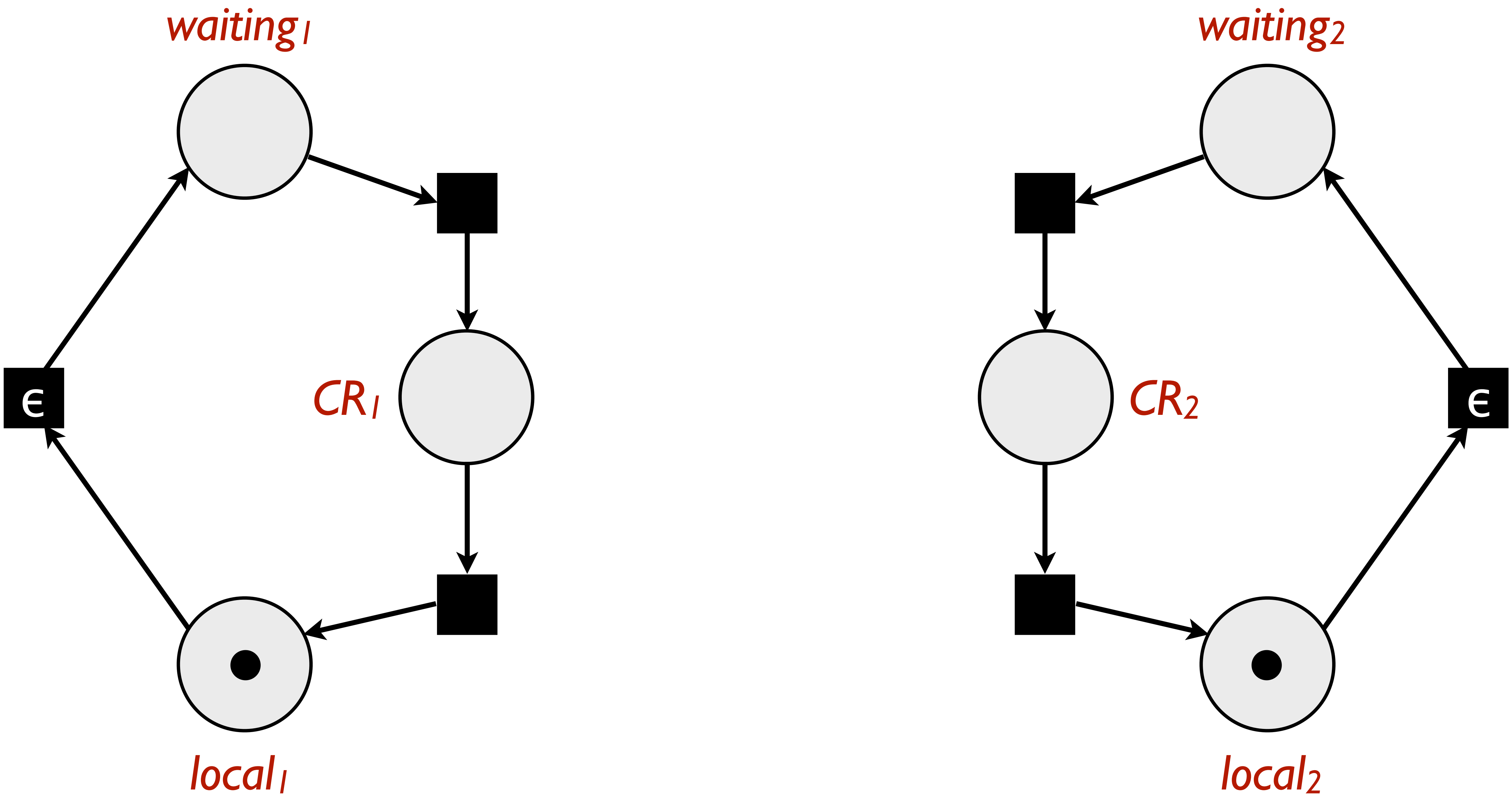
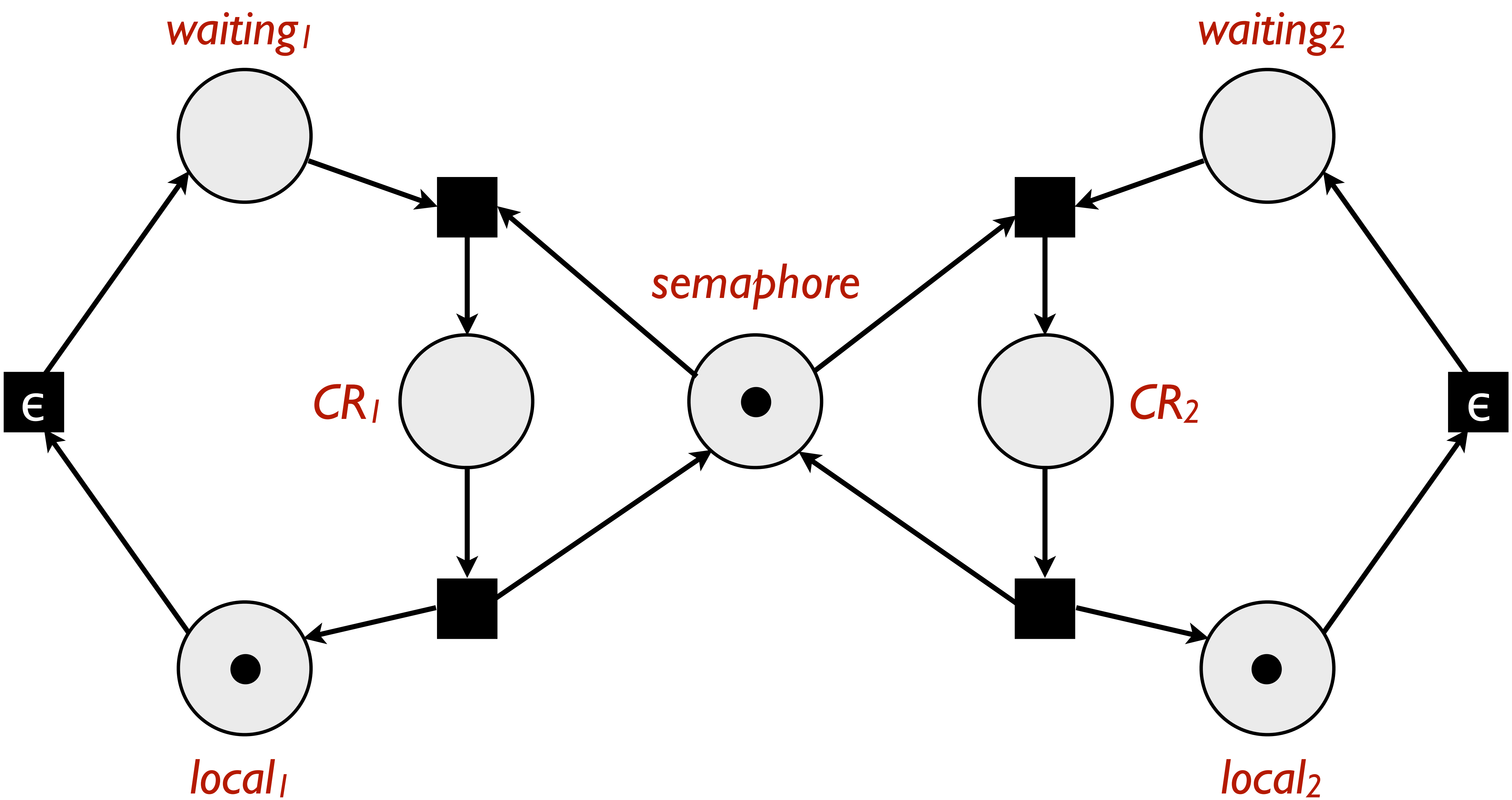


Mutual exclusion



Mutual exclusion



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Modelling power vs. analysability

- many properties of interest for concurrent systems can be automatically determined for Petri nets
  => but can be *very* expensive in the general case

- properties include:
  => *k*-boundedness (i.e. no place ever has more than *k* tokens)
  => liveness
  => reachability

Reachability problem

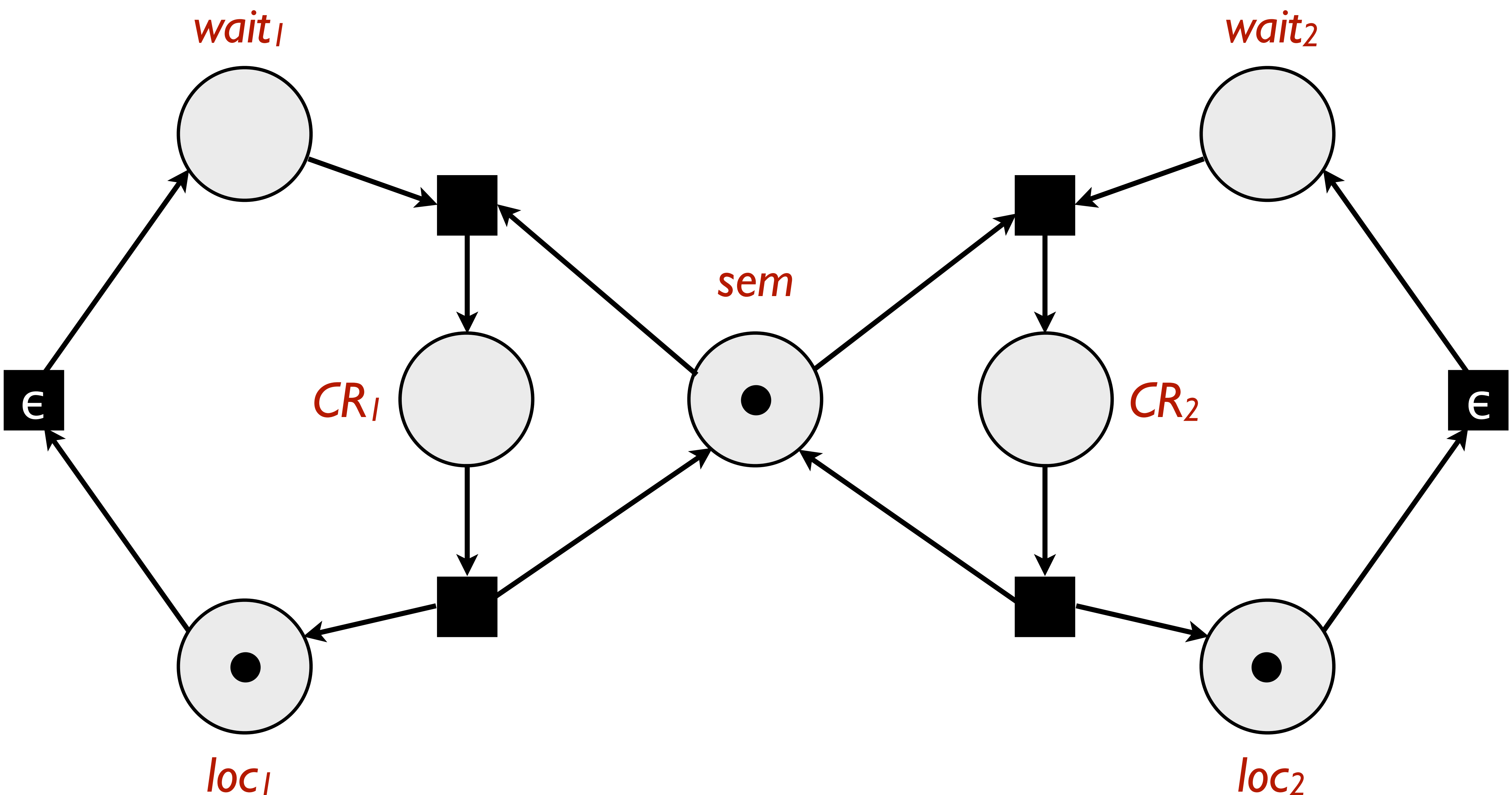
- the problem to decide whether some marking M can be derived from the initial marking

- starting point: construct a reachability graph from the initial marking

  => i.e. a *transition system* completely describing its behaviour
  => nodes denote markings
  => edges denote occurrences

- (more sophistication is needed when reachability graphs are not finite)

Reachability graph for our semaphore

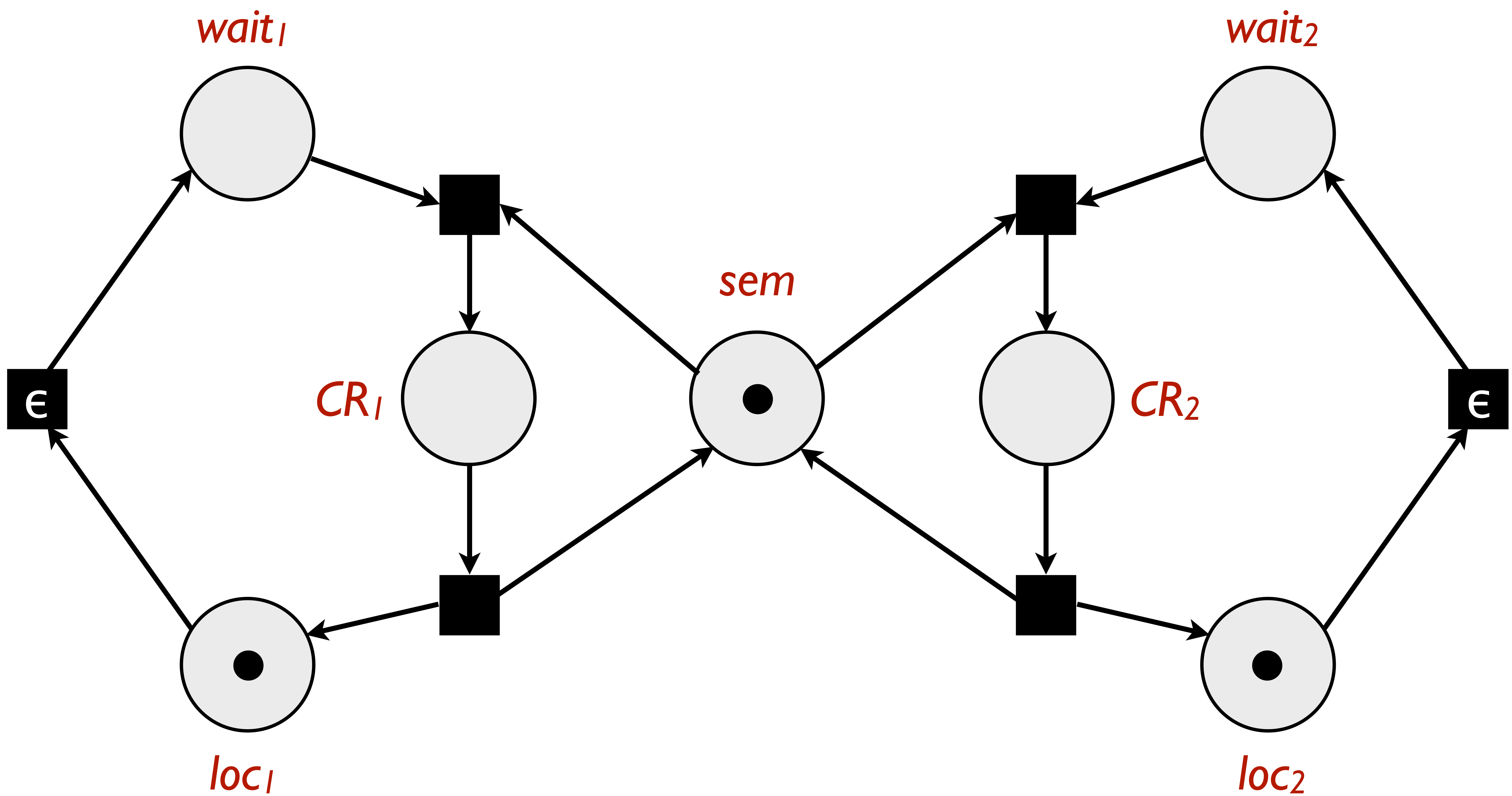


express marking M as a vector:

($M(\text{wait}_1)$ $M(\text{CR}_1)$ $M(\text{loc}_1)$ $M(sem)$ $M(\text{wait}_2)$ $M(\text{CR}_2)$ $M(\text{loc}_2)$)

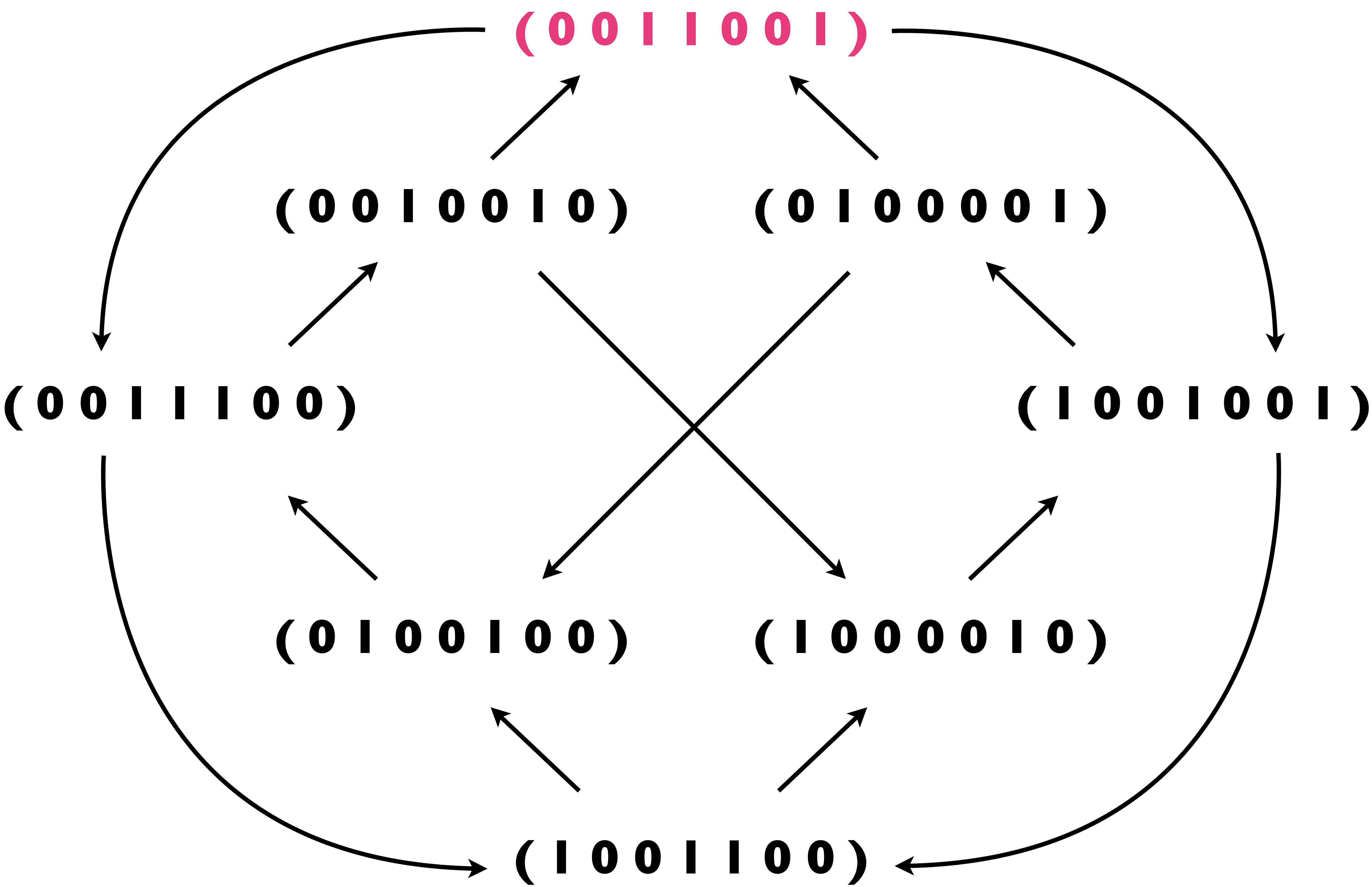
***i.e.* (0 0 1 1 0 0 1)**

Reachability graph for our semaphore



- *prove that **(0 1 0 0 0 1 0)** is unreachable*
- *prove that* $M(\mathrm{CR}_1)+M(\mathrm{CR}_2)+M(\mathrm{sem}) = 1$

Reachability graph for our semaphore



Deciding reachability is expensive

- reachability is an important analysis
- decidable, but expensive in the general case
  => EXPSPACE-hard
  => reachability graph not always finite
- part II of Reisig (2013) treats the problem with more sophistication than we have

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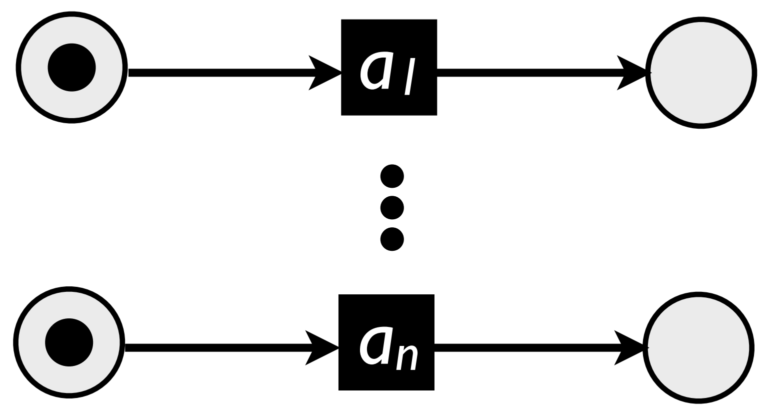
The problem of interleaving semantics

- consider the following Petri net:



- its reachability graph contains 2^n states

  => state explosion problem

  => due to interleaving of occurrences

  => unnecessary: ordering of occurrences here immaterial!

Interleaving vs. true concurrency semantics

- an interleaving semantics imposes a total ordering on sequences of occurrences
  => completely described by a reachability graph
  => nodes denote markings; edges denote occurrences
  => state explosion!

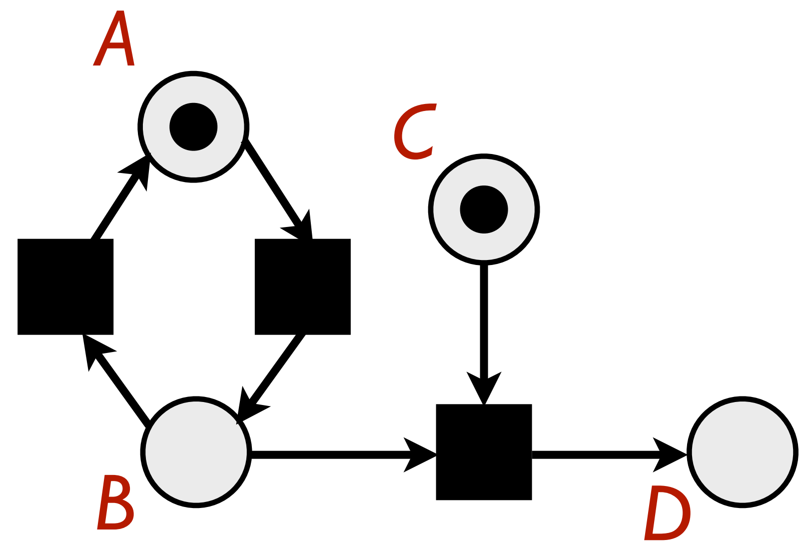
- a true concurrency semantics instead models time as a partial order
  => two or more occurrences can happen *simultaneously*
  => completely described by a so-called unfolding

Unfoldings are compact representations of concurrency

- an unfolding of a Petri net *N* is a Petri net that is more "tree like" - but represents the same behaviour

- idea: analyse the unfolding of a Petri net itself, rather than an underlying transition system (as in the interleaving semantics)

Example: an unfolding



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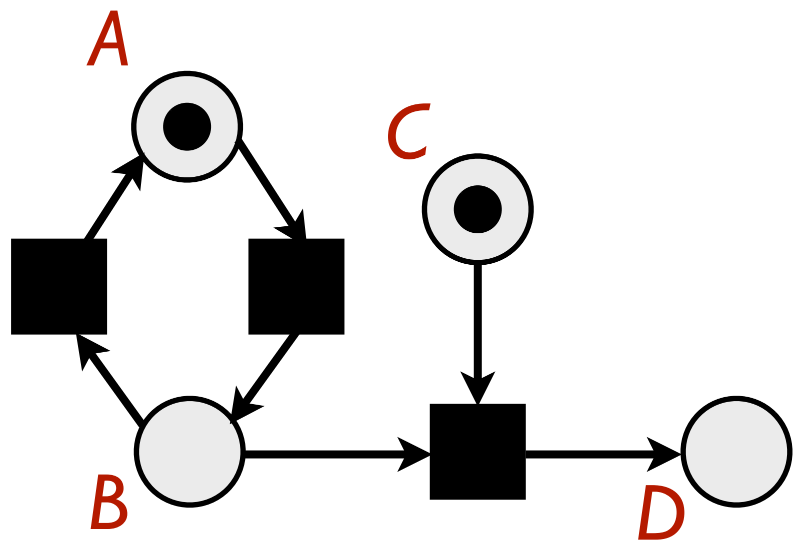

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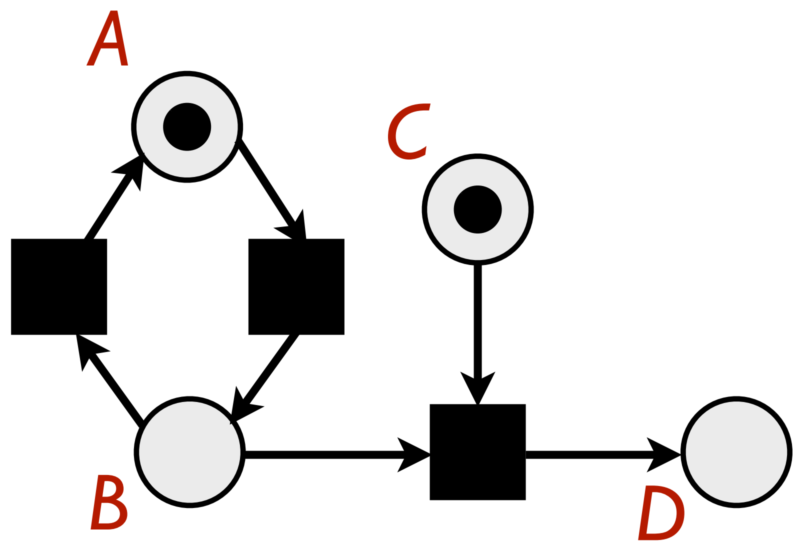

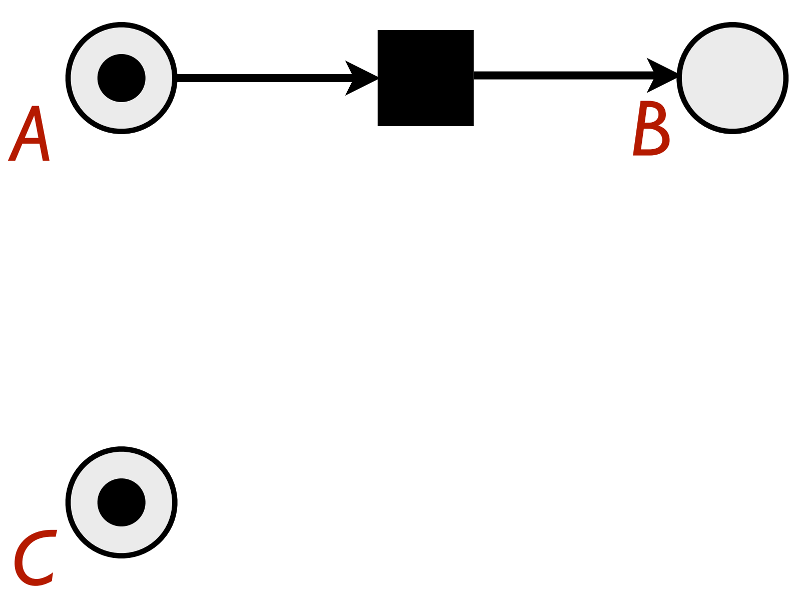

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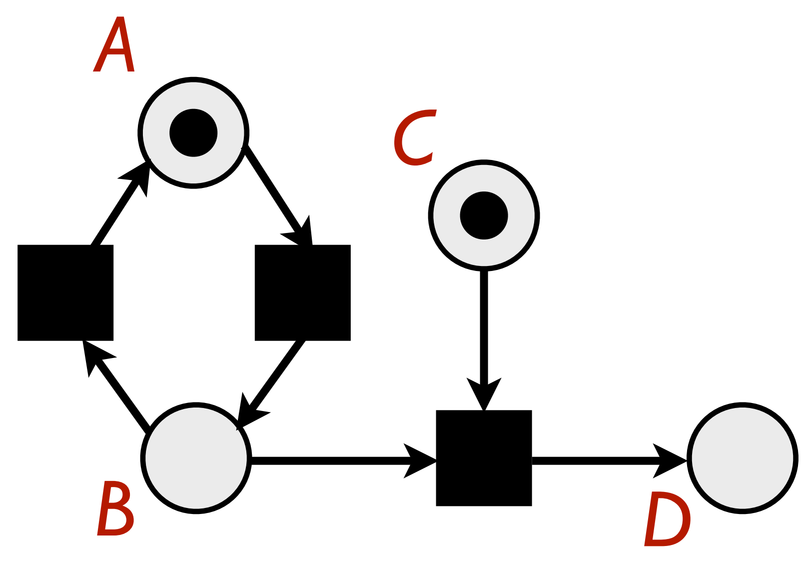

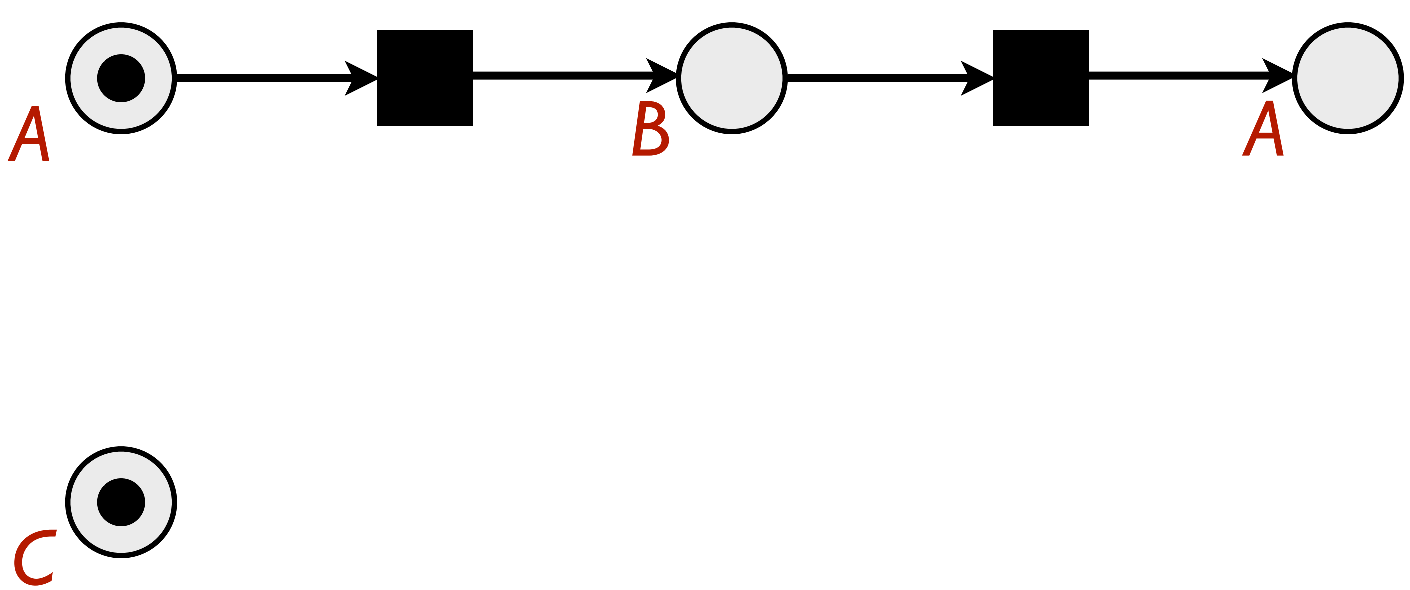

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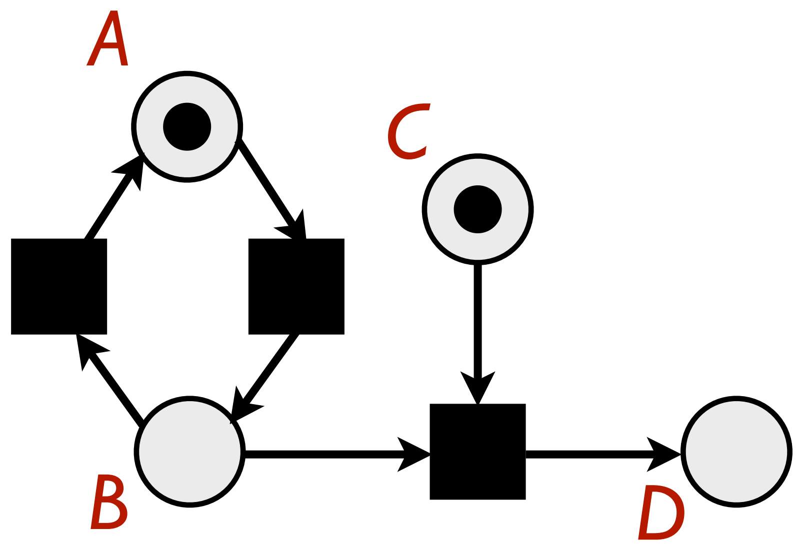

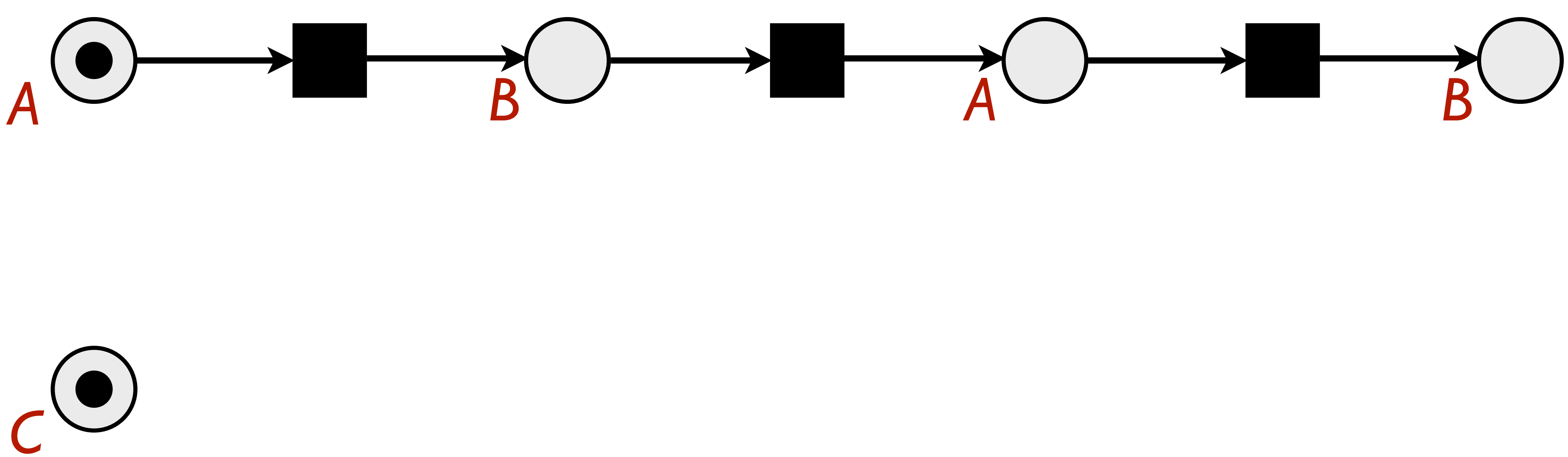

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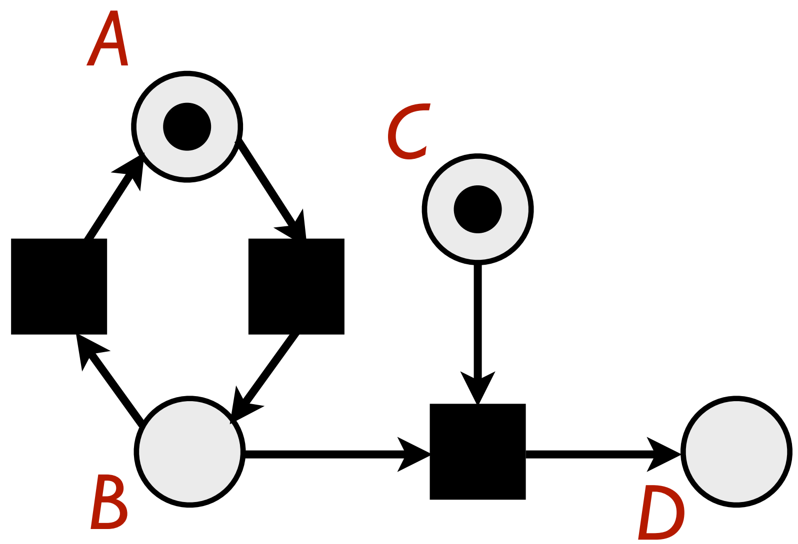

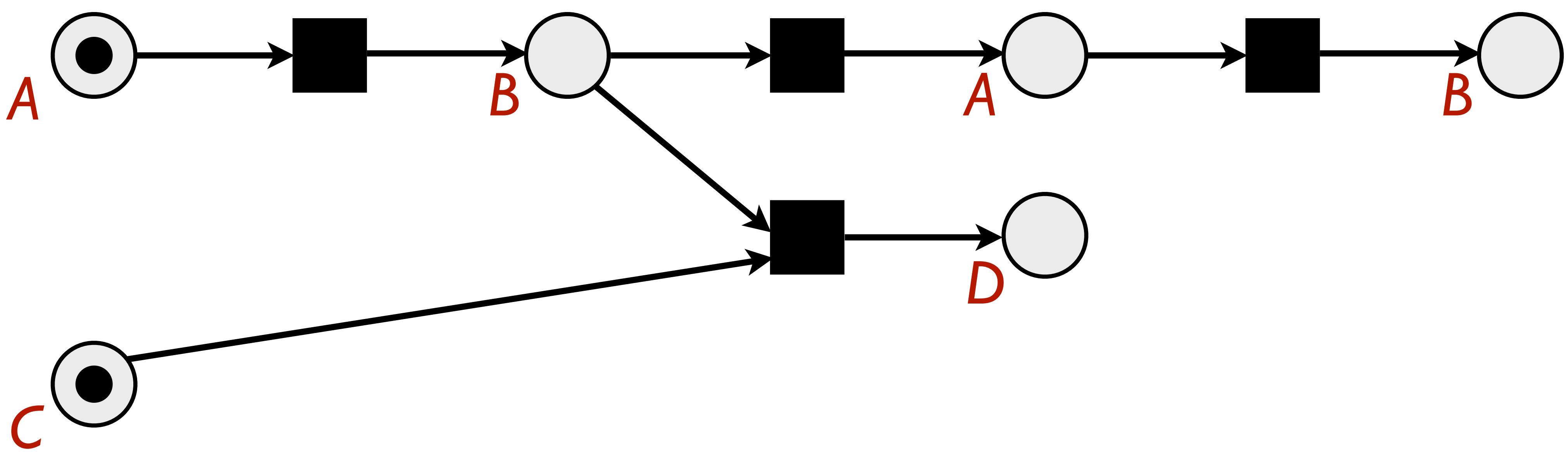

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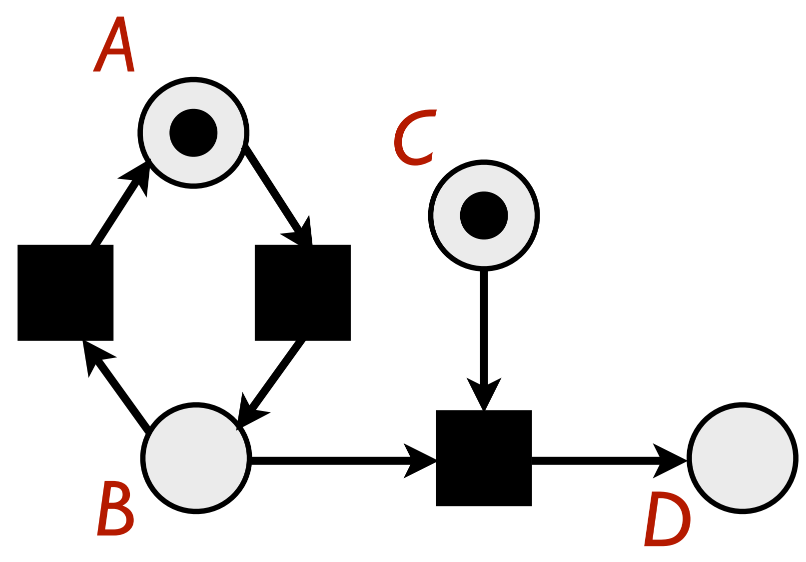

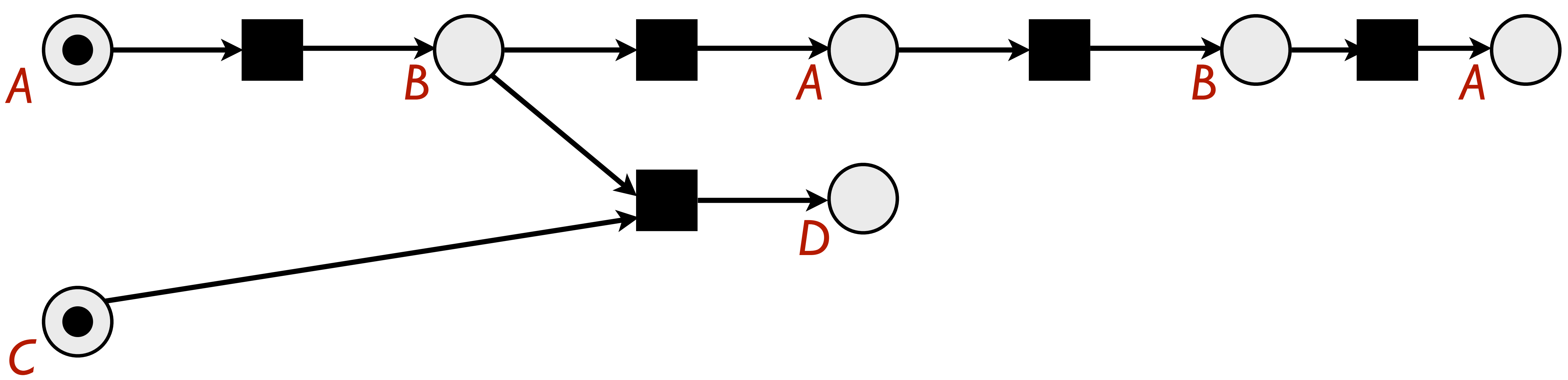

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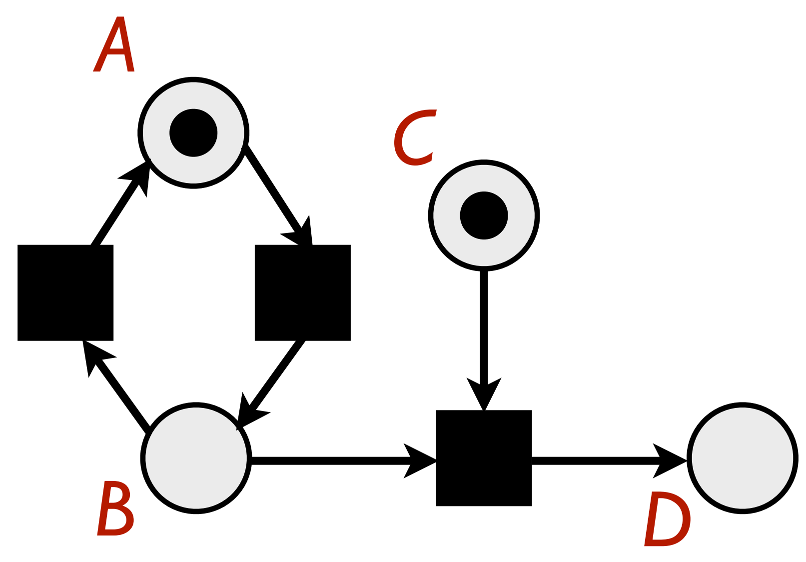
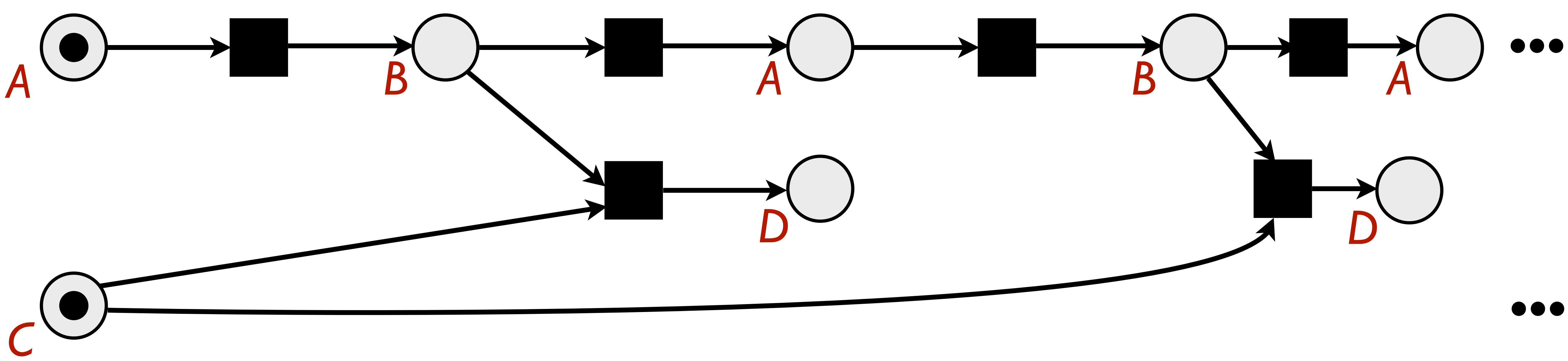




Constructing an unfolding

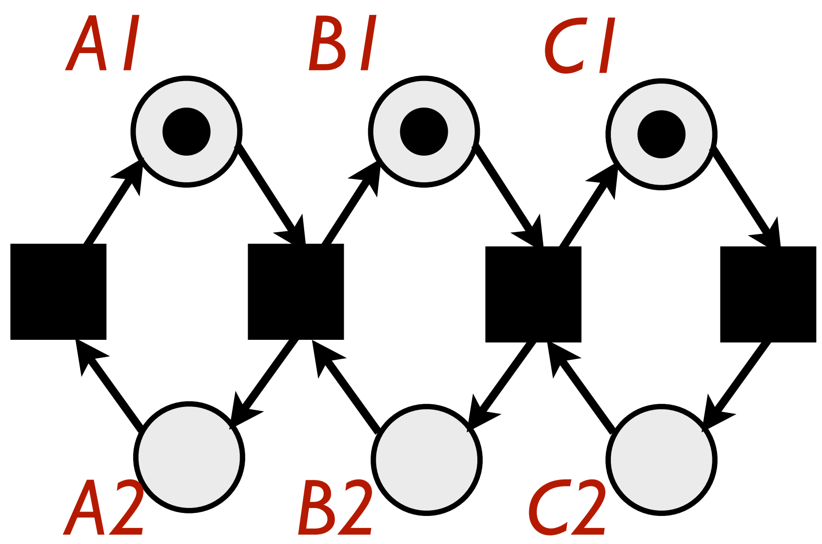
- assumption: Petri nets are 1-bounded
  => possible to generalise to other Petri net variants

- steps to construct an unfolding *N'* from a Petri net *N*:

  (1) initialise *N'* with the places in *N* containing tokens in the initial marking
  (2) if a reachable* marking in *N'* enables a transition *t* in *N*, then disjointly add *t* to *N'* and:
  => link it to the corresponding preset
  => disjointly add the postset of *t*
  (3) iterate step 2

  **checking reachability is far easier for this special net class*

Another example



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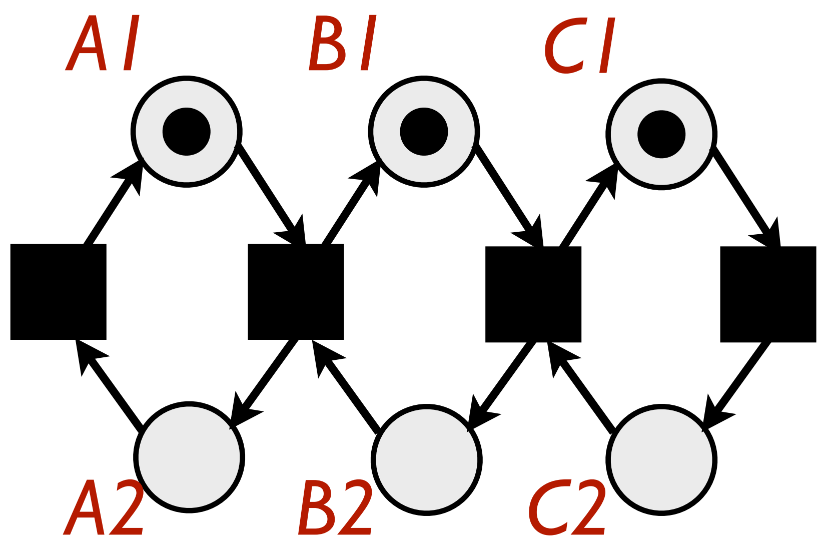

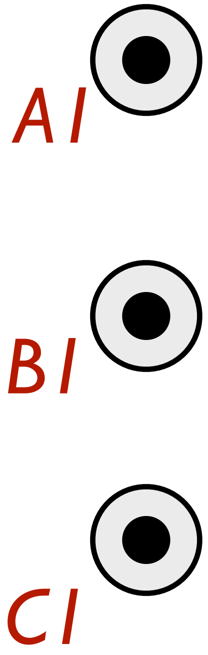

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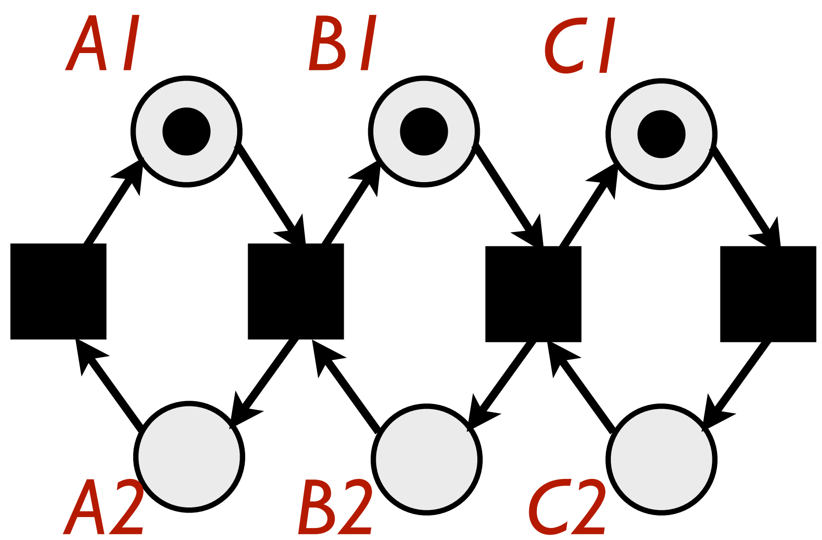

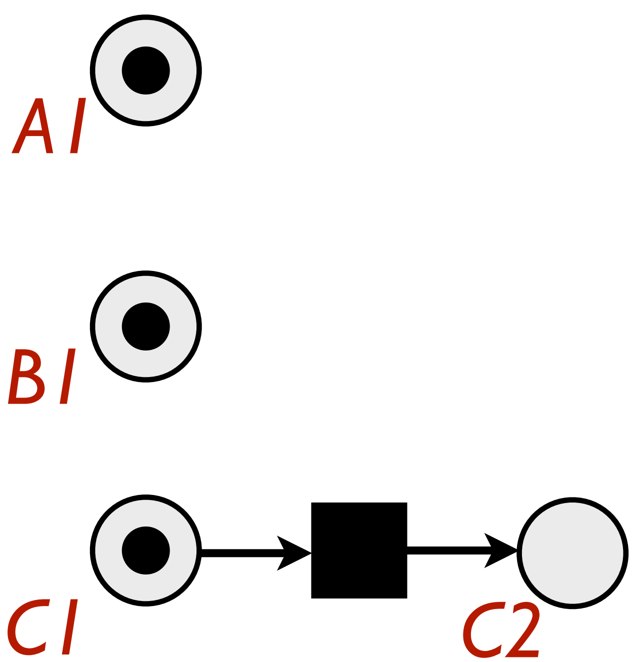

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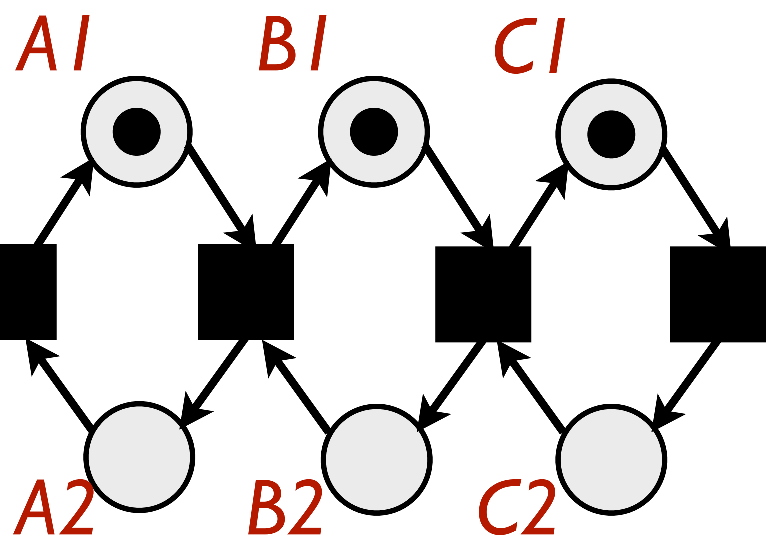

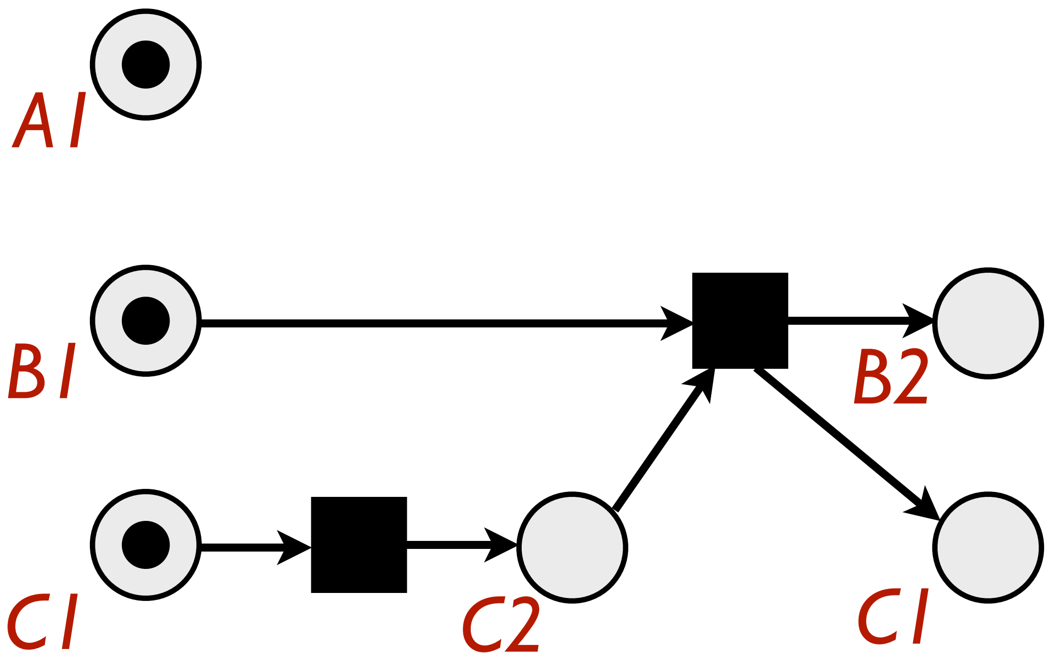

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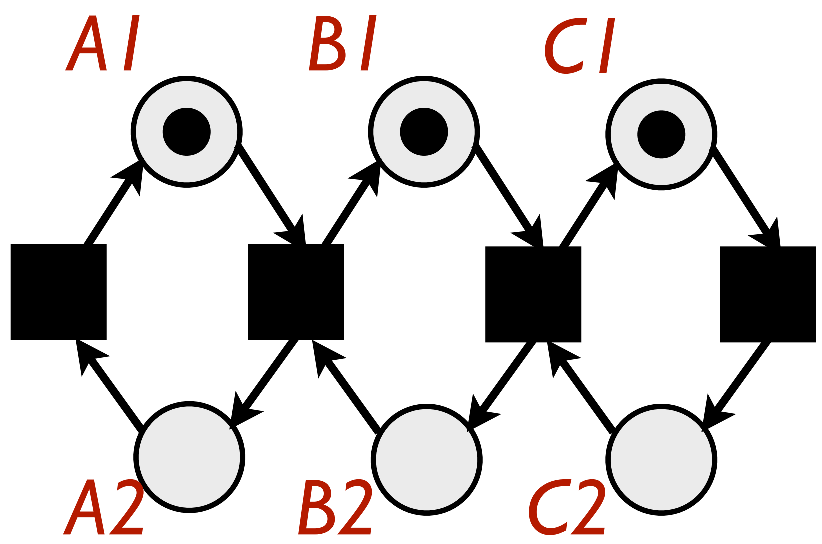

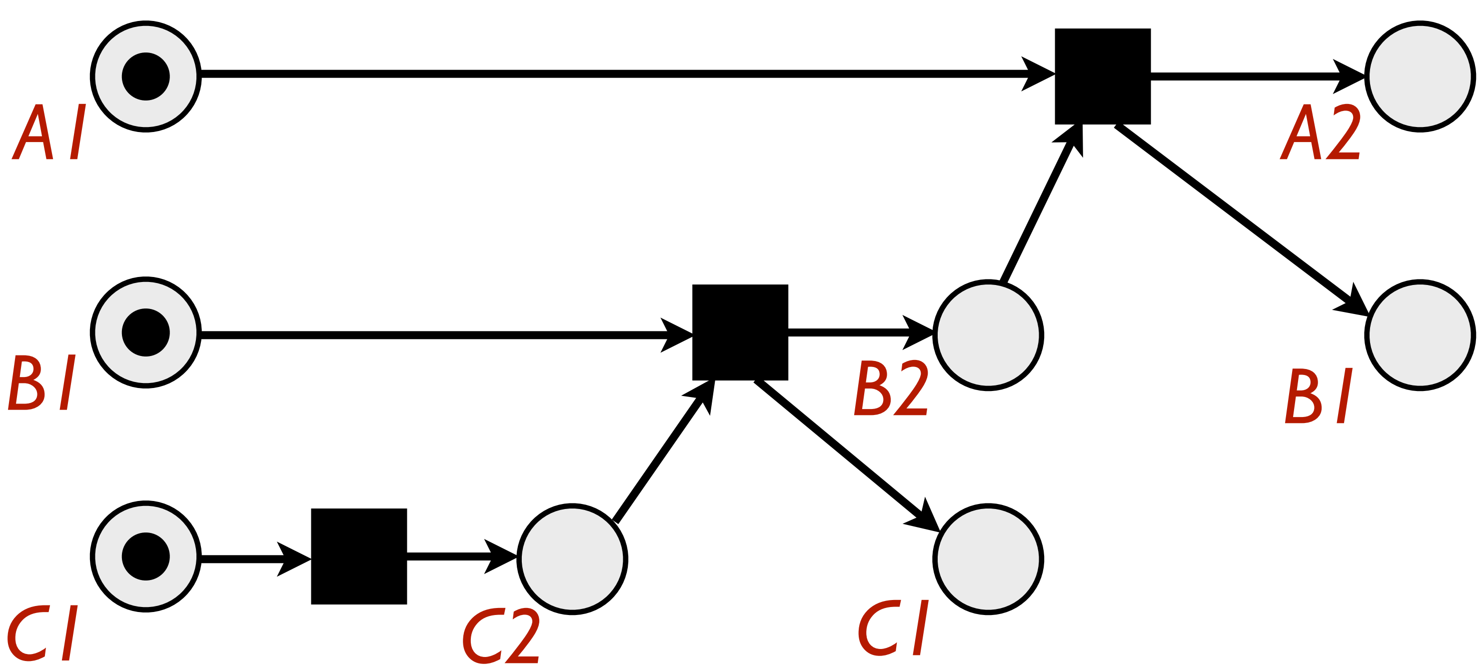

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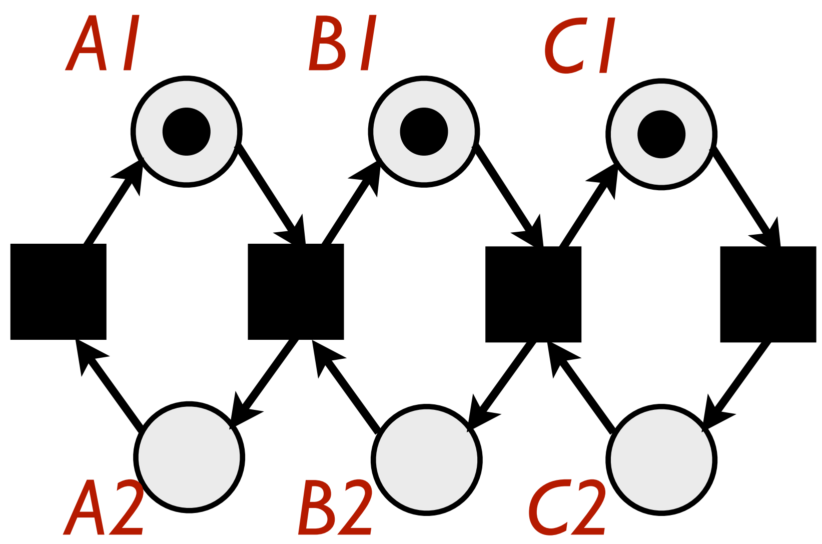
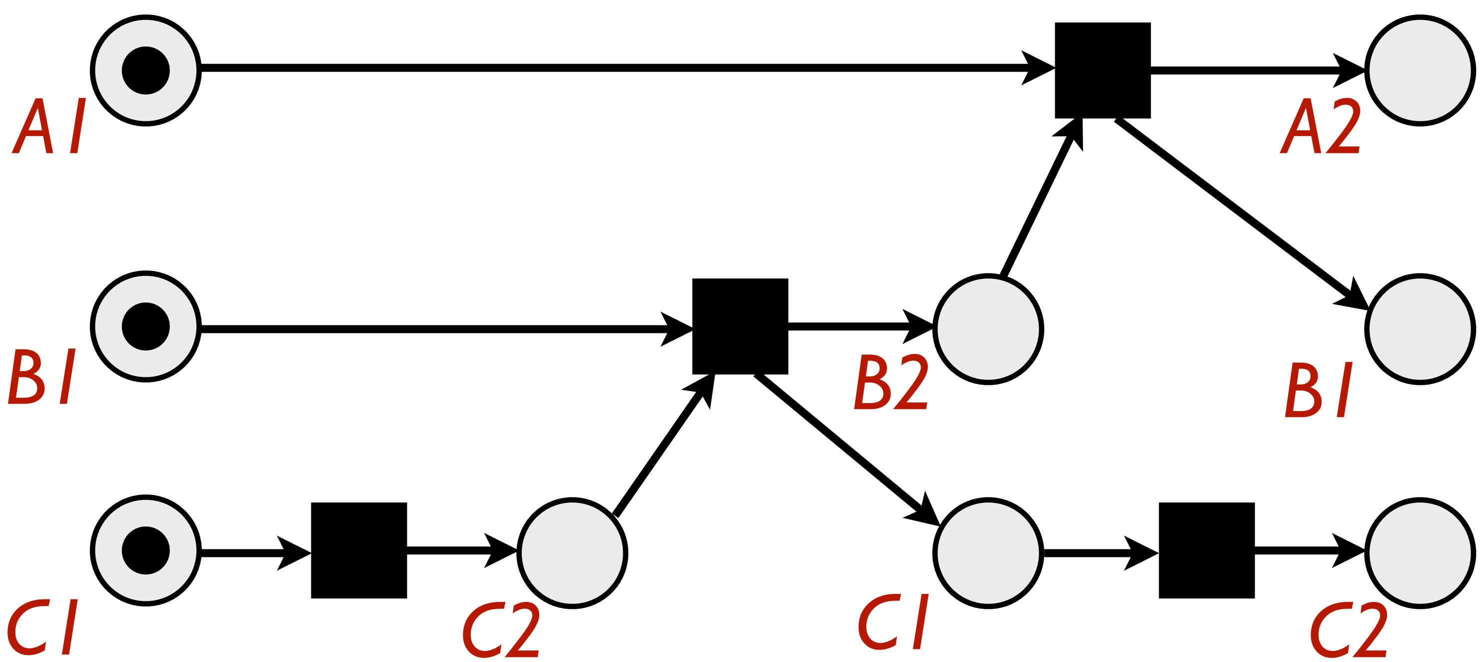
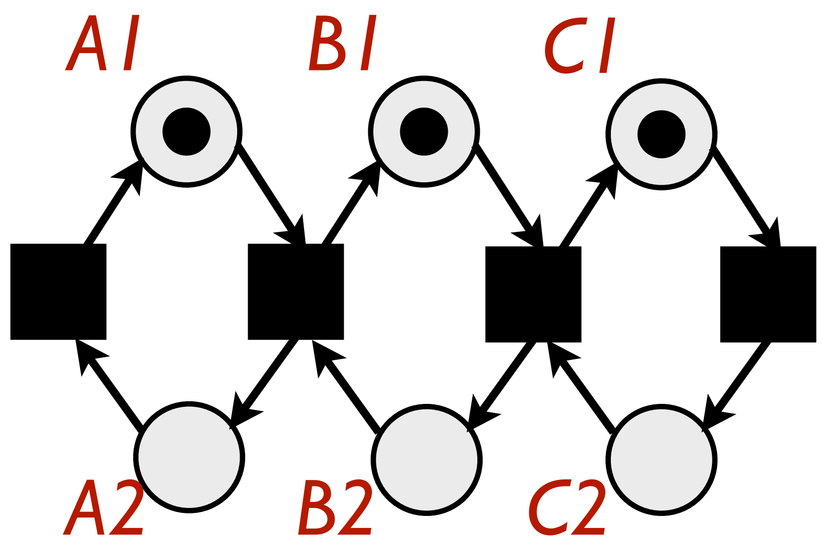
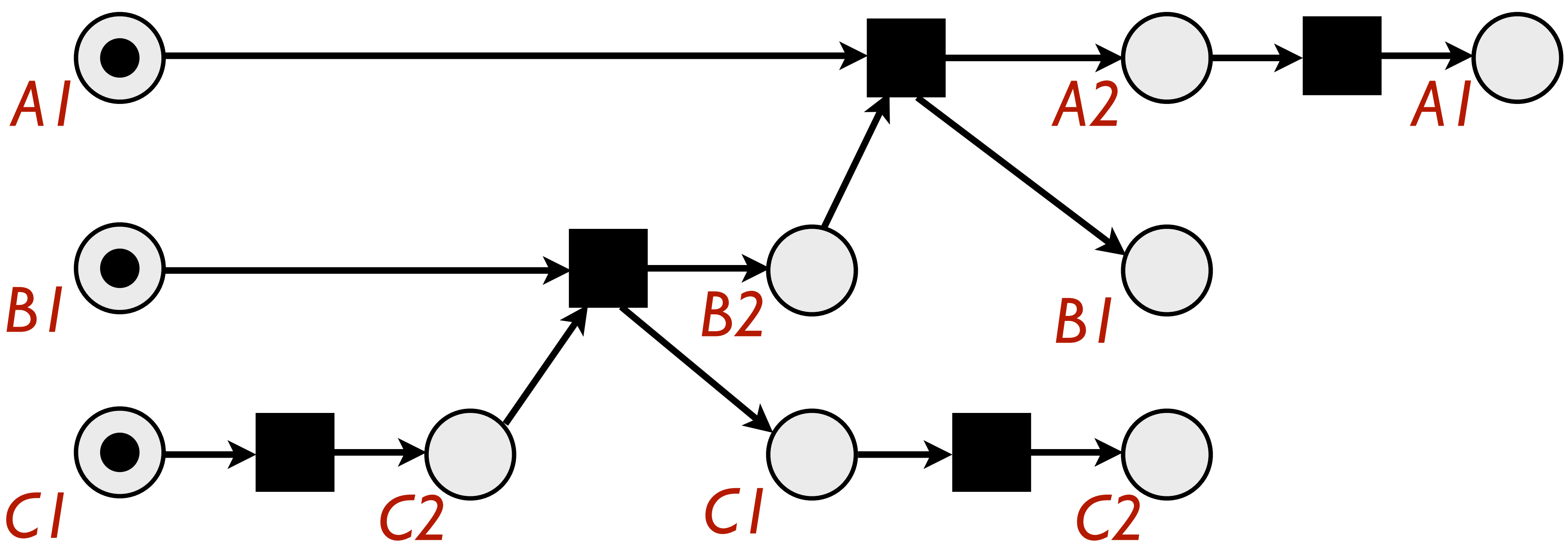




Another example





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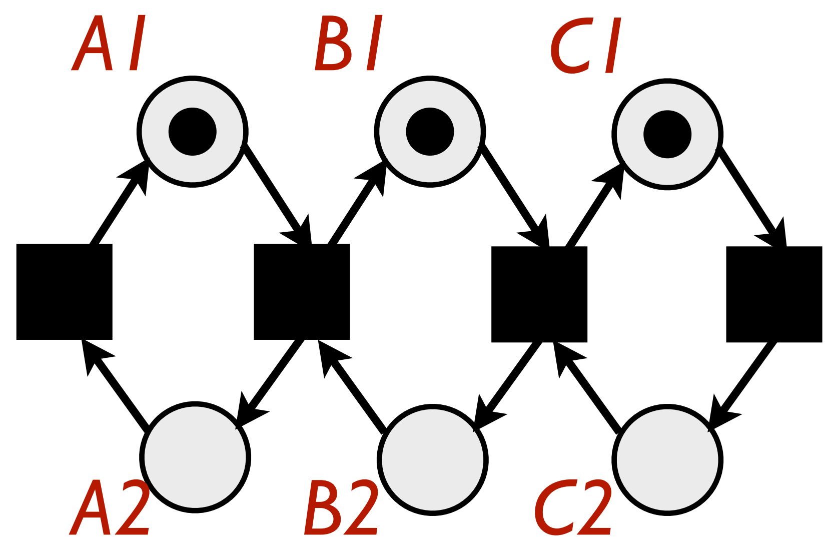
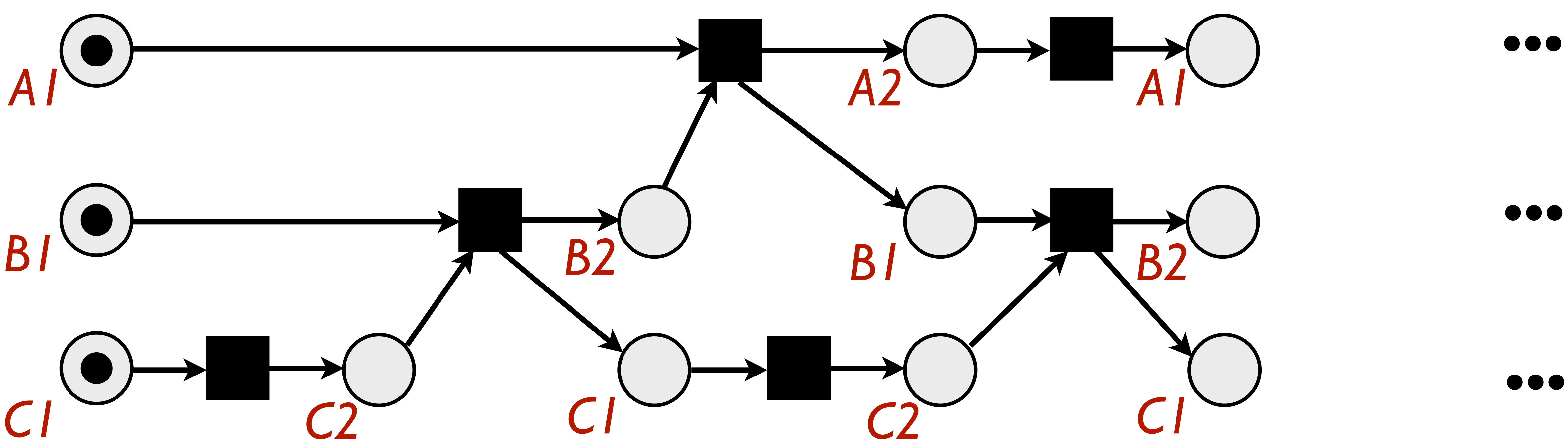
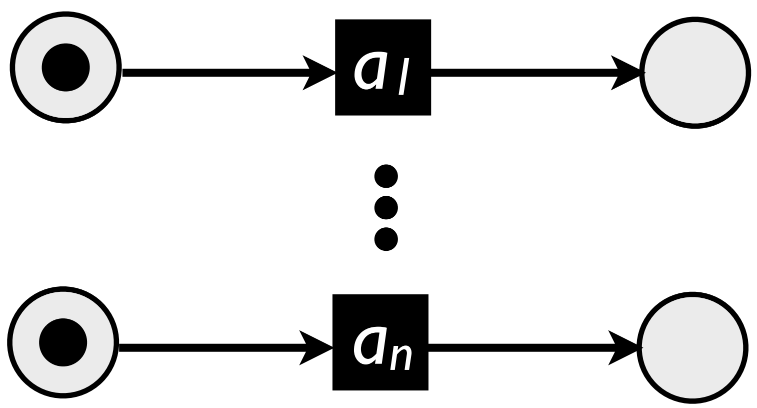




Returning to our small example

- construct an unfolding of the following Petri net:



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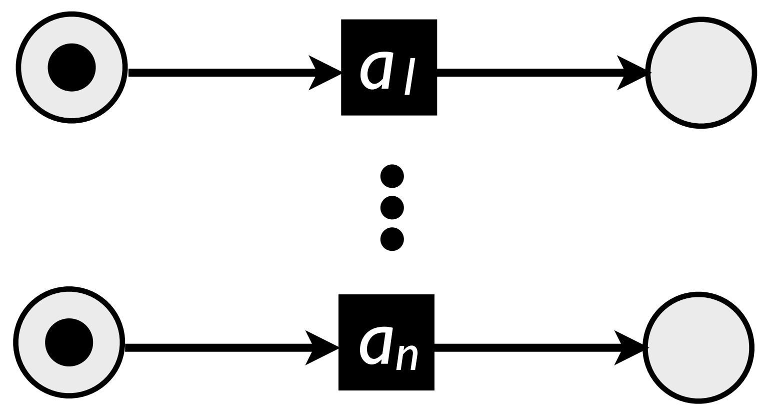


the unfolding is just the Petri net itself!
=> size O(n)
=> whereas interleaving yields 2^n reachable states

Petri net analysis using unfoldings

- suppose we want to know if some transition *t* in a Petri net *N* can occur (i.e. a liveness property)

- compute an answer by exploring the unfolding of *N* until either:
  => a transition labelled *t* is found; or
  => it can be concluded that no such transition occurs

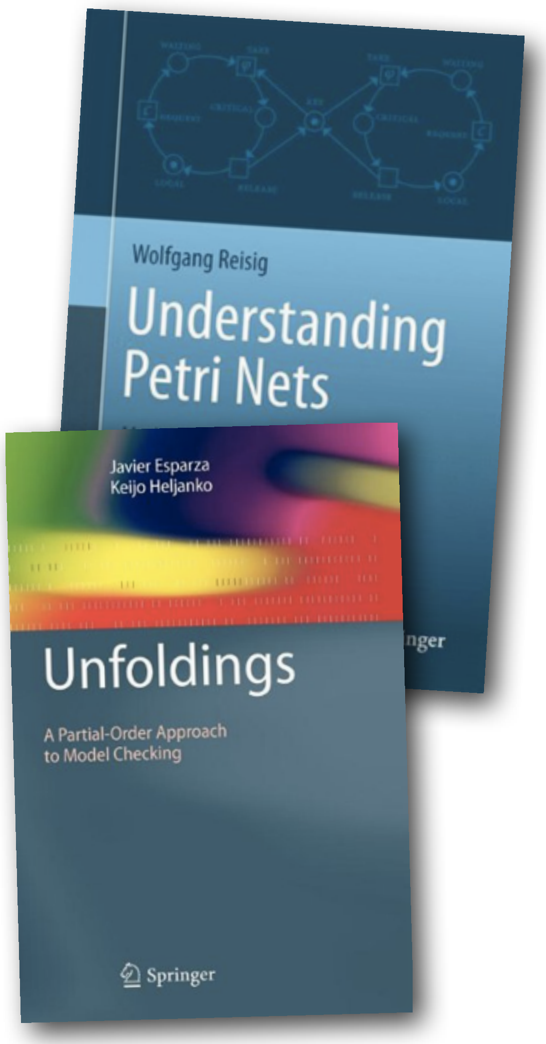
- important to note that only a finite prefix of the unfolding is explored
  => Esparza & Heljanko (2008) cover this
  important part (that we omit)

Next on the agenda

1. modelling concepts: *cookies for everyone!*
2. synchronisation problems as Petri nets
3. Petri net analyses
4. true concurrency semantics; unfoldings

Main sources for this lecture

- Understanding Petri Nets (2013)
  => by Wolfgang Reisig
  => chapters 1-3



- Unfoldings (2008)
  => by Javier Esparza & Keijo Heljanko
  => chapters 1-3

- both available online (see the course webpage)

Summary

- Petri nets facilitate a graphical, intuitive means of modelling concurrent and distributed systems

- automatic analyses exist for reachability, boundedness, liveness, ... but are expensive in the general case

- unfoldings (based on true concurrency) may give a more compact representation of concurrency than reachability graphs (based on interleaving)