Problem Sheet 10: Verification of Real-Time Systems Sample Solutions

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Starred exercises (∗) are more challenging than the others.

1 MTL Property Checking

i. Yes: it simply means that a holds at every position (if any) of accepted timed words.

ii. No: this requires that relative to every position (if any) of accepted timed words, a occurs 1 time unit in the future; but this cannot be the case for the last position of any (non-empty) timed word. (The only position that can be reasoned about relative to the end position *is* the end position, which is exactly 0 time units in the future.) A counterexample is the timed word $(a, 1.0)$ $(a, 2.0)$.

iii. Yes: the formula requires that *if* there is a future position 1 time unit in the future, *then* a holds there.

iv. Yes: the clock x is reset after reading a, then to reach an accepting state, c must occur within the range $(0, 1)$ because of the clock constraint $0 < x < 1$.

v. Yes: as above, noting that b must occur before c.

vi. Yes: as above. It expresses that after reading a, c must occur within the range $(0, 1)$, and until then, only a or b may occur (only the latter does).

vii. No. A counterexample is the timed word $(a, 1.0)$ $(b, 1.2)$ $(c, 1.3)$.

2 Region Automaton Construction

i. First, draw the clock regions associated with the timed automaton. Since there is only one clock x, and because the maximum constant in the clock constraints is 1, our diagram is very simple:

The initial and accepting state of the region automaton will be $(S1, x = 0)$. To determine the outgoing edges from this state, we first determine the *time successors* of the region $x = 0$. This is the set of clock regions that can be reached from $x = 0$ by letting time pass, i.e.

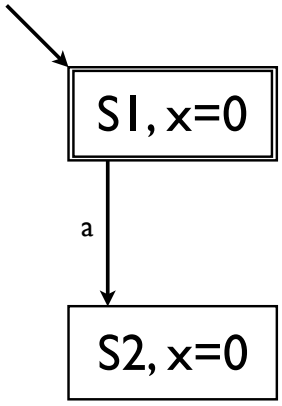
$$0 < x < 1$$

$$x = 1$$

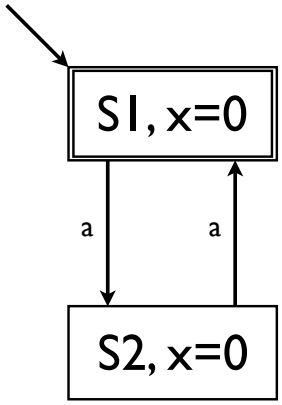
$$x > 1$$

(We don't break $x > 1$ into smaller clock regions because the largest constant in the clock constraints is 1.)

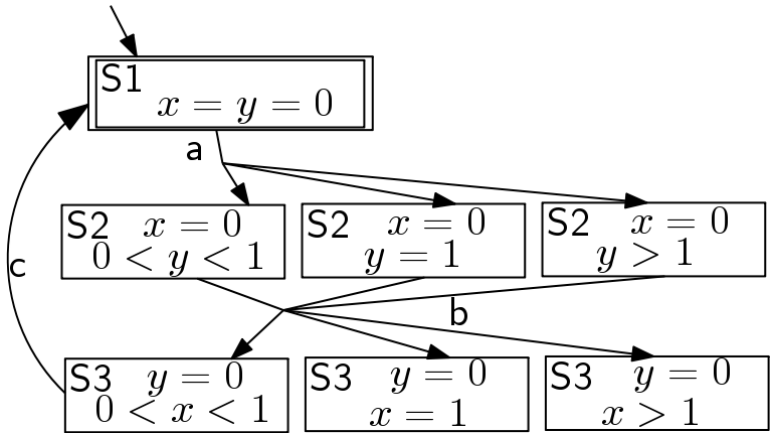
In the original timed automaton, we can reach state $S2$ from $S1$ when $x = 1$ and the next position of the timed word is a. One might think that we should therefore add an edge from $(S1, x = 0)$ to $(S2, x = 1)$ on a. However, the original automaton resets x to 0, so instead of adding an edge to $(S2, x = 1)$, we add an edge to $(S2, x = 0)$, i.e.



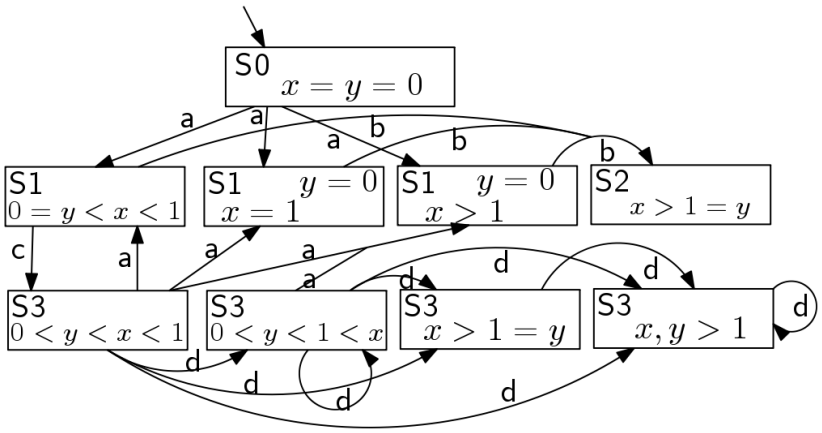
Through similar reasoning, for the other edge in the original timed automaton, we complete the region automaton (omitting the non-reachable states):



ii. Following the same process used for the previous question, we get the (somewhat larger!) region automaton below. You can construct it more efficiently by noting that at least one clock is reset on each transition (e.g. for the first transition, the corresponding regions will all have $x = 0$ but varying y).



iii. (∗)



3 Semantics of MTL Formulae

i. The empty word satisfies $\Box \Diamond > 0$ true, but the same is not true for non-empty words ($\Diamond > 0$ true does not hold relative to the final position, since there are no positions greater than 0 time units in the future).

ii. The formula $\Box \Diamond \geq 0$ true is satisfied by any word. For such a word w, the relation $w, i \models \Diamond \geq 0$ true must hold for all positions i in w, i.e. there is some $j \geq i$ such that $w, j \models$ true. Clearly this is the case because we can always take $j = i$.

iii. The formulae are not in general equivalent. Let $w = (p, 4)\ (q, 8)$ and $a = 1, b = 2, c = 3, d = 6$. Then $w \models \Diamond[a + c, b + d]\ q = \Diamond[4, 8]\ q$, but $w \not\models \Diamond[a, b]\ \Diamond[c, d]\ q = \Diamond[1, 2]\ \Diamond[3, 6]\ q$.