Software Verification

Assertion Inference

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Proving Programs Automatically

The Program Verification problem:

- Given: a program P and a specification S = [Pre, Post]
- Determine: if every execution of P, for any value of input parameters, satisfies S
- Equivalently: establish whether {Pre} P {Post} is (totally) correct

- A general and fully automated solution to the Program Verification problem is unachievable because the problem is undecidable
- One of the consequences of this intrinsic limitation is the impossibility of computing intermediate assertions fully automatically

(It is not an obvious consequence: formally, a reduction between undecidable problems)

Proving Programs Automatically

The Program Verification problem:

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- Equivalently: establish whether {Pre} P {Post} is (totally) correct

One way to put it, practically:

> Proving the correctness of a computer program requires knowledge about the program that is not readily available in the program text
>
> -- Chang & Leino

In this lecture, we survey techniques to automatically infer assertions in interesting special cases

The Assertion Inference Paradox

Correctness is consistency of implementation to specification

The paradox:

> if the specification is inferred from the implementation, what do we prove?

The Assertion Inference Paradox

The paradox:

if the specification is inferred from the implementation, what do we prove?

Possible retorts:

- The paradox only arises for correctness proofs; there are other applications (e.g. reverse-engineering legacy software)
- The result may be presented to a programmer for assessment
- Inferred specification may be inconsistent, thus denoting a problem

The Assertion Inference Paradox

The paradox:

if the specification is inferred from the implementation, what do we prove?

The paradox does not arise if we only infer intermediate assertions and not specifications (pre and postconditions)

- Intermediate assertions are a technical means to an end (proving correctness)
  - tools infer loop invariants
- The specification is a formal description of what the implementation should do
  - programmers write specifications

Invariants

Let us consider a general (and somewhat informal) definition of invariant:

Def. Invariant: assertion whose truth is preserved by the execution of (parts of) a program.

```
x: INTEGER
from x := 1  until ...  loop x := - x end
```

Some invariants:

- $-1 \le x \le 1$
- $x = -1 \lor x = 0 \lor x = 1$
- $x \ge -10$

Kinds of Invariants

Def. Invariant: assertion whose truth is preserved by the execution of (parts of) a program.

We can identify different types of invariants, according to what parts of the program preserve the invariant:

- Location invariant at x: assertion that holds whenever the computation reaches location x
- Program invariant: predicate that holds in any reachable state of the computation
- Class invariant: predicate that holds between (external) feature invocations
- Loop invariant: predicate that holds after every iteration of a loop body

$$\frac{\{P\}\ A\ \{I\} \qquad \{I \wedge \neg\ c\}\ B\ \{I\}}{\{P\}\ \textbf{from}\ A\ \textbf{until}\ c\ \textbf{loop}\ B\ \textbf{end}\ \{I \wedge c\}}$$

Kinds of Invariants

```
1:        x: INTEGER
2:
3:        from x := 1
4:        until ...
5:        loop x := - x end
```

- Location invariant at 2:

- Loop invariant:

- Program invariant:

Kinds of Invariants

```
1:      x: INTEGER
2:
3:      from x := 1
4:      until ...
5:      loop x := - x end
```

- Location invariant at 2:

  $x = 0$

- Loop invariant:

  $x = -1 \vee x = 1$

- Program invariant:

  $x \geq -10$

Focus on Loop Invariants

If we have loop invariants we can get (basically) everything else at little cost

- while getting loop invariants requires invention

In the following discussion we focus on loop invariants (and call them simply "invariants")

This focus is also consistent with the Assertion Inference Paradox

Focus on Loop Invariants

The various kinds of invariants are closely related by the inference rules of Hoare logic

- If Lx is a location invariant at x then:

  @x ⇒ Lx

  is a program invariant

- If P is a program invariant then it is also a location invariant at every location x

- If I is a loop invariant of:

  x: from ... until c loop ... end

  then I ∧ c is a location invariant at x+1

- If L is a location invariant at x+1:

  x: a := b + 3

  then L [b + 3 / a] is a location invariant at x

- Etc...

$$\frac{\{P\}\ A\ \{I\} \qquad \{I \wedge \neg c\}\ B\ \{I\}}{\{P\}\ \textbf{from}\ A\ \textbf{until}\ c\ \textbf{loop}\ B\ \textbf{end}\ \{I \wedge c\}}$$

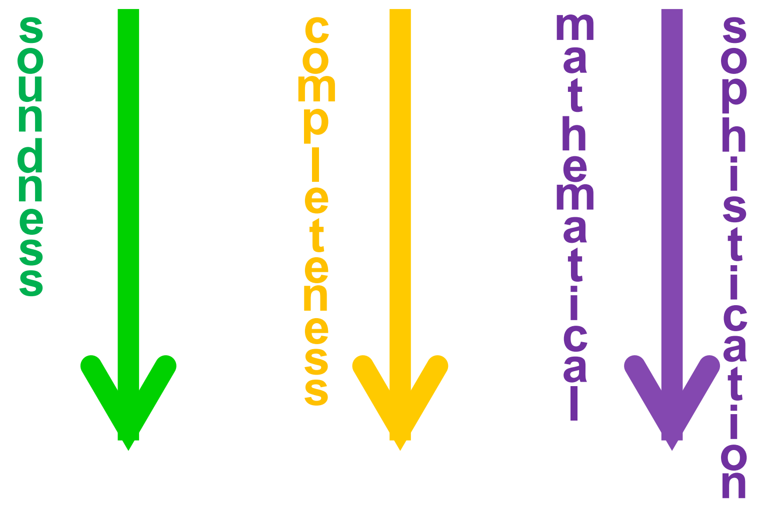
$$\{P\ [e\ /\ x]\}\ x := e\ \{P\}$$

Techniques for Invariant Inference

Classification of invariant inference techniques:

- Dynamic techniques
- Static techniques
  - statistical techniques
  - exact techniques

(Roughly) direction of increasing:



Exact Static Techniques for Invariant Inference

Static Invariant Inference: classification

Static exact techniques for invariant inference are further classified in categories:

- Direct
- Assertion-based
    - postcondition mutation
- Based on abstract interpretation
- Constraint-based
    - usually, template-based

Exact Static Techniques for Invariant Inference:

Postcondition-mutation Approach

The Role of User-provided Contracts

Techniques for invariant inference rarely take advantage of other annotations in the program text, such as contracts provided by the user

- Not every annotation can (or should, cf. Assertion Inference Paradox) be inferred automatically.

However, there is a close connection between a loop's invariant and its postcondition

The Role of User-provided Contracts

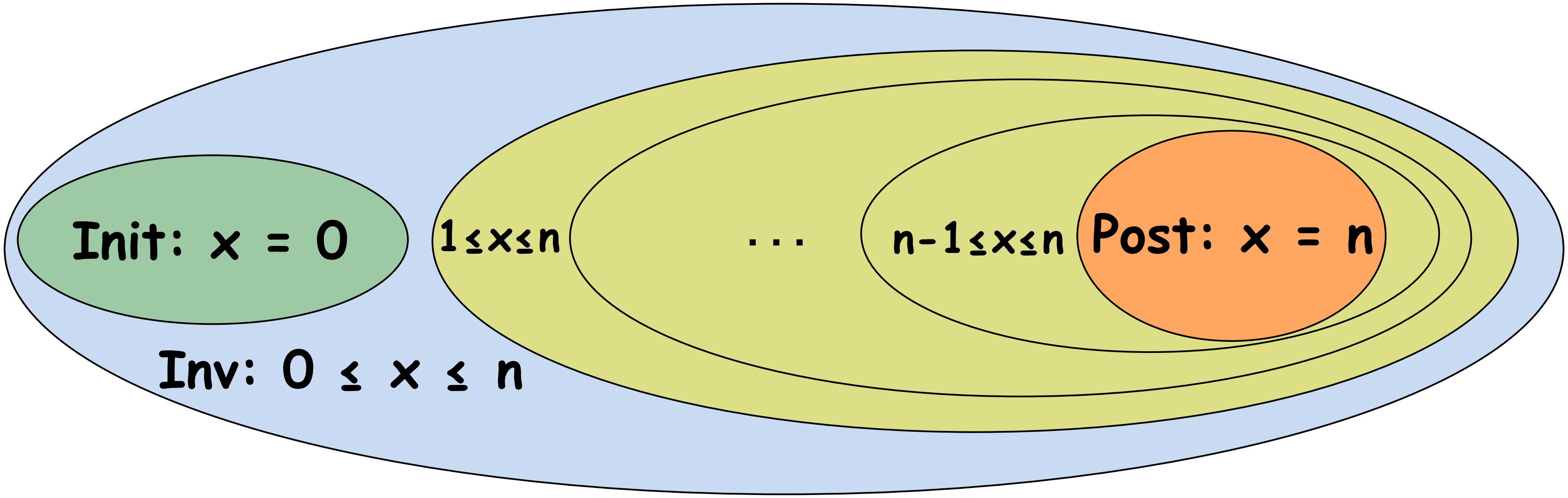
However, there is a close connection between a loop's invariant and its postcondition

Semantically, the invariant is a weakened form of the postcondition

- A larger set of program states

Example: from $x := 0$ until $x = n$ loop $x := x + 1$ end

- Post: $x = n$ (with $n > 0$)
- Invariant: $0 \le x \le n$



Invariants by Postcondition Mutation

- In a nutshell:

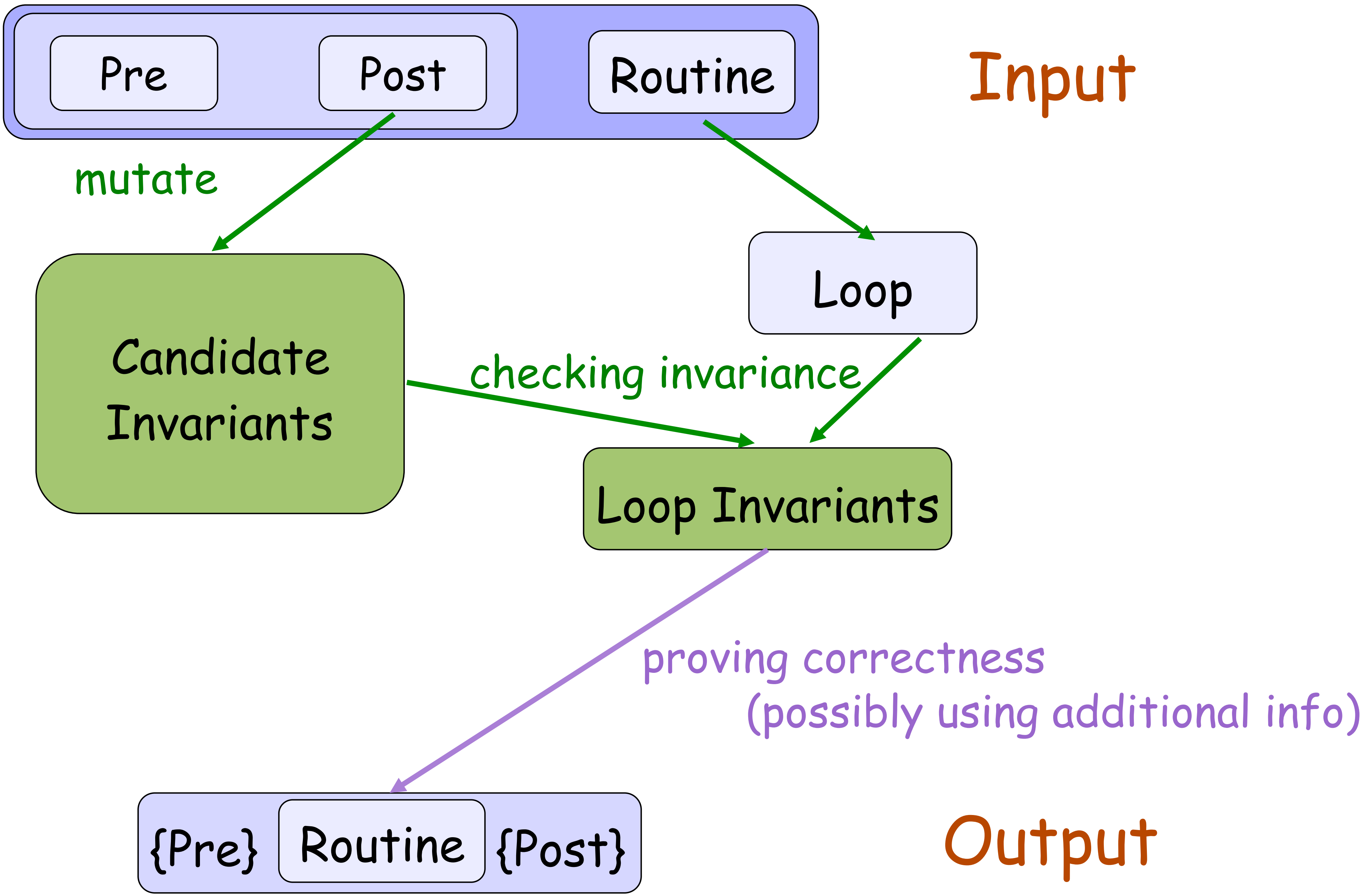
  Static verification of candidate invariants
  obtained by mutating postconditions

  - Assume the availability of postconditions
  - Mutate postconditions according to various heuristics
    - the heuristics mirror common patterns that link postconditions to invariants
    - each mutated postcondition is a candidate invariant
  - Verify which candidates are indeed invariants
    - With an automatic program prover such as Boogie
  - Retain all verified invariants

- 2009 – gin-pink

- 2013 - DynaMate

Loop invariant inference



Maximum value of an array

```
max (A: ARRAY [T] ; n: INTEGER): T
  require A.length = n ≥ 1
  local i: INTEGER
  do
    from i := 0 ;  Result := A[1];
    until i = n
    loop
      i := i + 1
      if  Result ≤ A[i]  then  Result := A[i]  end
    end
  ensure       ( ∀ 1 ≤ j ≤ n ⇒ A[j] ≤ Result )  and
                  ( ∃ 1 ≤ j ≤ n ∧ A[j] = Result )
```

Maximum value of an array

max (A: ARRAY [T] ; n: INTEGER): T
  require $A.length = n \geq 1$
  ensure $(\forall\ 1 \leq j \leq n \Rightarrow A[j] \leq Result)$ and
      $(\exists\ 1 \leq j \leq n \wedge A[j] = Result)$

- Constant relaxation: replace "constant" n by "variable" i
- Term dropping: remove second conjunct

Invariant: $\forall\ 1 \leq j \leq i \Rightarrow A[j] \leq Result$

Maximum value of an array (cont'd)

```
max (A: ARRAY [T] ; n: INTEGER): T
    require A.length = n ≥ 1
    ensure ( ∀ 1 ≤ j ≤ n ⇒ A[j] ≤ Result ) and
                        ( ∃ 1 ≤ j ≤ n ∧ A[j] =
Result )
```

- Term dropping: remove first conjunct

Invariant: $\exists\ 1 \le j \le n \wedge A[j] = Result$

Maximum value of an array (2nd version)

```
max_v2 (A: ARRAY [T] ; n: INTEGER): T
  require A.length = n ≥ 1
  local i: INTEGER
  do
    from i := 1 ;  Result := A[1];
    until i > n
    loop
      if  Result ≤ A[i]  then  Result := A[i]  end
      i := i + 1
    end
  ensure ∀ 1 ≤ j ≤ n ⇒ A[j] ≤ Result
```

Maximum value of an array (2nd version)

```
max_v2 (A: ARRAY [T] ; n: INTEGER): T
    require A.length = n ≥ 1
    ensure ∀ 1 ≤ j ≤ n ⇒ A[j] ≤ Result
```

- Constant relaxation: replace "constant" n by "variable" i

$$\forall\ 1 \le j \le i \Rightarrow A[j] \le Result$$

- Variable aging:

  use expression representing the previous value of i: i - 1

Invariant: $\forall\ 1 \le j \le i - 1 \Rightarrow A[j] \le Result$

Postcondition Mutation Heuristics

Constant relaxation

- replace "constant" by "variable"
  - cannot/may be changed by any of the loop bodies

Uncoupling

- replace subexpression appearing twice by two subexpressions
  - for example: subexpression = variable id

Term dropping

- remove a conjunct

Variable aging

- replace subexpression by another expression representing its previous value

Invariant Inference: the Algorithm

Goal: find invariants of loops in procedure proc

For each:

- post: postcondition clause of proc
- loop: outer loop in proc

compute all mutations M of post w.r.t. loop

- considering postcondition clauses separately implements term dropping

Result: any formula in M which can be verified as invariant of any loop in proc

Array Partitioning

```
partition (A: ARRAY [T]; n: INTEGER; pivot: T): INTEGER
  require A.length = n ≥ 1
  local l, h: INTEGER
  do
    from l := 1 ; h := n        until l = h
    loop
      from until l = h or A[l] > pivot loop l := l + 1 end
      from until l = h or pivot > A[h] loop h := h - 1 end
      A.swap (l, h)
    end
    if pivot ≤ A[l] then l := l - 1 end ;  h := l ; Result := h
  ensure  (∀ 1≤k≤ Result ⇒ A[k] ≤ pivot) and
          (∀ Result<k≤n ⇒ A[k] ≥ pivot)
```

Array Partitioning

partition (A: ARRAY [T]; n: INTEGER; pivot: T): INTEGER
  require A.*length* = n ≥ 1
  ensure (∀ $1 \le k \le$ Result ⇒ A[k] ≤ pivot) and
    (∀ Result$< k \le n$ ⇒ A[k] ≥ pivot)

- Uncoupling: replace first occurrence of Result by l
  and second by h
  (∀ $1 \le k \le l$ ⇒ A[k] ≤ pivot) and (∀ $h < k \le n$ ⇒ A[k] ≥ pivot)
- Variable aging: use expression representing the previous
  value of l: $l - 1$

Invariant:
  (∀ $1 \le k \le l - 1$ ⇒ A[k] ≤ pivot) and (∀ $h < k \le n$ ⇒ A[k] ≥ pivot)

Array Partitioning

$partition\ (A: ARRAY\ [T];\ n: INTEGER;\ pivot: T): INTEGER$
$\quad \textbf{require}\ A.length = n \geq 1$
$\quad \textbf{ensure}\ (\forall\ 1 \leq k \leq Result \Rightarrow A[k] \leq pivot)\ \text{and}$
$\qquad\qquad (\forall\ Result < k \leq n \Rightarrow A[k] \geq pivot)$

- Term dropping: remove first conjunct

$$\forall\ Result < k \leq n \Rightarrow A[k] \geq pivot$$

- Constant relaxation: replace "constant" Result by "variable" h

Invariant: $\forall\ h < k \leq n \Rightarrow A[k] \geq pivot$

Postcondition Mutation in DynaMate

DynaMate is a tool that combines postcondition mutation with template-based techniques for invariant inference and with automated testing. It finds loop invariants to verify Java/JML programs.

Experiments on 28 methods of java.util:

# proved methods	% proved proof obligations	# invariants post mutations	# other invariants	time
25	97 %	10	15	45 min.

Limitations of the approach

Some invariants are not mutations of the postcondition

- “completeness” of the postcondition
- integration with other techniques
- more heuristics

Combinatorial explosion

- predefined mutations, time out

Dependencies

- especially with nested loops
- dynamic checking

Limitations of automated reasoning techniques

Exact Static Techniques for Invariant Inference:

Constraint-based Approach

Constraint-based Invariant Inference

- In a nutshell:

  encode semantics of iteration as constraints on a template invariant

  - Choose a template invariant expression
    - template defines a (infinte) set of assertions
  - Encode the loop semantics as a set of constraints on the template
    - initiation + consecution
  - Solve the constraints
    - this is usually the complex part
  - Any solution is an invariant

- E.g.: 2003 -- Henny Sipma et al. 2004 -- Zohar Manna et al.
  2007 -- Tom Henzinger et al.

Constraint-based Inv. Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Template invariant expression:

$$T = c \cdot x + d \cdot n + e \leq 0$$

- Constraints encoding loop semantics:
  - Initiation: "T holds for the initial values of x and n"

$$T\,[0/x;\, n_0/n] \equiv c \cdot 0 + d \cdot n_0 + e \leq 0 \equiv d \cdot n_0 + e \leq 0$$

Constraint-based Inv. Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Constraints encoding loop semantics:

  Consecution: "if T holds and one iteration of the loop is executed, T still holds"

  $T\,[x/x;\ n/n] \wedge (\neg(x \geq n) \wedge x' = x + 1 \wedge n' = n) \Rightarrow T\,[x'/x;\ n'/n]$

- Solving the constraints requires to eliminate occurrences of x, x', n, n'
  - For linear constraints we can use Farkas's Lemma

Farkas's Lemma (1902)

Let S be a system of linear inequalities over *n* real variables:

$$S \triangleq \begin{bmatrix} a_{11}x_1 + \cdots + a_{1n}x_n + b_1 & \leqslant & 0 \\ \vdots & \vdots & \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n + b_m & \leqslant & 0 \end{bmatrix}$$

and let Ψ be a linear inequality:

$$\psi \triangleq c_1x_1 + \cdots + c_nx_n + d \leqslant 0$$

Then S ⇒ Ψ is valid iff S is unsatisfiable or there exist *m* +1 real nonnegative coefficients $\lambda_0, \lambda_1, \ldots, \lambda_m$ such that:

$$c_j = \sum_{i=1}^{m} \lambda_i a_{ij} \quad (1 \leqslant j \leqslant m) \qquad d = -\lambda_0 + \sum_{i=1}^{m} \lambda_i b_i$$

Constraint-based Inv. Inference: Example

Use Farkas's lemma to turn the consecution constraint:

$$T[x/x;\, n/n] \wedge x < n \wedge x' = x + 1 \wedge n' = n \Rightarrow T[x'/x;\, n'/n]$$

into a constraint over c, d, and e only.

$$\begin{array}{c|cccccl}
\lambda_1 & cx & +dn & & & +e & \leqslant 0 \\
\lambda_2 & x & -n & & & +1 & \leqslant 0 \\
\lambda_3 & -x & & +x' & & -1 & \leqslant 0 \\
\lambda_4 & x & & -x' & & +1 & \leqslant 0 \\
\lambda_5 & & -n & & +n' & & \leqslant 0 \\
\lambda_6 & & n & & -n' & & \leqslant 0 \\
\hline
 & & & cx' & +dn' & +e & \leqslant 0
\end{array}$$

Constraint-based Inv. Inference: Example

$$
\begin{array}{c|cccccc}
\lambda_1 & cx & +dn & & & +e & \leqslant 0 \\
\lambda_2 & x & -n & & & +1 & \leqslant 0 \\
\lambda_3 & -x & & +x' & & -1 & \leqslant 0 \\
\lambda_4 & x & & -x' & & +1 & \leqslant 0 \\
\lambda_5 & & -n & & +n' & & \leqslant 0 \\
\lambda_6 & & n & & -n' & & \leqslant 0 \\
\hline
 & & & cx' & +dn' & +e & \leqslant 0
\end{array}
$$

$$
\Phi \triangleq \exists \lambda_0, \ldots, \lambda_6 \begin{bmatrix} \lambda_0, \ldots, \lambda_6 \geqslant 0 \\ \lambda_1 c + \lambda_2 - \lambda_3 + \lambda_4 = 0 \\ \lambda_1 d - \lambda_2 - \lambda_5 + \lambda_6 = 0 \\ \lambda_3 - \lambda_4 = c \\ \lambda_5 - \lambda_6 = d \\ -\lambda_0 + \lambda_1 e + \lambda_2 - \lambda_3 + \lambda_4 = e \end{bmatrix}
$$

Constraint-based Inv. Inference: Example

$$\Phi \quad \triangleq \quad \exists \lambda_0, \ldots, \lambda_6 \left[\begin{array}{c} \lambda_0, \ldots, \lambda_6 \geqslant 0 \\ \lambda_1 c + \lambda_2 - \lambda_3 + \lambda_4 = 0 \\ \lambda_1 d - \lambda_2 - \lambda_5 + \lambda_6 = 0 \\ \lambda_3 - \lambda_4 = c \\ \lambda_5 - \lambda_6 = d \\ -\lambda_0 + \lambda_1 e + \lambda_2 - \lambda_3 + \lambda_4 = e \end{array} \right]$$

Finally, eliminate existential quantifiers from Φ to get the constraint:

$$c \leq 0 \ \vee \ (c + d = 0 \wedge e \leq 0)$$

(Quantifier elimination is also quite technical, but there are tools that do that for us)

Constraint-based Inv. Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Any solution [c, d, e] to:
  - Initiation and Consecution:

$$(d \cdot n_0 + e \leq 0) \wedge (c \leq 0 \;\vee\; (c + d = 0 \wedge e \leq 0))$$

  determines an invariant of the loop.

  For example, substituting the following values in the template leads to invariants:

  - [0, -1, 0] ---> $n \geq 0$
  - [1, 0, 0] ---> $x \geq 0$
  - [1, -1, 0] ---> $x - n \leq 0$

Constraint-based Inv. Inference: Summary

- Main issues:
  - choice of invariant templates for which effective decision procedures exist
    - interesting research topic per se, on the brink of undecidability
  - heuristics to extract the "best" invariants from the set of solutions
- Advantages:
  - sound & complete (w.r.t. the template)
  - exploit heterogeneous decision procedures together
  - fully automated (possibly except for providing the template)
    - providing the template introduces a "natural" form of user interaction
- Disadvantages:
  - suitable mathematical decision theories are usually quite sophisticated
    - hence, hard to extend and customize
  - exact constraint solving is usually quite expensive
  - mostly suitable for algebraic/numeric/scalar invariants
    - requires integration with other techniques to achieve full functional correctness proofs

Dynamic Techniques for Invariant Inference

Dynamic Invariant Inference

- In a nutshell:

  testing of candidate invariants

  - Choose a set of test cases
  - Perform runtime monitoring of candidate invariants
  - If some test run violates a candidate, discard the candidate
  - The surviving candidates are guessed invariant

- Daikon tool, 1999 -- Mike Ernst et al.
- CITADEL: Daikon for Eiffel, 2008 -- Nadia Polikarpova
- AutoInfer for Eiffel (Yi "Jason" Wei et al.)

Dynamic Invariant Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Test cases: $\{\, n = k \mid 0 \leq k \leq 1000 \,\}$
- Candidate invariants:
  - $\{\, x \geq c \mid -1000 \leq c \leq 1000 \,\}$,
    $\{\, n \geq c \mid -1000 \leq c \leq 1000 \,\}$
  - $\{\, x = c \cdot n + d \mid -500 \leq c, d \leq 500 \,\}$
  - $\{\, x < n, x \leq n, x = n, x \neq n, x \geq n, x > n \,\}$
  - $\{\, x \pm n \geq c \mid -500 \leq c \leq 500 \,\}$
  - ...

Dynamic Invariant Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Survivors (after loop iterations) :
  - $\{ x \geq -c \mid 0 \leq c \leq 1000 \}$,
    $\{ n \geq -c \mid 0 \leq c \leq 1000 \}$
  - $x \leq n$
  - $\{ x + n \geq c \mid -500 \leq c \leq 500 \}$
  - ...

Dynamic Invariant Inference: Summary

- Main issues:
  - choose suitable test cases
  - handle huge sets of candidate invariants (runtime overhead)
  - estimate soundness/quality of survivor predicates
  - select heuristically the "best" survivor predicates
- Advantages:
  - straightforward to implement (at least compared to other techniques)
  - guessing is often rather accurate in practice (possibly with some heuristics)
  - customizable and rather flexible:
    in principle, whatever you can test you can check for invariance
- Disadvantages:
  - unsound (educated guessing)
  - without heuristics, large amount of useless, redundant predicates
  - sensitive to choice of test cases
  - some complex candidate invariants are difficult to implement efficiently

Exact Static Techniques for Invariant Inference:

Direct Approach

Direct Static Invariant Inference

- In a nutshell:

  solve the fixpoint equations underlying the program

  - v(i): value of variable v at step i of the computation
  - Encode the semantics of loops explicitly and directly as recurrence equations over v(i)
  - Solve recurrence equations
  - Eliminate step parameter i to obtain invariant

- 1973 -- Shmuel Katz & Zohar Manna
- 2005 -- Laura Kovacs et al.

Direct Static Invariant Inference: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- $x(i)$, $n(i)$
- Recurrence relations:

$$x(i) = \begin{cases} 0 & i = 0 \\ x(i-1)+1 & 0 < i \leqslant n_0 \\ x(i-1) & i > n_0 \end{cases} \qquad n(i) = \begin{cases} n_0 \geqslant 0 & i = 0 \\ n(i-1) & i > 0 \end{cases}$$

Direct Static Invariant Inference: Example

$$x(i) = \begin{cases} 0 & i = 0 \\ x(i-1)+1 & 0 < i \leqslant n_0 \\ x(i-1) & i > n_0 \end{cases} \qquad n(i) = \begin{cases} n_0 \geqslant 0 & i = 0 \\ n(i-1) & i > 0 \end{cases}$$

- Solving recurrence relations:
  - $x(i) = \min(n_0, i) \geq 0$
  - $n(i) = n_0$
- Eliminating step parameter i:
  - $x(i) - n(i) = \min(n_0, i) - n_0 \leq 0$, or:
  - $x - n \leq 0$, hence:
  - $0 \leq x \leq n$

Direct Static Invariant Inference: Summary

- Main issues:
  - in its bare form, more a set of guidelines than a technique
  - step parameter elimination is tricky
- Advantages:
  - since semantics is represented explicitly, obtained invariants are often powerful
  - benefits from the programmer's ingenuity
  - additional information about the program can be plugged in
- Disadvantages:
  - solving recurrence equations can be very difficult (when possible at all)
  - typically restricted to algebraic/numeric/scalar invariants

APPENDIX – Additional Material

Exact Static Techniques for Invariant Inference:

Approach Based on Abstract Interpretation

Abstract Interpretation for Invariants

- In a nutshell:

  symbolic execution over an abstract domain
  with guarantee of termination

  - Consider the over-approximation of the value of variables over some coarse abstract domain (instead of their exact values)
  - Symbolically execute the program over the abstract domain
  - Iterate loops until termination
    - termination guaranteed by the nature of the abstract domain or by heuristic cut-offs (widening)
  - The final expression is an invariant

- 1976 -- Michael Karr
- 1977, 1978 -- Patrick & Radhia Cousot, Nicolas Halbwachs
- ...

Abstract Interpretation for Inv.: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Abstract interval domain:
  conjunction of inequalities in the form
  $$v \le c \text{ or } c \le v$$
  for any program variable v and integer constant c
- Initially: $S(0) \triangleq \{\, 0 \le x, x \le 0, 0 \le n \,\}$
- After one loop iteration: $S(1) \triangleq \{\, 1 \le x, x \le 1, 0 \le n \,\}$
- Set of abstract states reached in at most one loop iteration:
  $$S(0) \vee S(1) = \{\, 0 \le x, x \le 1, 0 \le n \,\}$$

Abstract Interpretation for Inv.: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
      from x := 0
      until x ≥ n
      loop x := x + 1 end
end
```

- Initially: $S(0) \triangleq \{ 0 \leq x, x \leq 0, 0 \leq n \}$
- Set of abstract states reached in at most one loop iteration:
  $S(0) \vee S(1) = \{ 0 \leq x, x \leq 1, 0 \leq n \}$
- $S(0) \vee S(1)$ does not subsume $S(0)$
  - no fixpoint, keep on iterating
- Abstract states after at most k loop iterations:

  $$S(0) \vee \ldots \vee S(k) = \{ 0 \leq x, x \leq k, 0 \leq n \}$$

- No convergence as: $S(0) \vee \ldots \vee S(k)$ does not subsume $S(0) \vee \ldots \vee S(k\text{-}1)$

Abstract Interpretation for Inv.: Example

```
dummy_routine (n: NATURAL)
local x: NATURAL do
        from x := 0
        until x ≥ n
        loop x := x + 1 end
end
```

- Abstract states after at most k loop iterations:

  $$S(0) \lor \ldots \lor S(k) = \{ 0 \le x, x \le k, 0 \le n \}$$

- Apply heuristic over-approximation:
  - relax $S(0) \lor \ldots \lor S(k)$ by dropping inequalities with growing bounds:
    $S' = \text{widen}\{ 0 \le x, x \le k, 0 \le n \} = \{ 0 \le x, 0 \le n \}$
- S' is a fixpoint of the loop iteration
- $0 \le x \land 0 \le n$ is a loop invariant
  - a very weak one, but more sophisticated choices of abstract domain and/or heuristic over-approximation would yield the "desired" $0 \le x \le n$

Abstract Interpretation for Inv.: Summary

- Main issues:
  - effective choice of abstract domain
    - trade off: accuracy vs. computational efficiency
  - smart choice of heuristic widening
- Advantages:
  - the abstract interpretation framework is quite general and customizable to many different program properties
  - fully automated
  - sound
  - scalable: efficient implementations are possible
- Disadvantages:
  - incompleteness, from two sources:
    - invariants can be inexpressible in the abstract domain
    - even if they are expressible, heuristic widening loses completeness
  - requires integration with other techniques to achieve full functional correctness proofs

Statistical Static Techniques for Invariant Inference

Statistical Static Invariant Inference

The goal of the analysis is usually different than for other classes of invariant inference techniques:

- inferring likely specific behavioral specification (e.g., temporal properties)
- inferring likely violations of invariance behavioral properties
- no functional invariants

```
   ...
x: create a_node
   ...
y: dispose a_node
```

Examples of inferred behavioral specs:

- Location x performs allocation (A) of some resource
- Location y performs deallocation (D) of some resource
- Every allocated resource is eventually deallocated

Statistical Static Invariant Inference

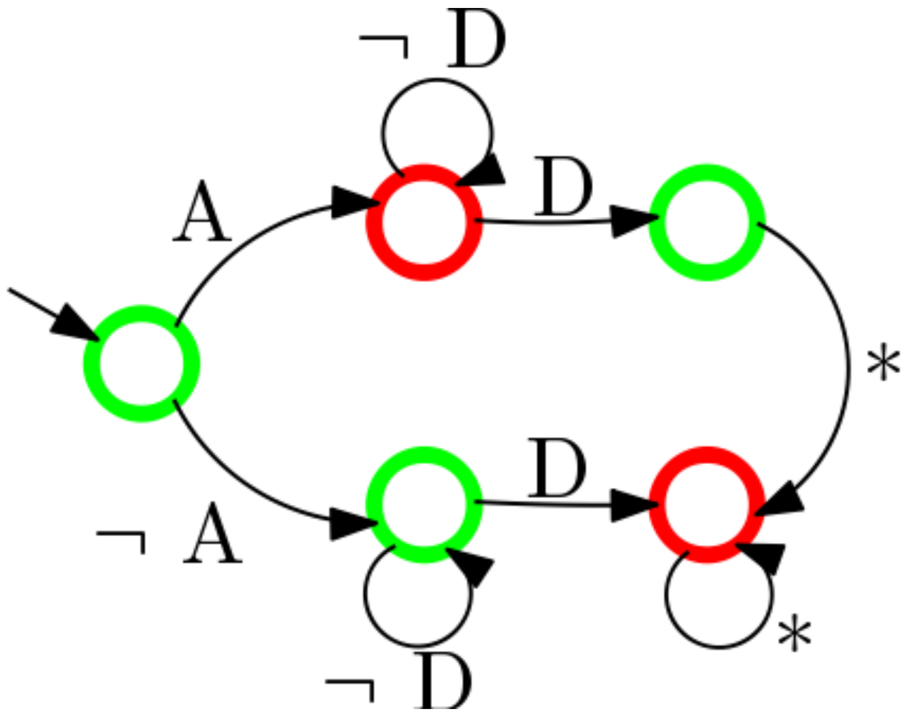
- In a nutshell:

  learning of a statistical model

  - Classify all possible behaviors (w.r.t. goal properties) of code
  - Assign prior probabilities to different behaviors
    - possibly including additional knowledge
  - Compute cumulative probabilities of code
    - e.g., through simple symbolic execution
  - Report likely inferred specifications or likely invariance violations
    - those with the highest cumulative probabilities

- 2001, 2006 -- Dawson Engler et al.
- 2002 -- James Larus et al.
- ...

Statistical Invariant Inference: Example



- Prior probabilities:
  - Pr [green] = 0.9 (probability that the sequence is ok)
  - Pr [red] = 0.1 (probability that the sequence gives a bug)
- Cumulative probabilities, e.g.:
  - Pr [A, ¬D, D] = $\Pi_f f(A, \neg D, D)$ = 0.9 x ... x ...
    - f are the various elements of knowledge

Statistical Invariant Inference: Summary

- Main issues:
  - choose suitable behavioral properties
  - compute efficiently complex products of probabilities for a huge number of candidate behaviors
  - classify possible behaviors by their probabilities
  - introduce effective *ad hoc* factors
- Advantages:
  - reasonably robust w.r.t. the choice of prior probabilities
  - customizable: can incorporate very specific probability factors
  - can handle large "real" programs
- Disadvantages:
  - unsound
  - mostly limited to simple behavioral properties
  - best suited for "systems" code (where full functional correctness is usually not a concern)