Assignment 11: Bisimulations and Coalgebra

ETH Zurich

1 Bisimulations

1.1 Tasks

1. Consider two CCS processes p and q, depicted as follows:

   (a) Are p and q strongly bisimilar? Justify your answer and provide the strong bisimulation relation, if applicable.

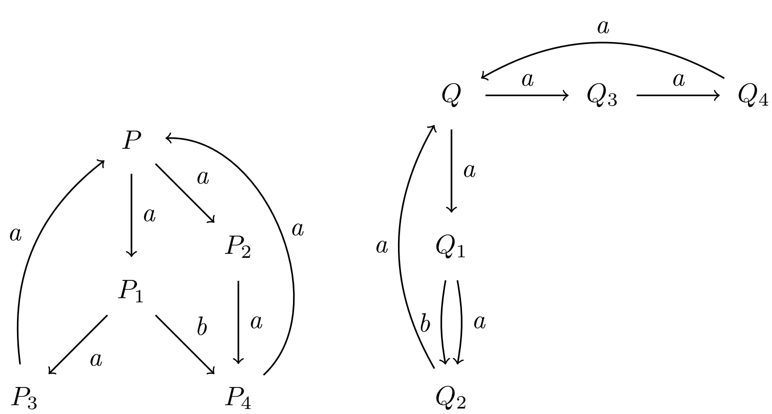
   (b) Are p and q weakly bisimilar? Justify your answer and provide the weak bisimulation relation, if applicable.

   (c) Consider the following CCS processes:

$$\begin{aligned} p' &= p + a\,.\,p_0 \\ p_0 &= b\,.\,p_0 \\ q' &= \overline{a}\,.\,\mathbf{0}, \end{aligned}$$

   where a and $\overline{a}$ are complementary actions, $\mathbf{0}$ stands for the process that cannot perform any action, and p is the process illustrated above.
   Draw the LTSs corresponding to $p' \mid q'$ and $(p' \mid q')\backslash\{a\}$, respectively. Are these LTSs weakly or strongly bisimilar?

2. Consider the following labelled transition system:



   Show that $P \sim Q$ by finding a strong bisimulation $\mathcal{R}$ such that $P\,\mathcal{R}\,Q$.

3. Suppose we have the following definitions of processes

$$\begin{aligned} \mathrm{S} &\stackrel{\mathsf{def}}{=} a.\overline{b}.\mathrm{S} \\ \mathrm{T} &\stackrel{\mathsf{def}}{=} \overline{a}.e.b.\mathrm{T} \\ \mathrm{ST} &\stackrel{\mathsf{def}}{=} (\mathrm{S} \mid \mathrm{T}) \setminus \{a, b\} \end{aligned}$$

Further we have

$$\begin{aligned} \mathrm{U} &\stackrel{\mathsf{def}}{=} e.x.y.\mathrm{U} \\ \mathrm{V} &\stackrel{\mathsf{def}}{=} \overline{x}.\overline{y}.\mathrm{V} \\ \mathrm{UV} &\stackrel{\mathsf{def}}{=} (\mathrm{U} \mid \mathrm{V}) \setminus \{x, y\} \end{aligned}$$

Your task is to

(a) Represent ST and UV as LTSs.

(b) Show that ST and UV are weakly bisimilar.

(c) Suppose we further have $\mathrm{UV}' \stackrel{\mathsf{def}}{=} (\mathrm{U} \mid \mathrm{V}) \setminus \{y\}$. Show that ST and UV$'$ are not weakly bisimilar.

4. Consider the labeled transition system describing the behavior of a process P:

$$\begin{tikzcd} \mathrm{P} \arrow[r, bend left=15, "b"] & \mathrm{P}_1 \arrow[l, bend left=15, "\bar{b}"] \arrow[r, bend left=15, "b"] & \mathrm{P}_2 \arrow[l, bend left=15, "\bar{b}"] \end{tikzcd}$$

Furthermore, consider the CCS process Q defined by the following equations:

$$\begin{aligned} \mathrm{Q} &\stackrel{\mathsf{def}}{=} (\mathrm{Q}_1 \mid \mathrm{Q}_2) \setminus \{a\} \\ \mathrm{Q}_1 &\stackrel{\mathsf{def}}{=} a.\bar{b}.\mathrm{Q}_1 \\ \mathrm{Q}_2 &\stackrel{\mathsf{def}}{=} b.\bar{a}.\mathrm{Q}_2 \end{aligned}$$

(a) Draw a labeled transition system that describes the behavior of process Q.

(b) (a) Are the processes P and Q strongly bisimilar?

(b) Are the processes P and Q weakly bisimilar?

Justify your answers: if yes, give a strong (weak) bisimulation $\mathcal{R}$ such that $\mathrm{P}\,\mathcal{R}\,\mathrm{Q}$; if no, argue why not.

2 Bisimulations up-to

Consider the following non-deterministic automata with state space in **Set** and actions labelled in an alphabet A:

$$\begin{tikzcd} \mathrm{x} & \mathrm{z} \arrow[l, "\mathrm{a}"] \arrow[r, bend left=15, "\mathrm{a}"] & \overline{y} \arrow[l, bend left=15, "\mathrm{a}"] \\ \arrow[from=1-1, to=1-3, bend right, "\mathrm{a}"'] \end{tikzcd} \qquad \begin{tikzcd} \mathrm{u} \arrow[r, bend left=15, "\mathrm{a}"] \arrow[rr, bend right, "\mathrm{a}"'] & \mathrm{w} \arrow[l, bend left=15, "\mathrm{a}"] & \overline{v} \arrow[l, "\mathrm{a}"'] \end{tikzcd}$$

The overlined states $\overline{y}$ and $\overline{v}$ represent accepting states.

2.1 Task

1. Discuss, informally, whether x and u in the above figures are bisimilar. How about their language equivalence?

2. Apply the generalized powerset construction and derive the deterministic automata $(o_x^\sharp, t_x^\sharp)$ and $(o_u^\sharp, t_u^\sharp)$ corresponding to x and u, respectively. Identify a bisimulation relation R stating the language equivalence of x and u.

3. A *bisimulation up-to union* is a relation R on $\mathscr{P}(\mathbf{Set})$ such that whenever $Z\ R\ W$ it holds that:

$$1.\ o^\sharp(Z) = o^\sharp(W) \quad \text{and} \quad 2.\ \text{for all } a \in A,\ t^\sharp(Z)(a)\ u(R)\ t^\sharp(W)(a).$$

   By $u(R)$ we represent the smallest equivalence relation such that:

$$\frac{Z\ R\ W}{Z\ u(R)\ W} \qquad \frac{Z_1\ u(R)\ W_1 \quad Z_2\ u(R)\ W_2}{Z_1 \cup Z_2\ u(R)\ W_1 \cup W_2}.$$

   Moreover, we know that any bisimulation up-to union is contained in a bisimulation. Identify a bisimulation up-to union showing that x and u in the figure are language equivalent. What do you observe?