Concepts of Concurrent Computation
Spring 2015
Lecture 9: Petri Nets

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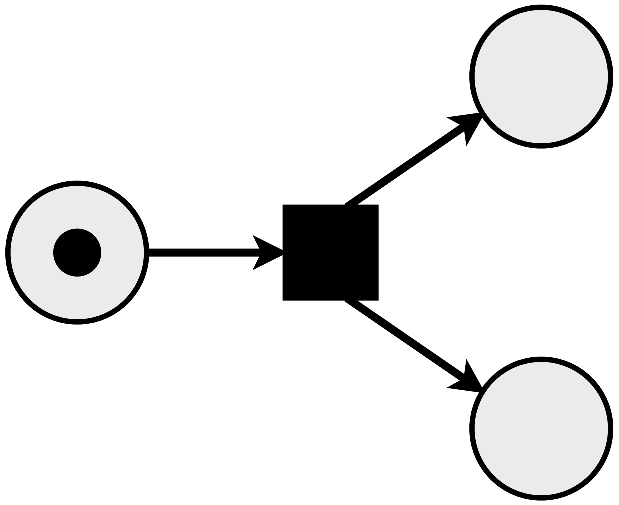
Petri nets

- Petri nets are mathematical models for describing systems with concurrency and resource sharing

- they facilitate many automatic analyses of interest for concurrent systems

- rich, intuitive graphical notation for choice, concurrent execution, interaction with the environment, ...



Petri nets - the origins

- proposed by Carl Adam Petri in his famous thesis *Kommunikation mit Automaten* (1962)

- aimed for a system architecture that could be expanded indefinitely
  => *no central components*
  => *in particular, no central, synchronising clock*
  => *actions with locally confined causes/effects*

- original presentation omitted the graphical representation

Today's agenda

1. modelling concepts: *cookies for everyone!*
2. synchronisation problems as Petri nets
3. Petri net analyses
4. true concurrency semantics; unfoldings

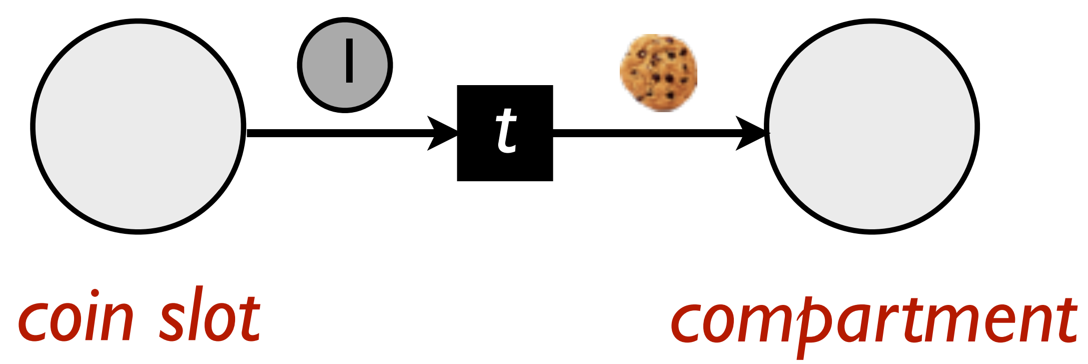
Let’s design a cookie vending machine

coin slot

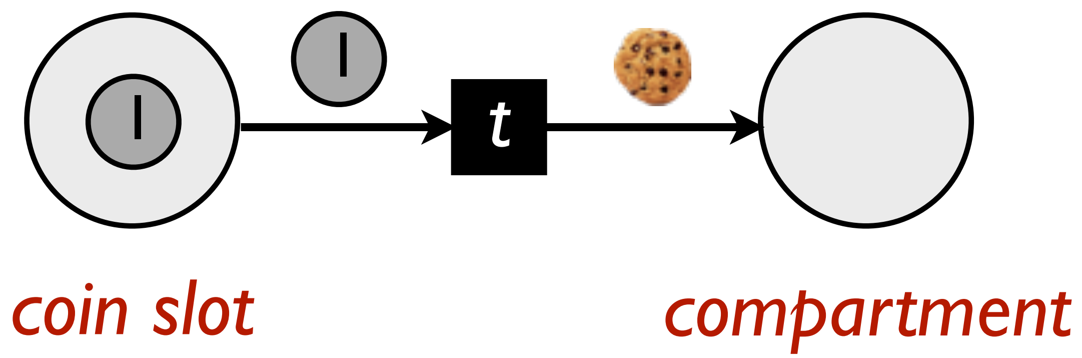
compartment

Let’s design a cookie vending machine

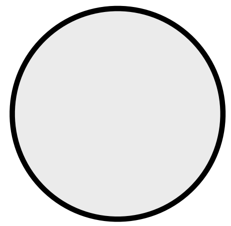
Let's design a cookie vending machine



Let’s design a cookie vending machine

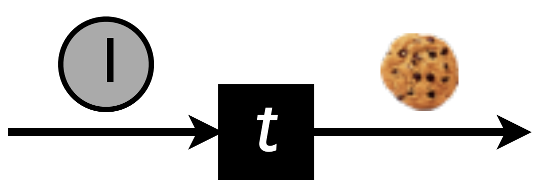


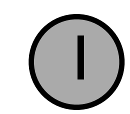
Terminology

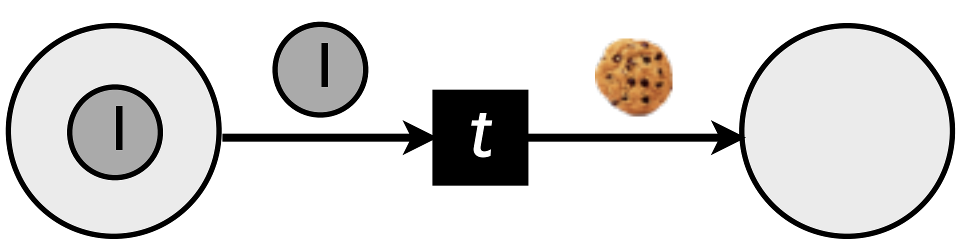


place

tokens

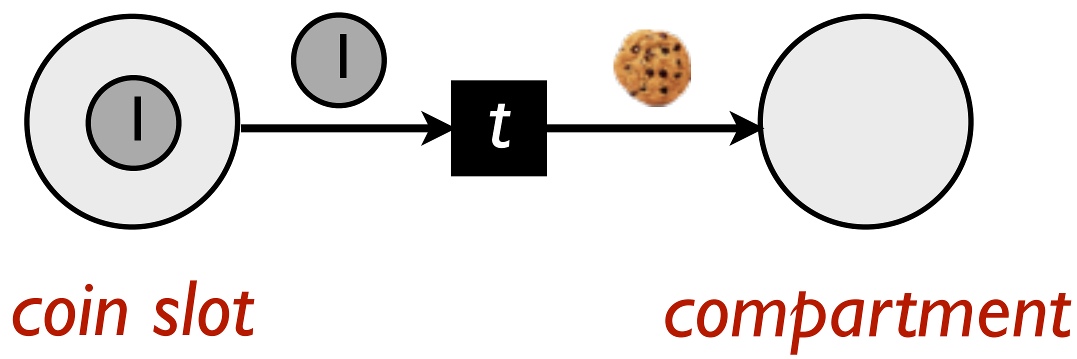


transition (with precondition  *)*



marking (distribution of tokens)

Let’s design a cookie vending machine



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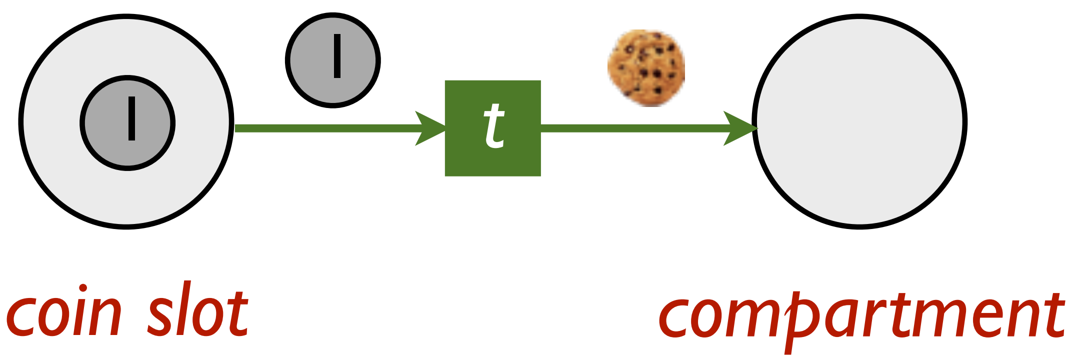
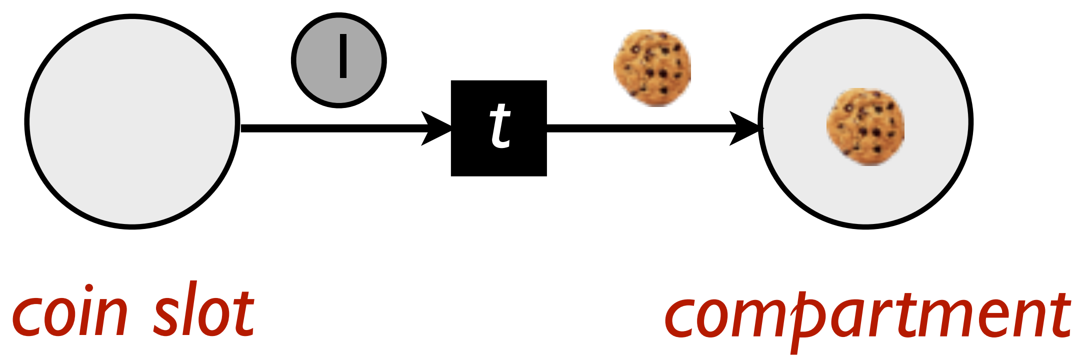


transition t is enabled
it can occur and change the marking

Let’s design a cookie vending machine



transition t is <u>enabled</u>
it can <u>occur</u> and change the <u>marking</u>

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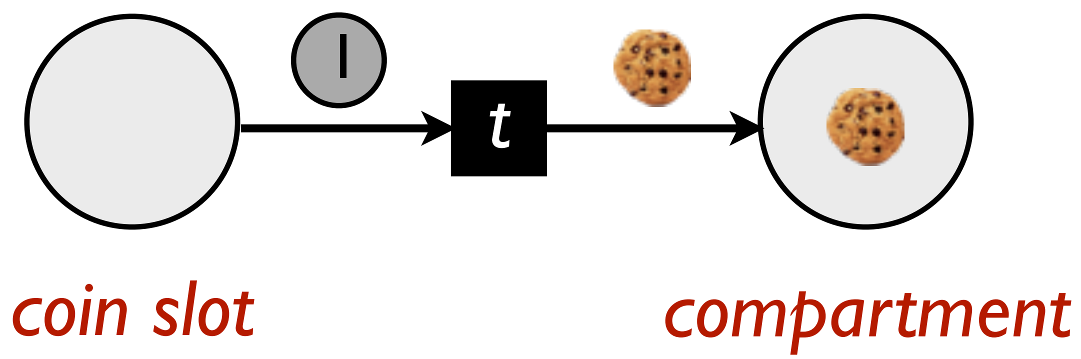
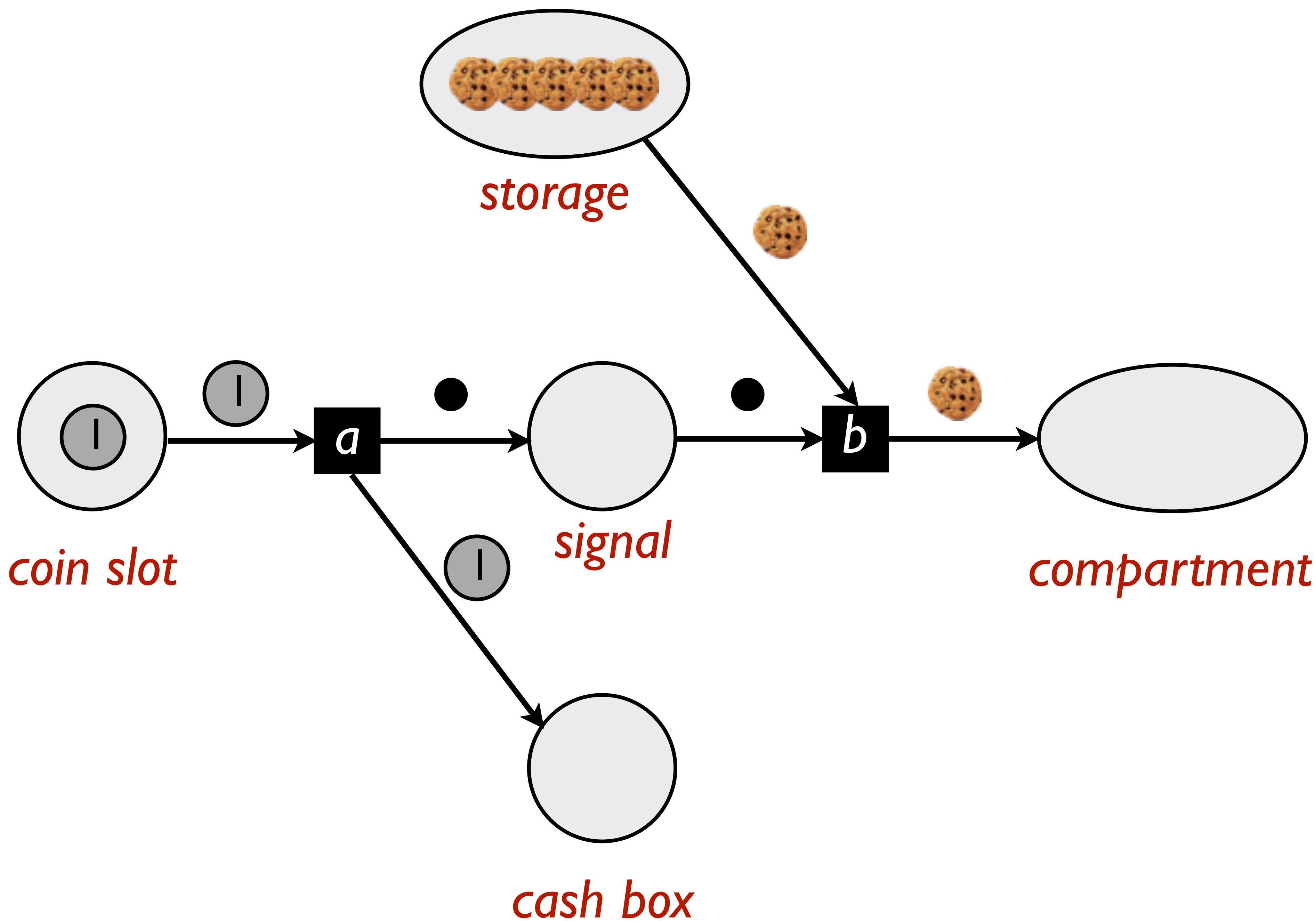


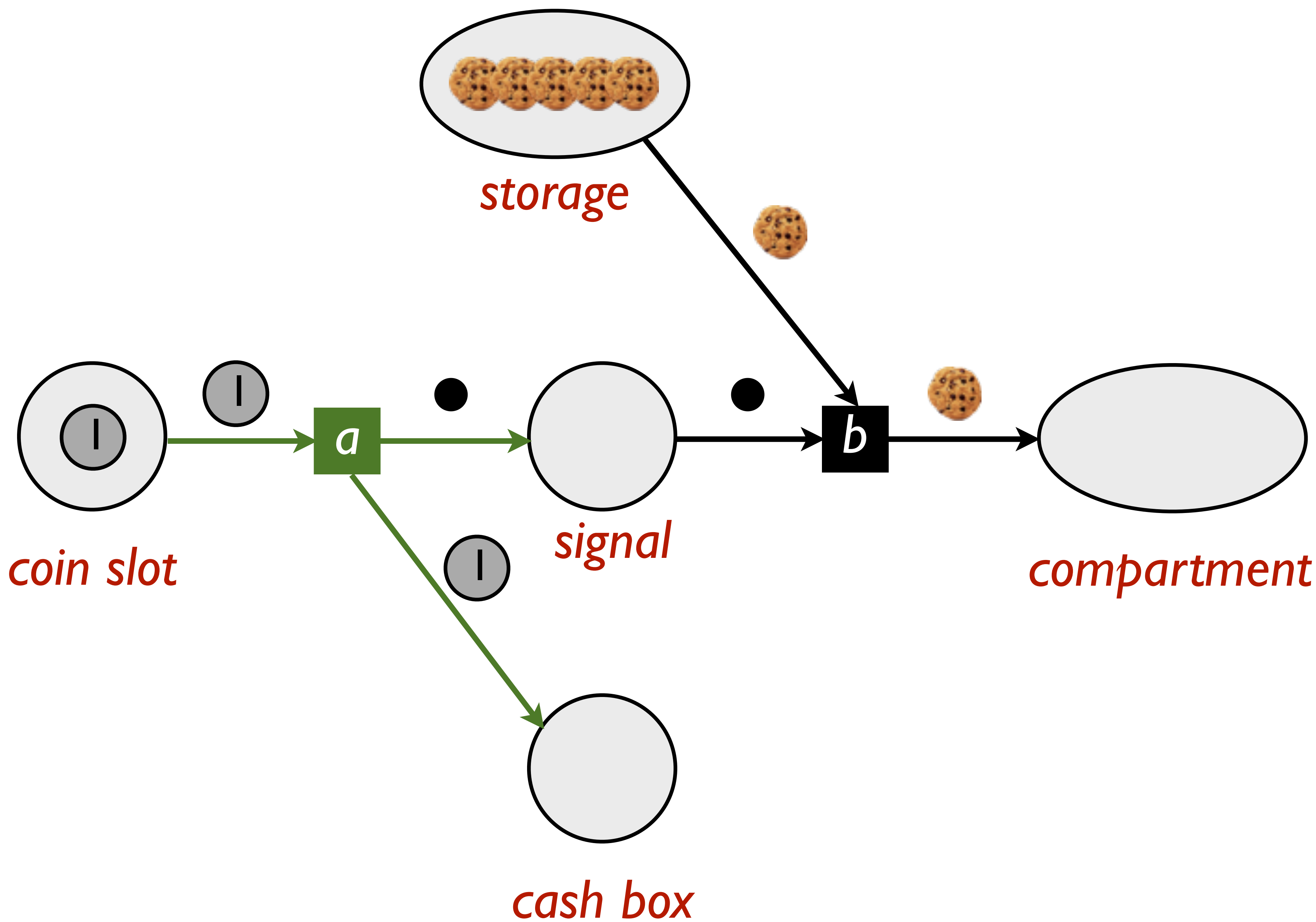
transition t is <u>enabled</u>
it can <u>occur</u> and change the <u>marking</u>

cash box?
finitely many cookies?

Let's look inside



Let's look inside



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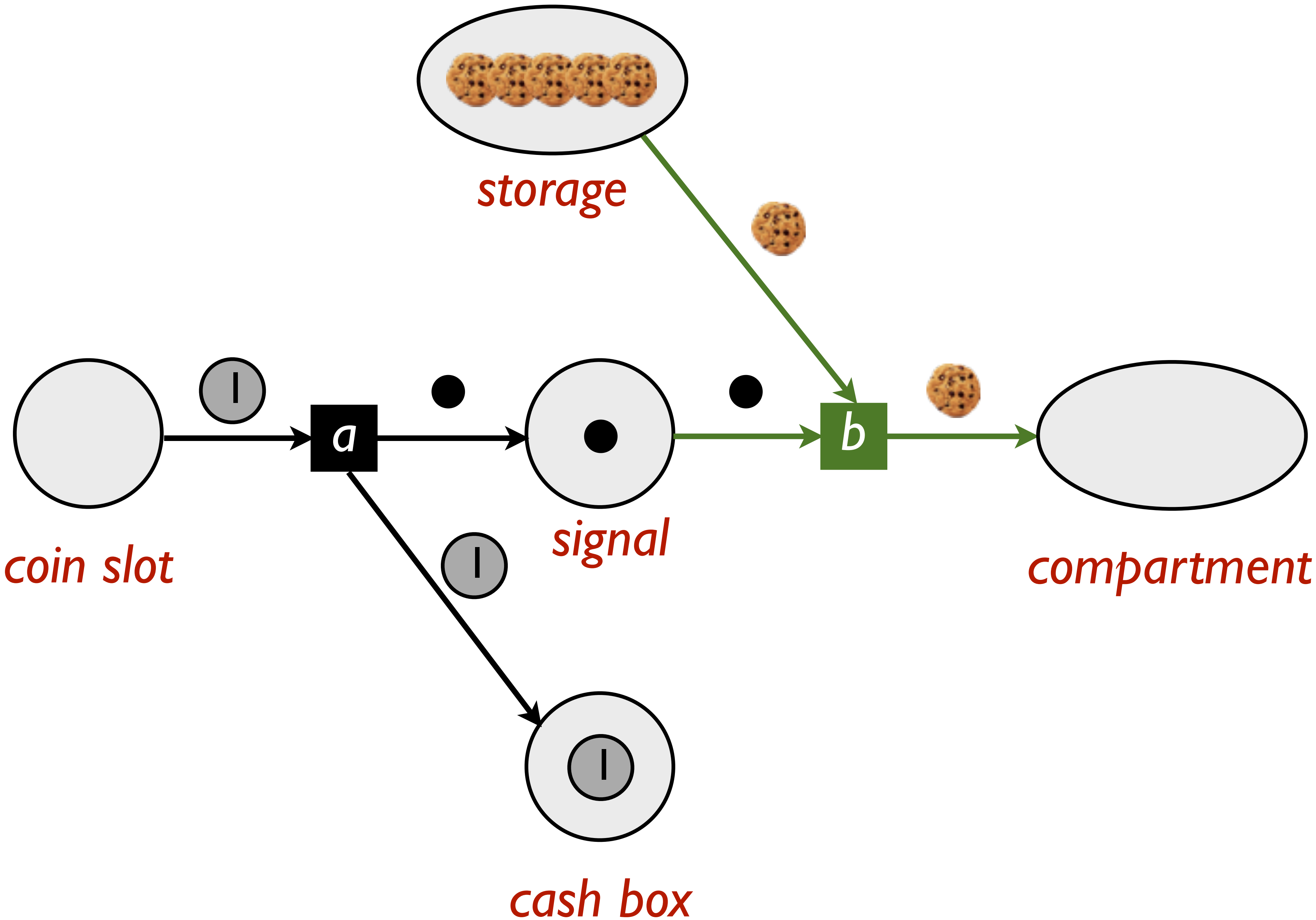

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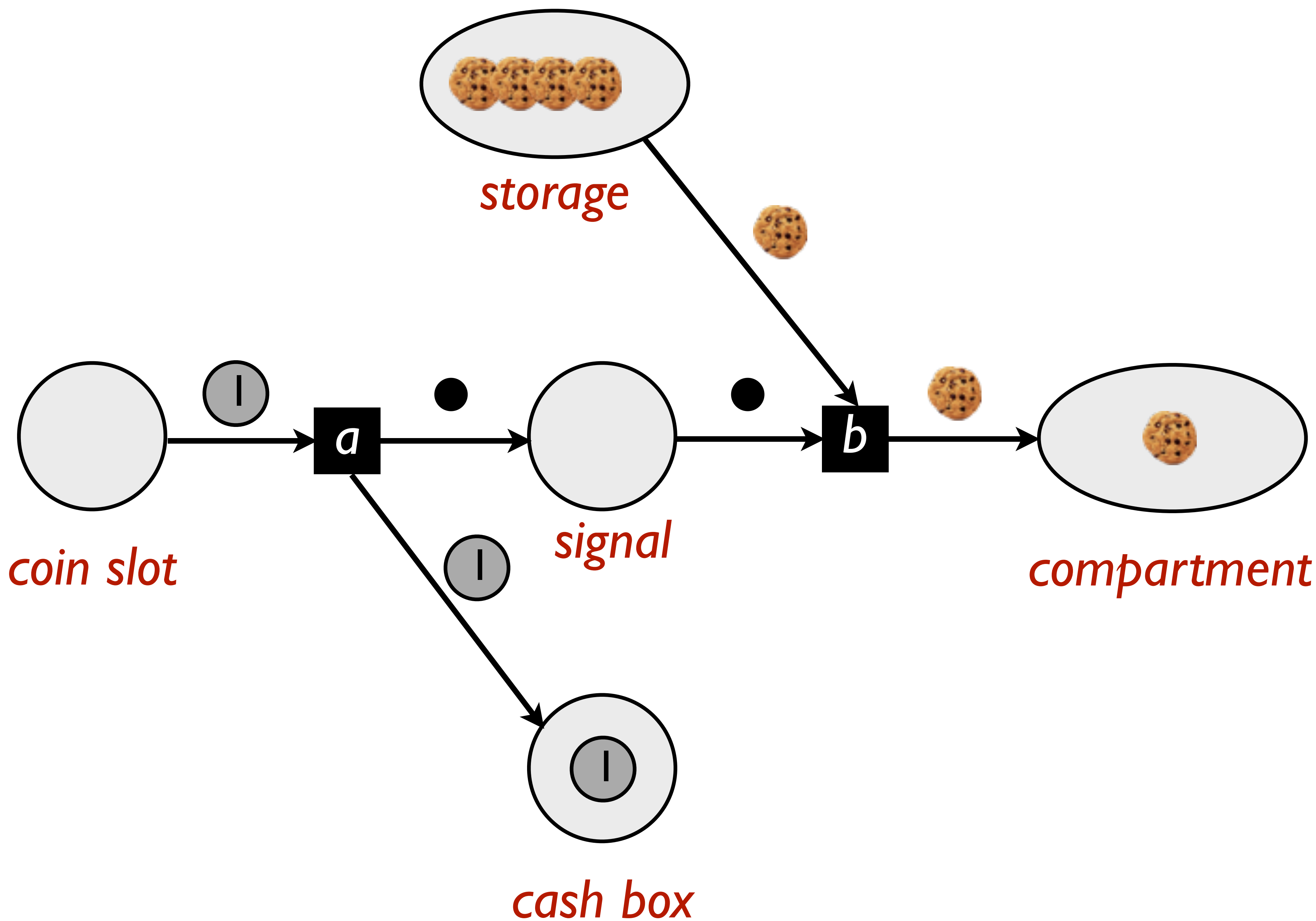

Let's open it up to the world

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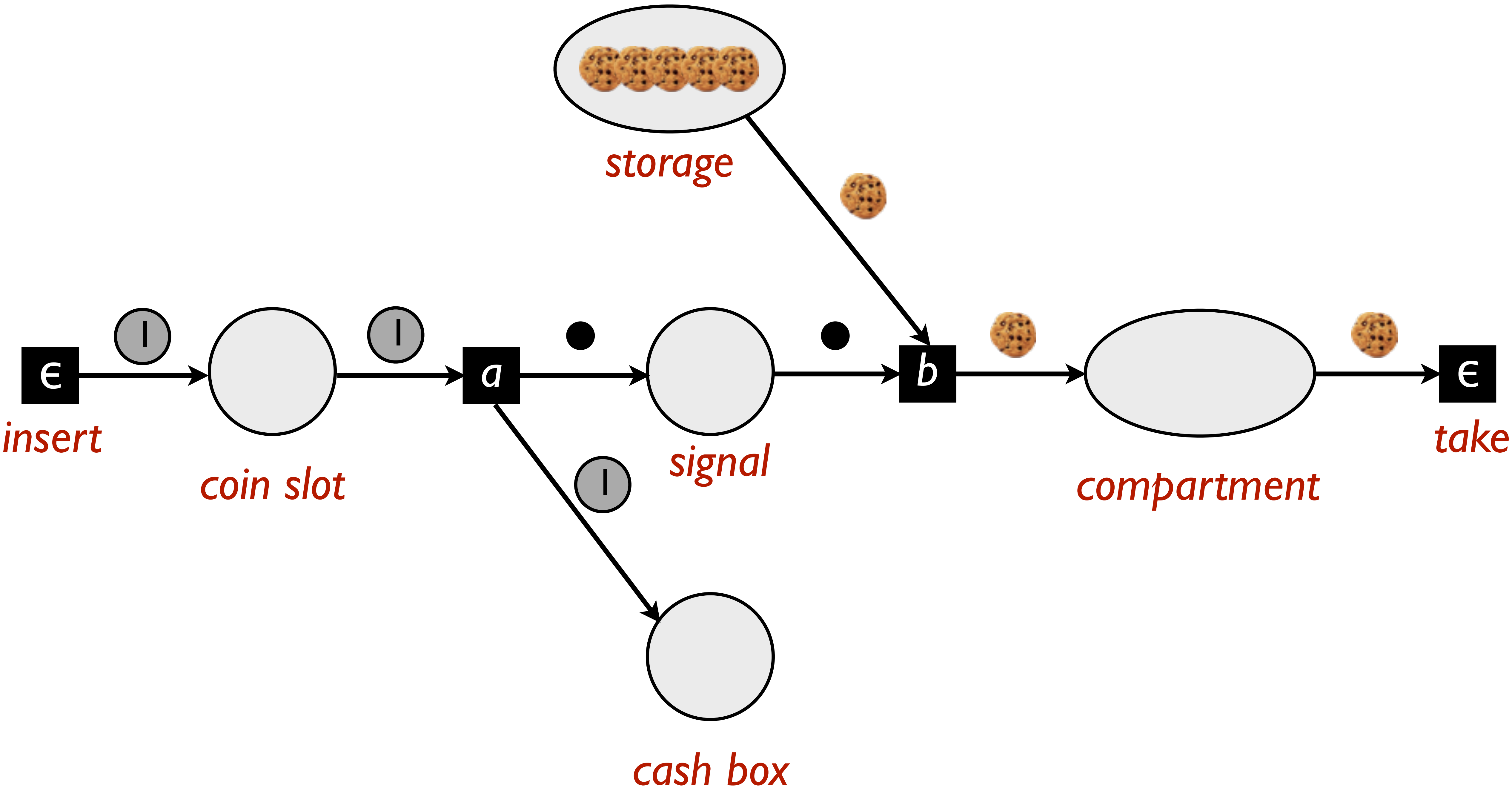
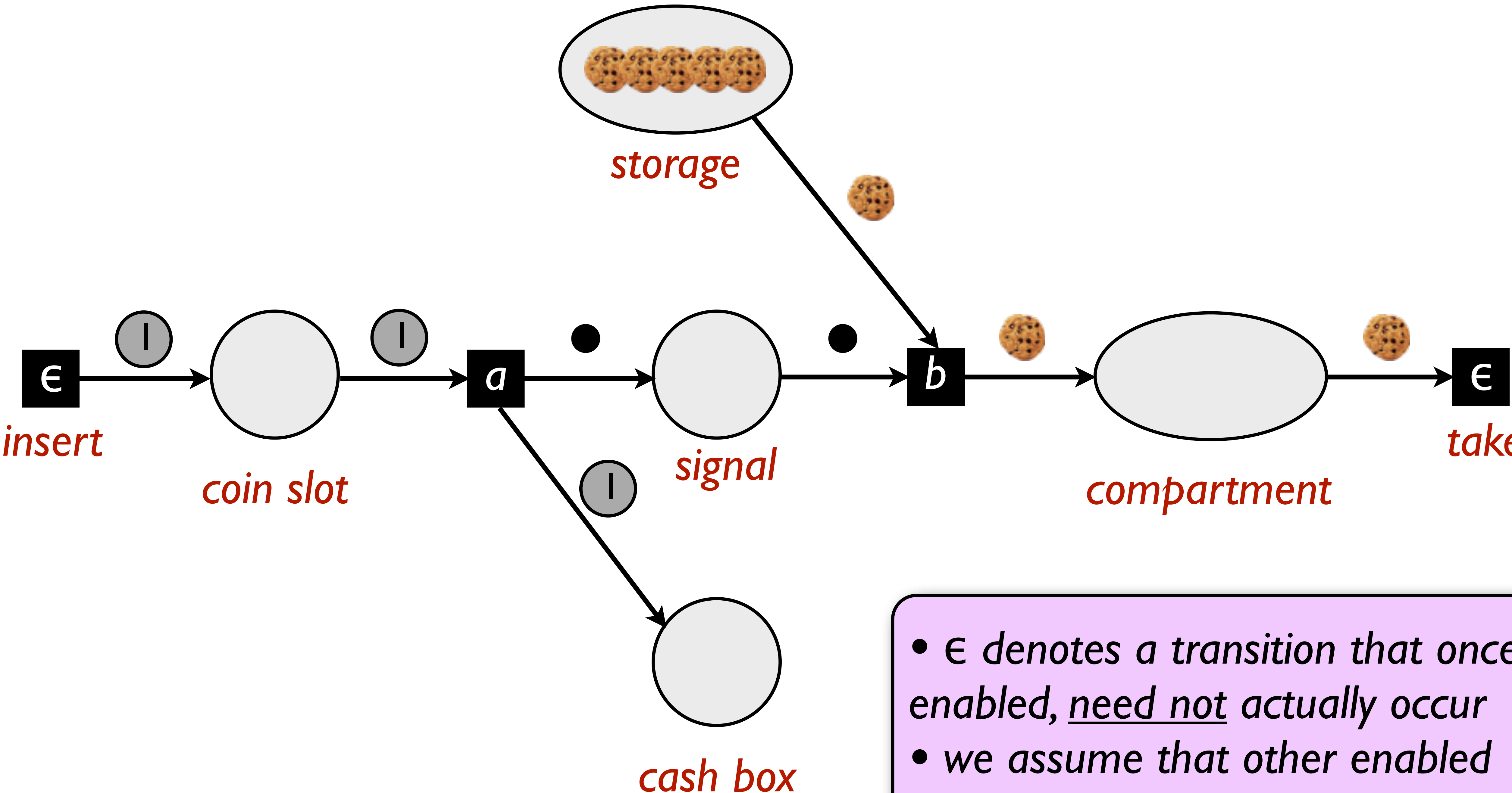
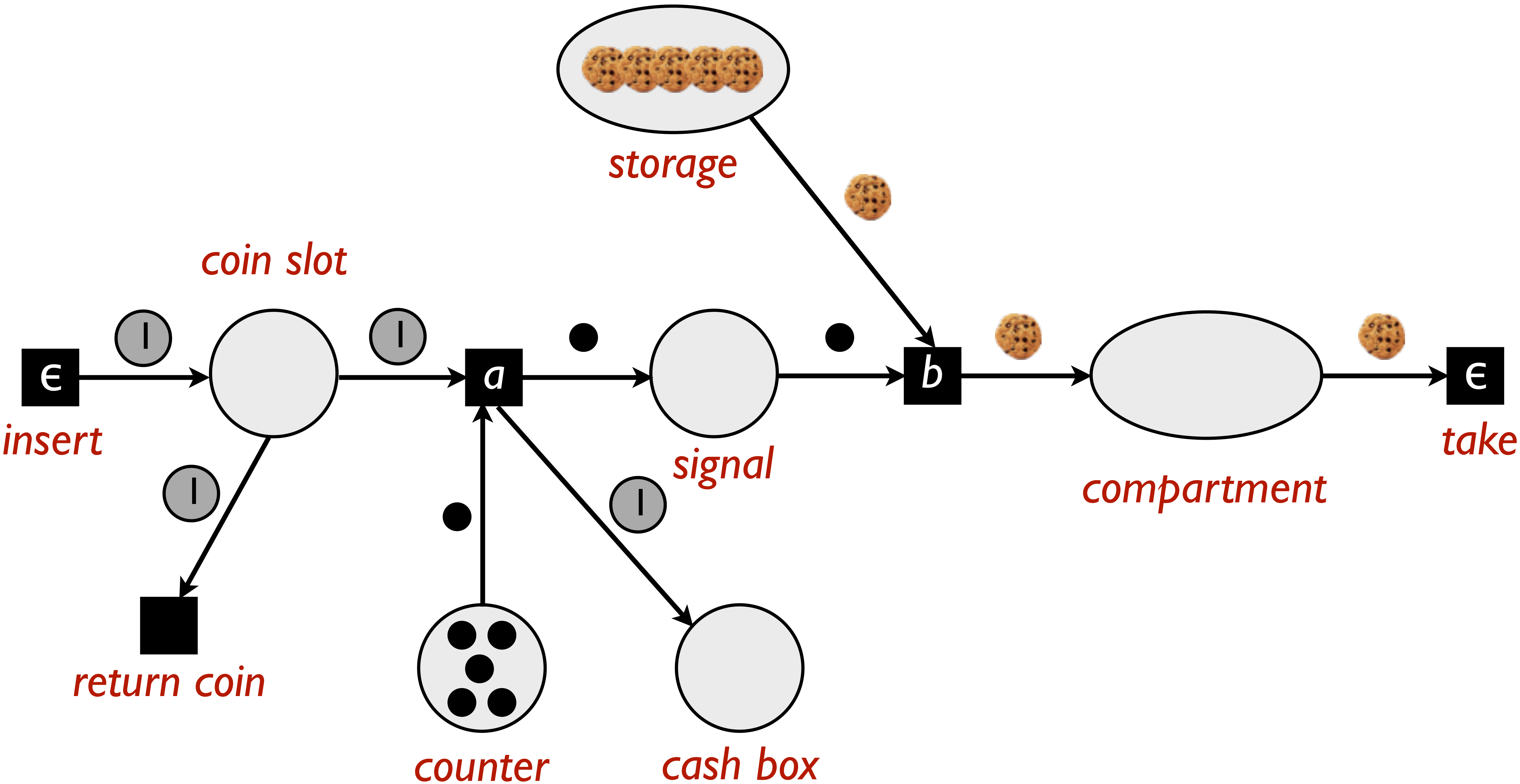


Let's open it up to the world

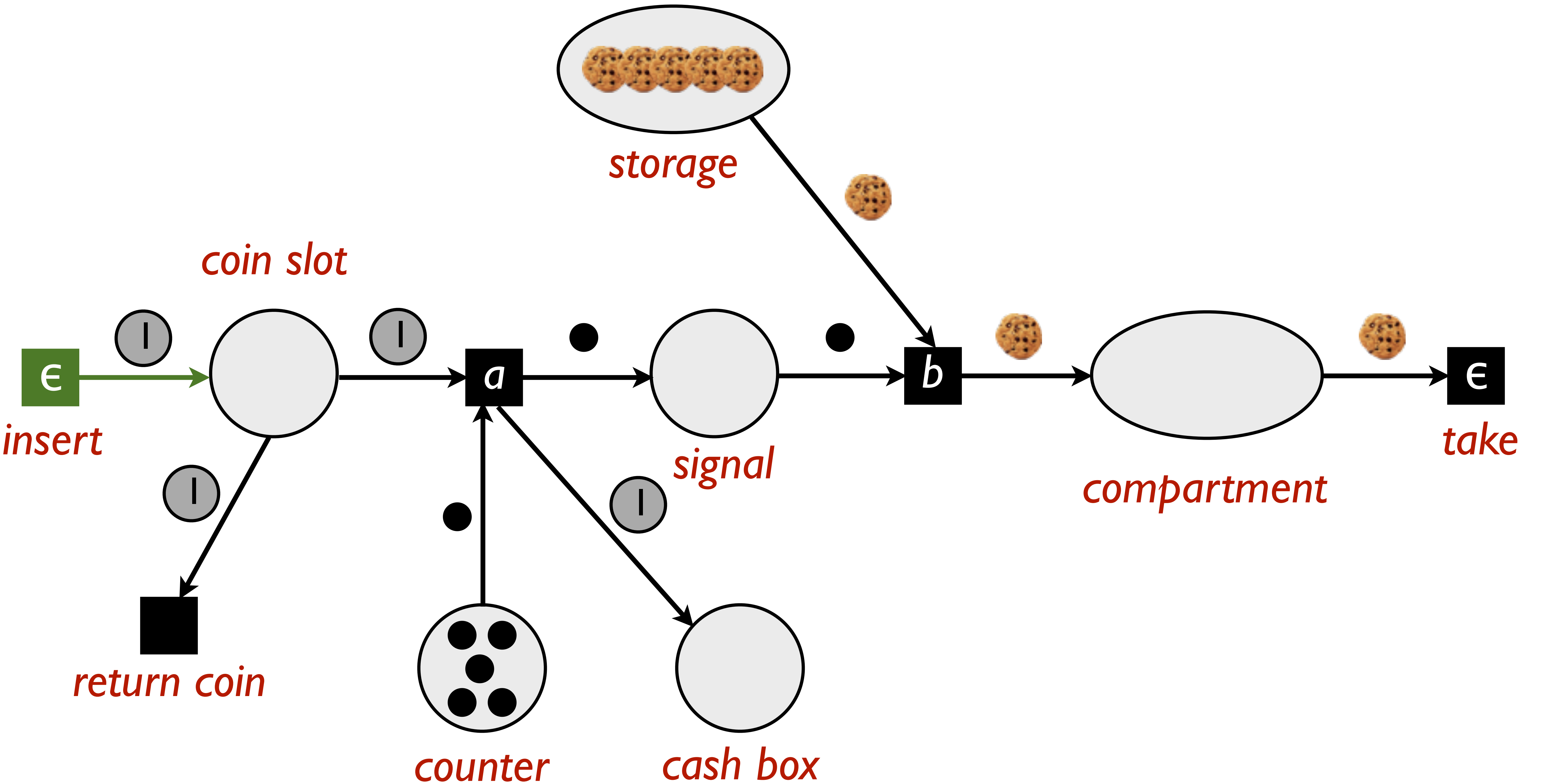


- ϵ denotes a transition that once enabled, need not actually occur
- we assume that other enabled transitions occur eventually

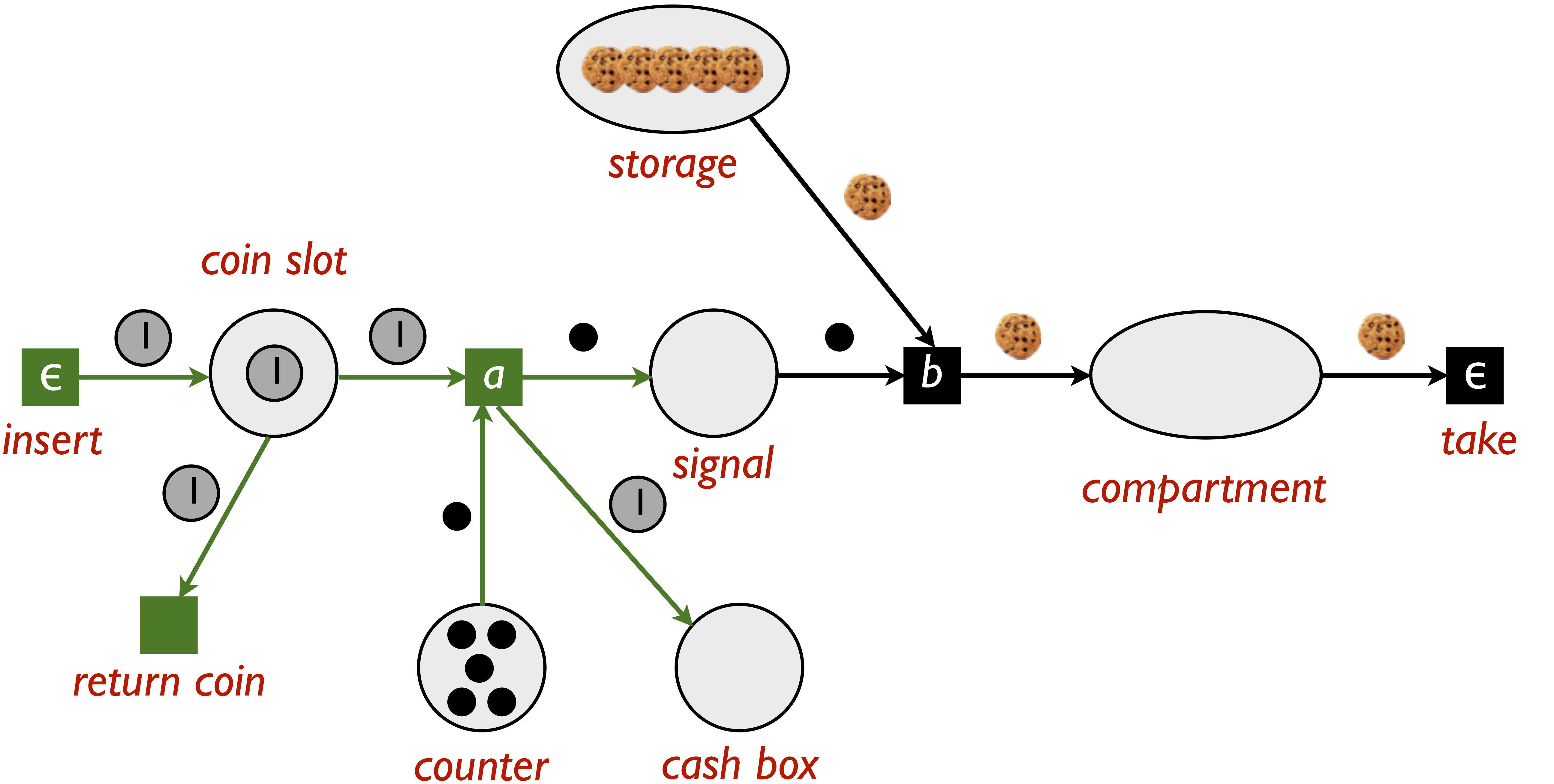
The ultimate cookie machine



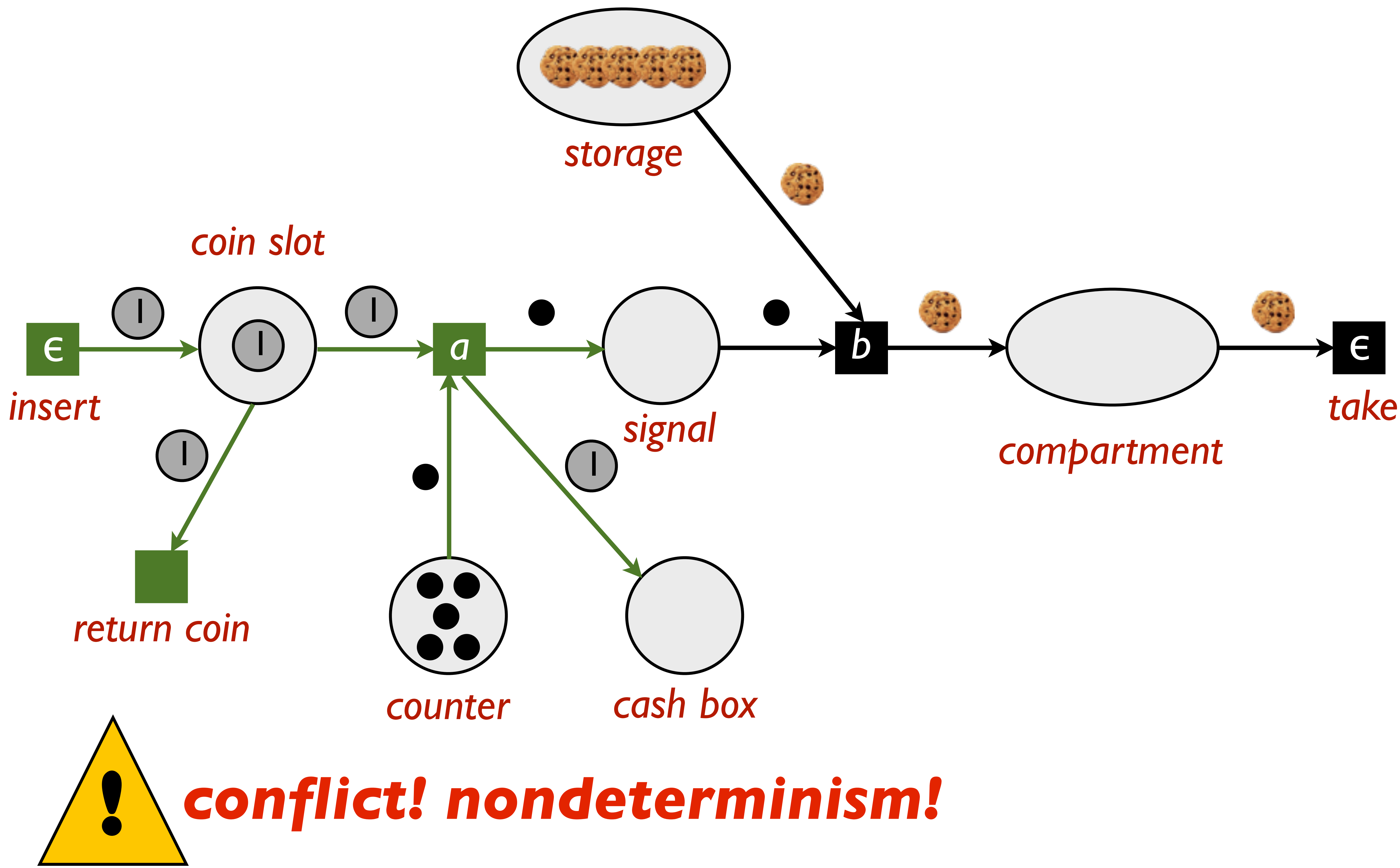
The ultimate cookie machine



The ultimate cookie machine

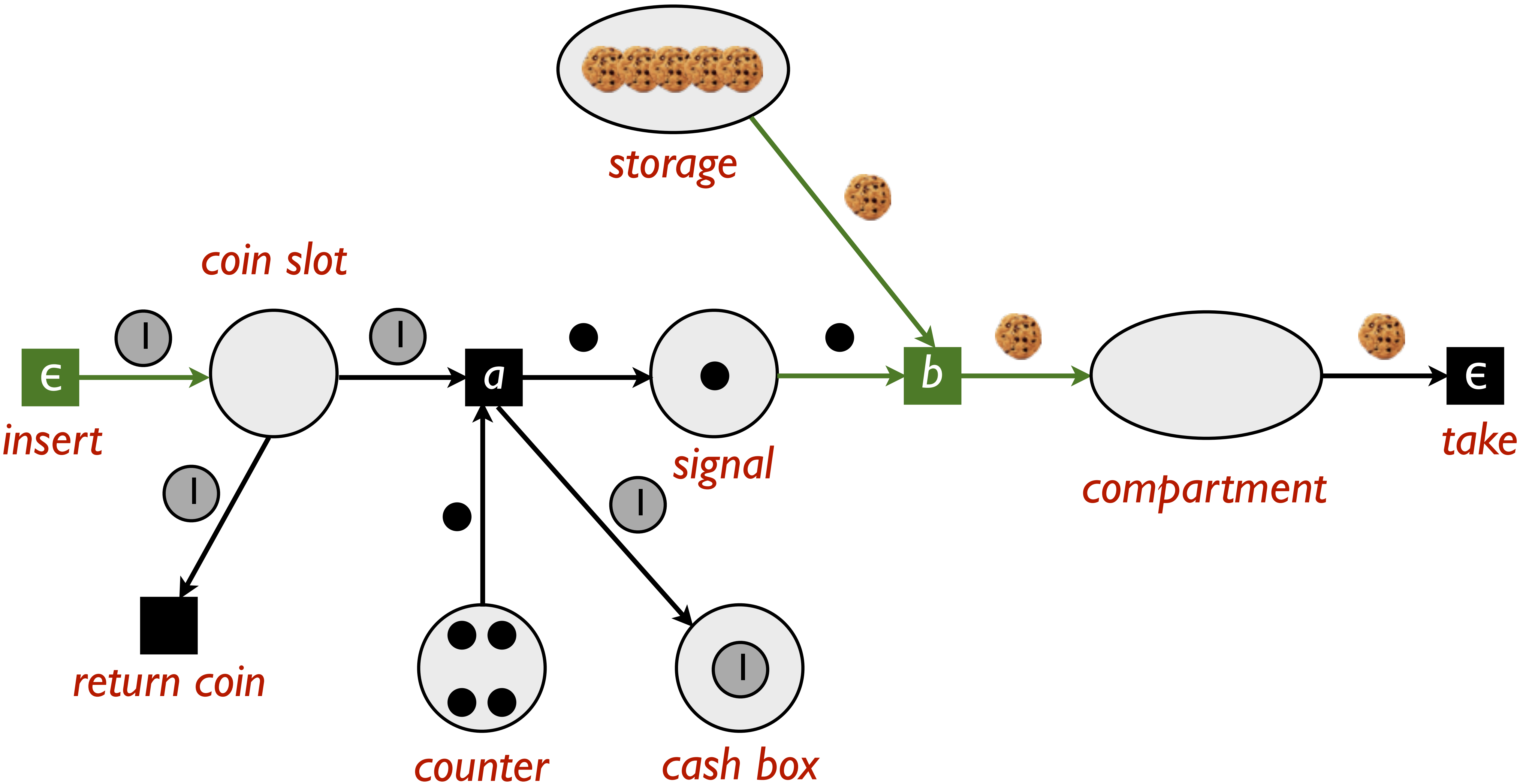


The ultimate cookie machine

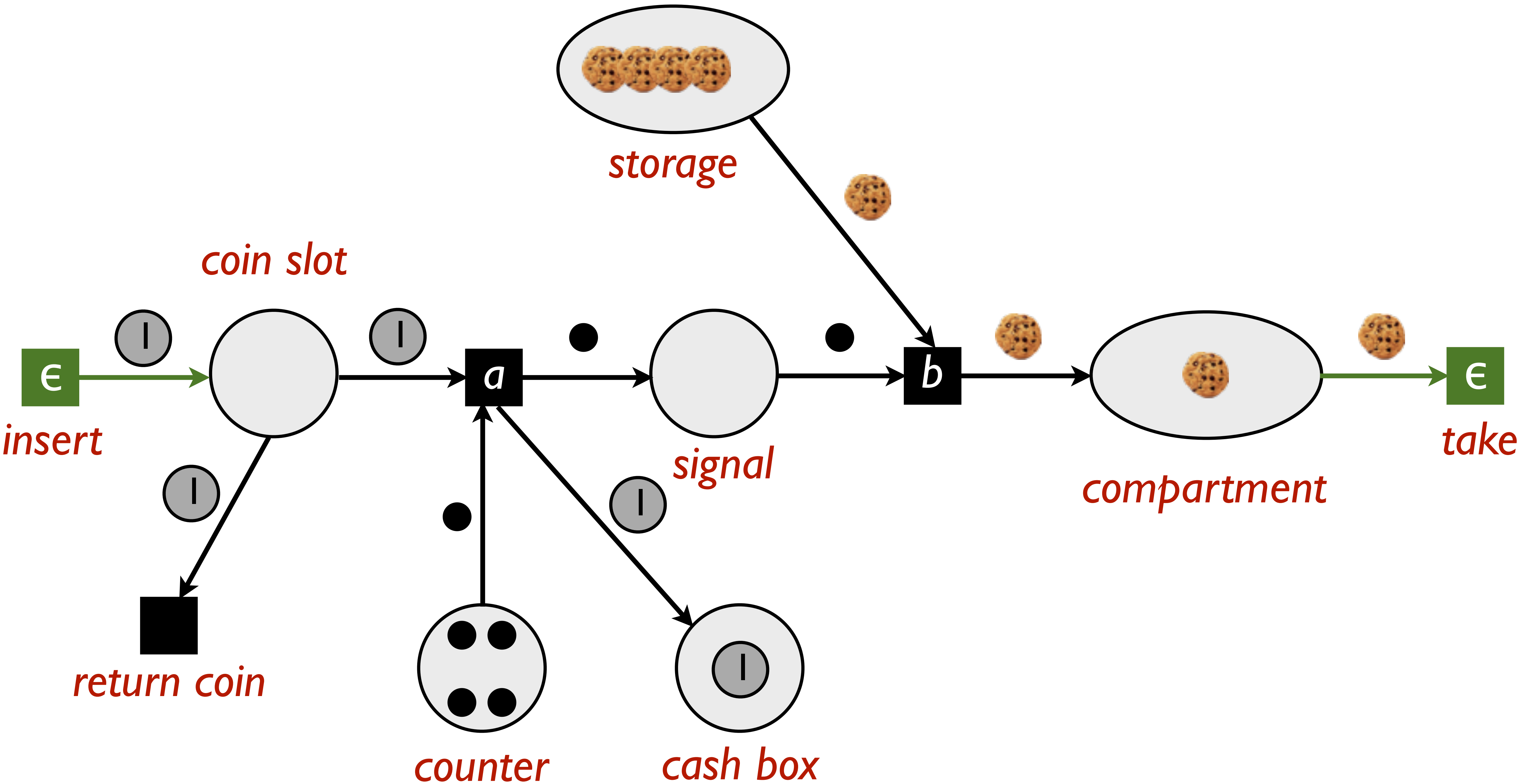


conflict! nondeterminism!

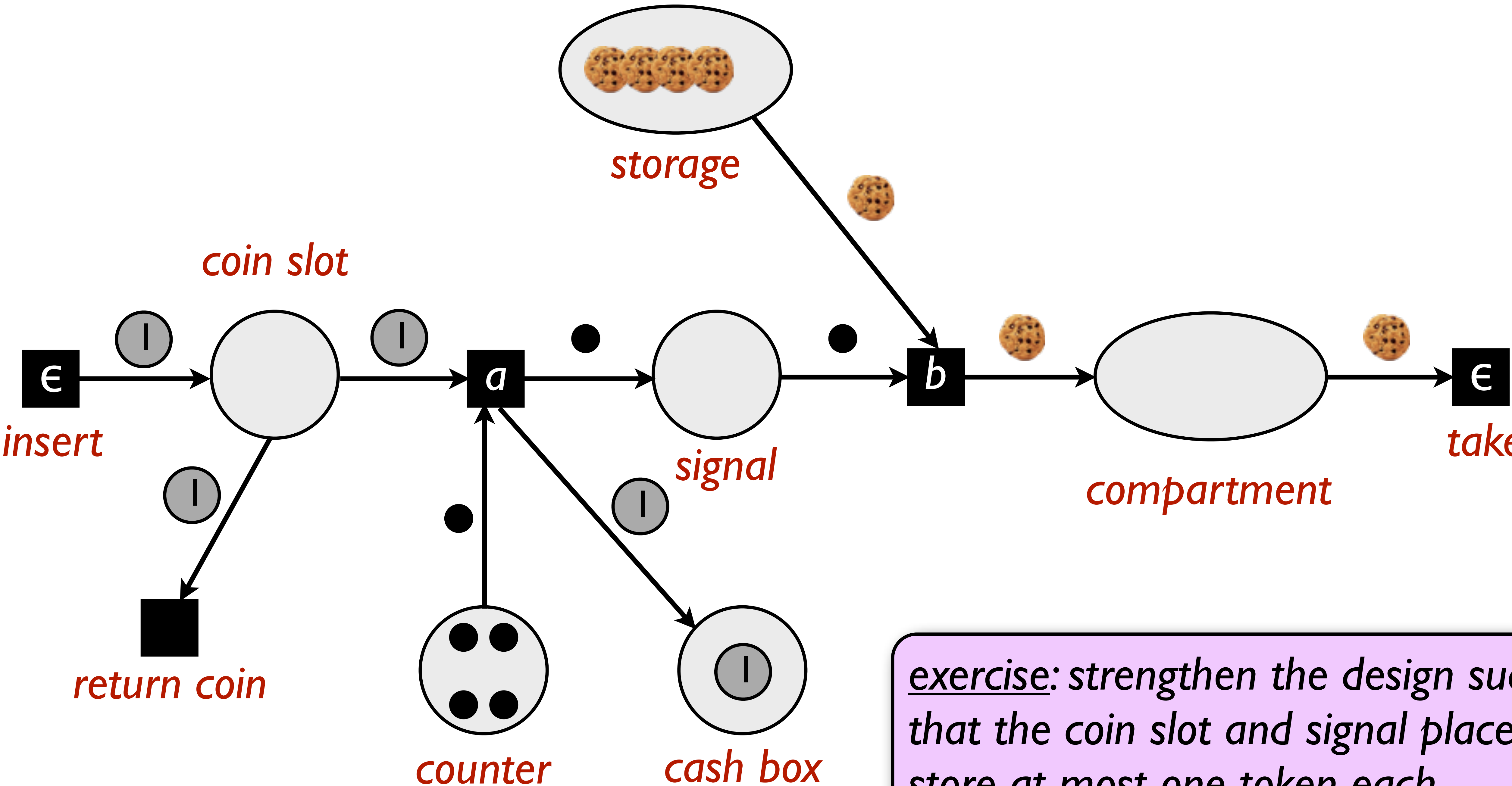
The ultimate cookie machine



The ultimate cookie machine



The ultimate cookie machine



exercise: strengthen the design such that the coin slot and signal places store at most one token each

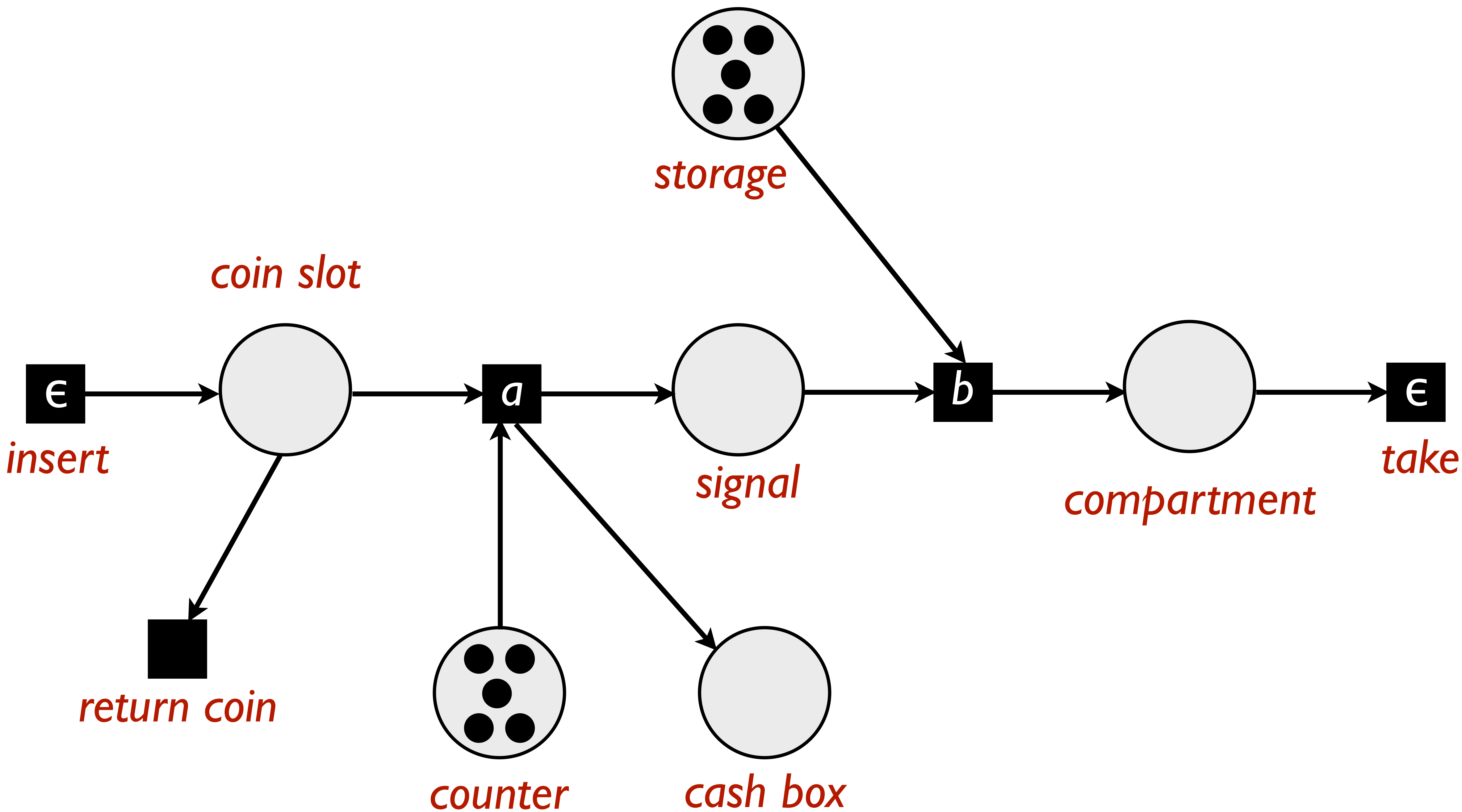
Elementary Petri nets

- if we are interested in only control flow, we can use a special case - elementary Petri nets - where all tokens are simply black dots

- assume all edges to be labelled by: “●“

- henceforth, we assume all Petri nets to be elementary

Elementary cookie vending machine



Petri nets: definition

- an (elementary) Petri net consists of a net structure:

  $$N = (P, T, F)$$

  with finite sets P and T of places and transitions, F an edge relation $F \subseteq (P \times T) \cup (T \times P)$ and an initial marking $M_0: P \rightarrow \mathbf{N}$

- markings have the form $M: P \rightarrow \mathbf{N}$; each place p holds $M(p)$ tokens

Petri nets: definition

- the preset of a transition t is the set of places p connected by edges from p to t (postset defined analogously)

- a transition is enabled if $M(p) \geq 1$ for all places p in the preset

- an enabled transition can occur, removing a token from each place in the preset and adding one to each place in the postset

Next on the agenda

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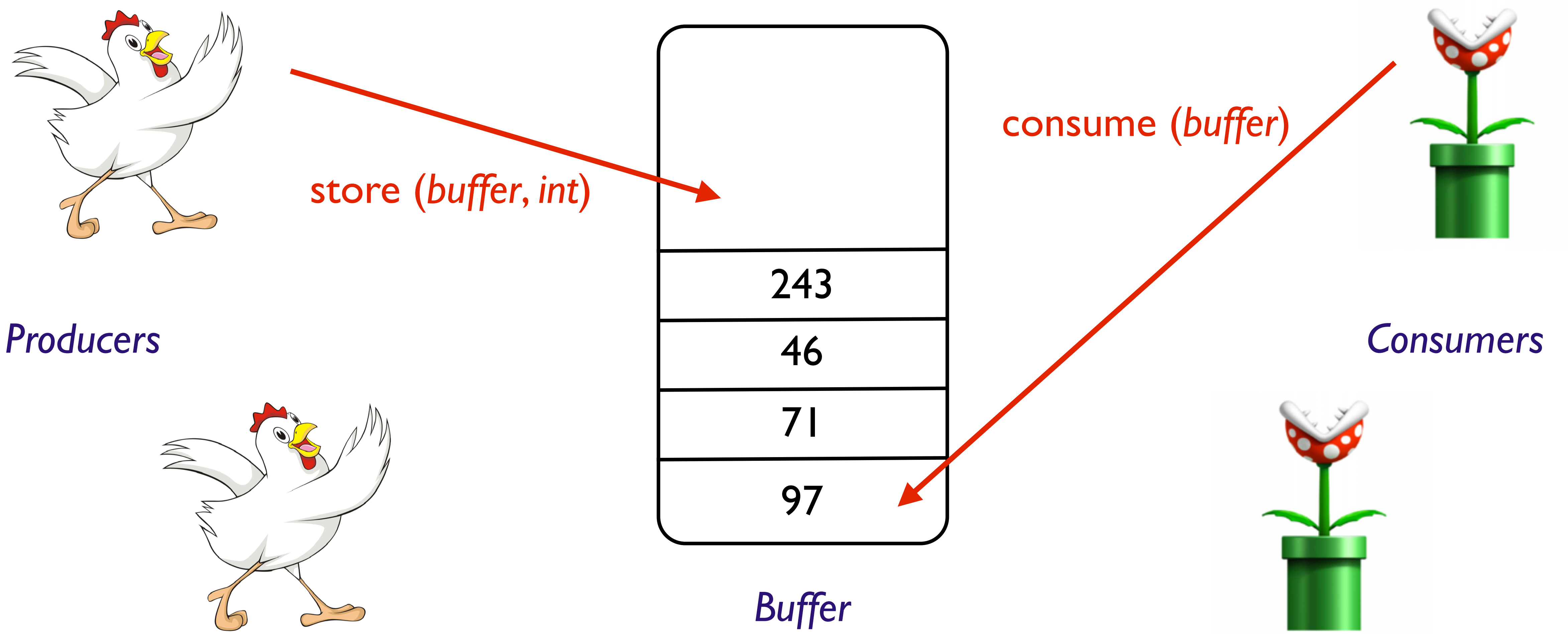
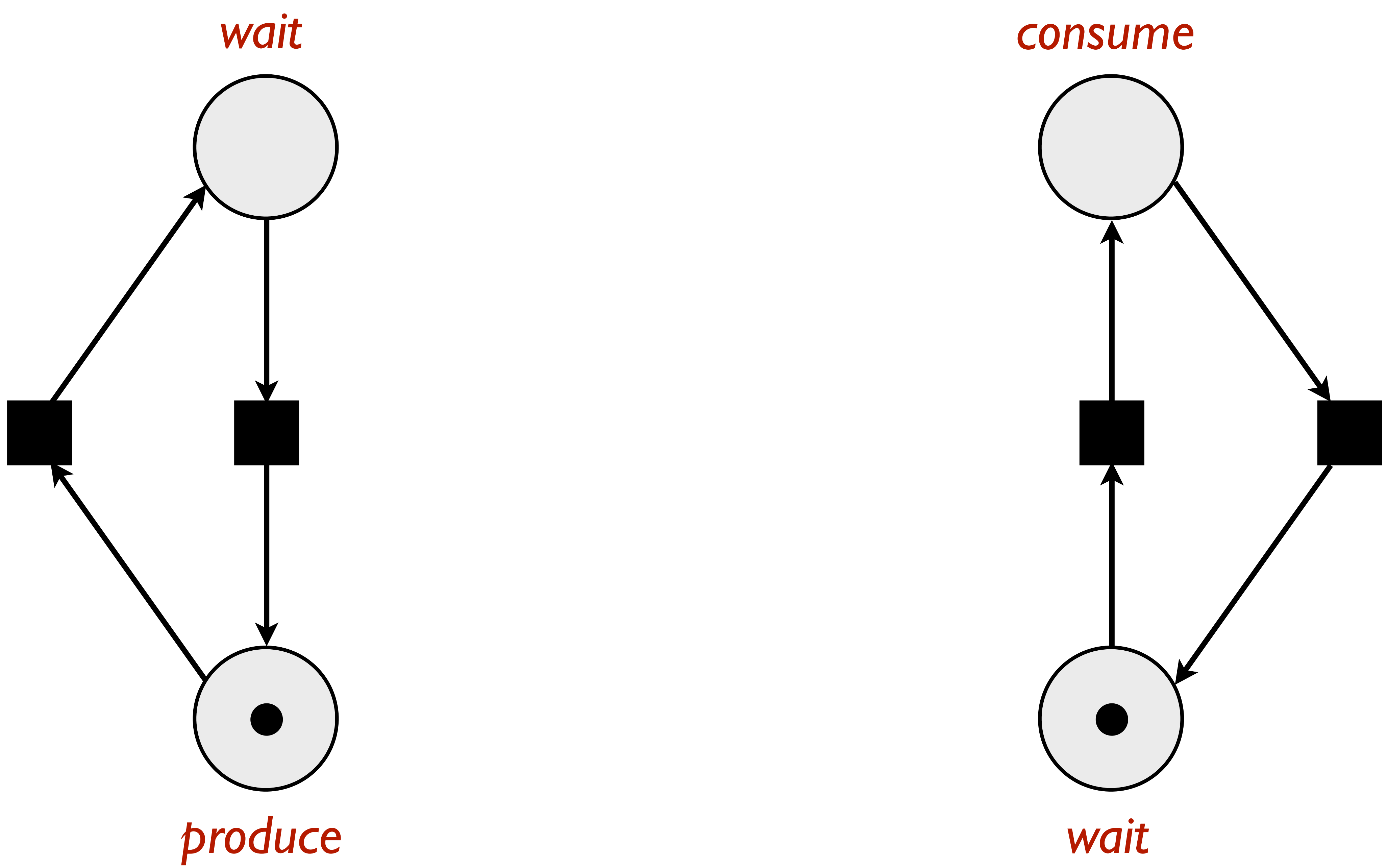
2. synchronisation problems as Petri nets
3. Petri net analyses
4. true concurrency semantics; unfoldings

Producer-consumer problem



Producer-consumer problem



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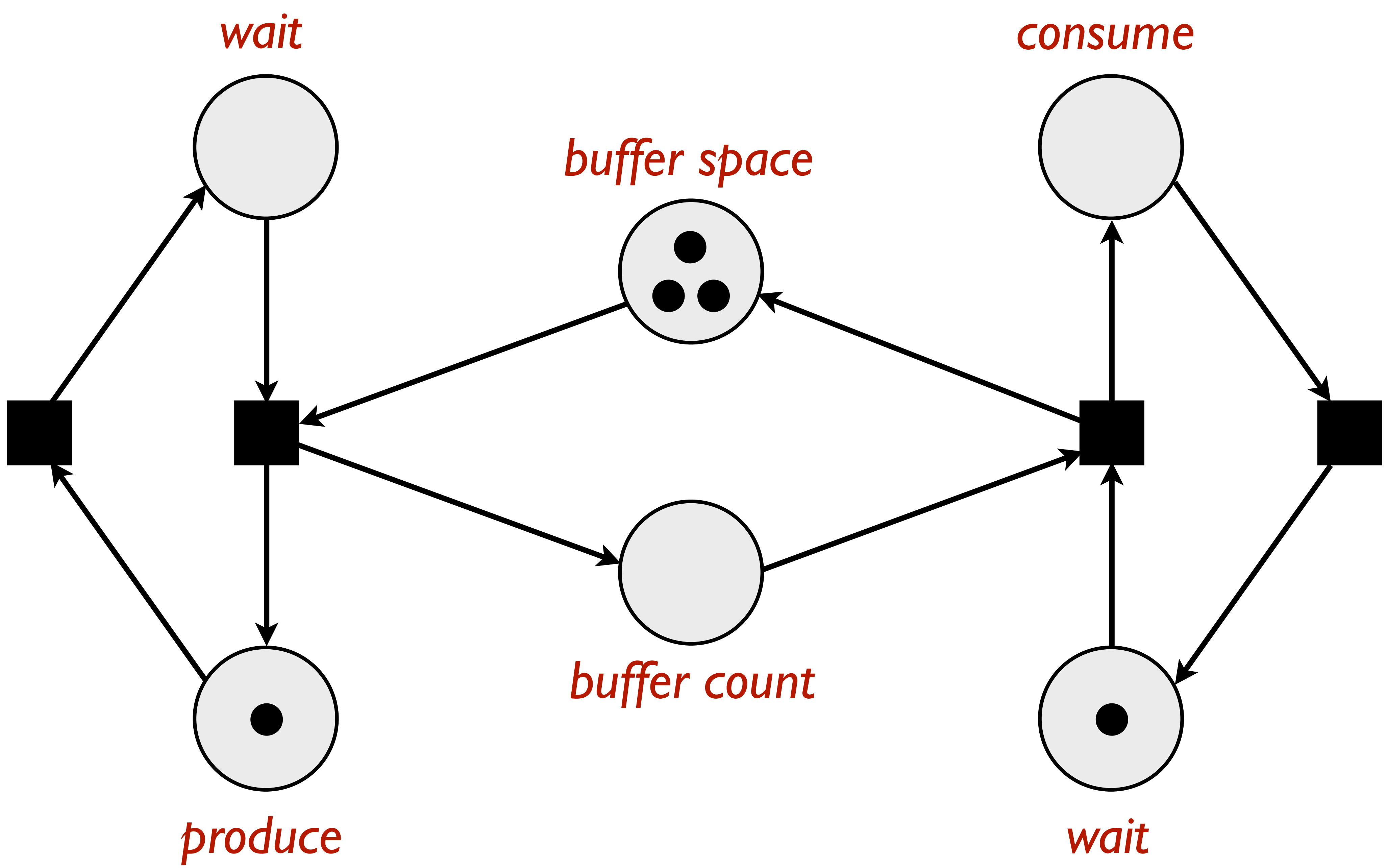

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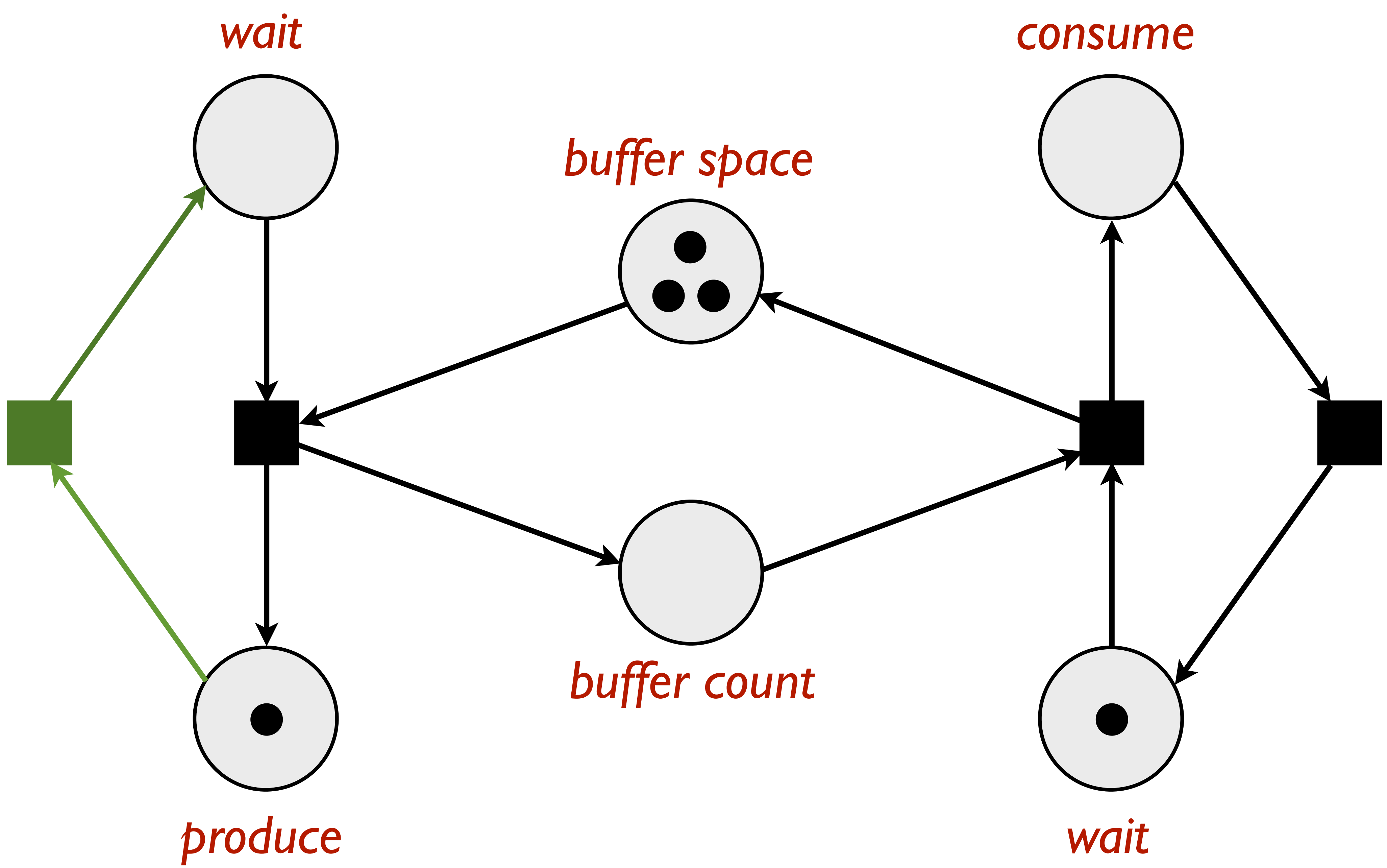

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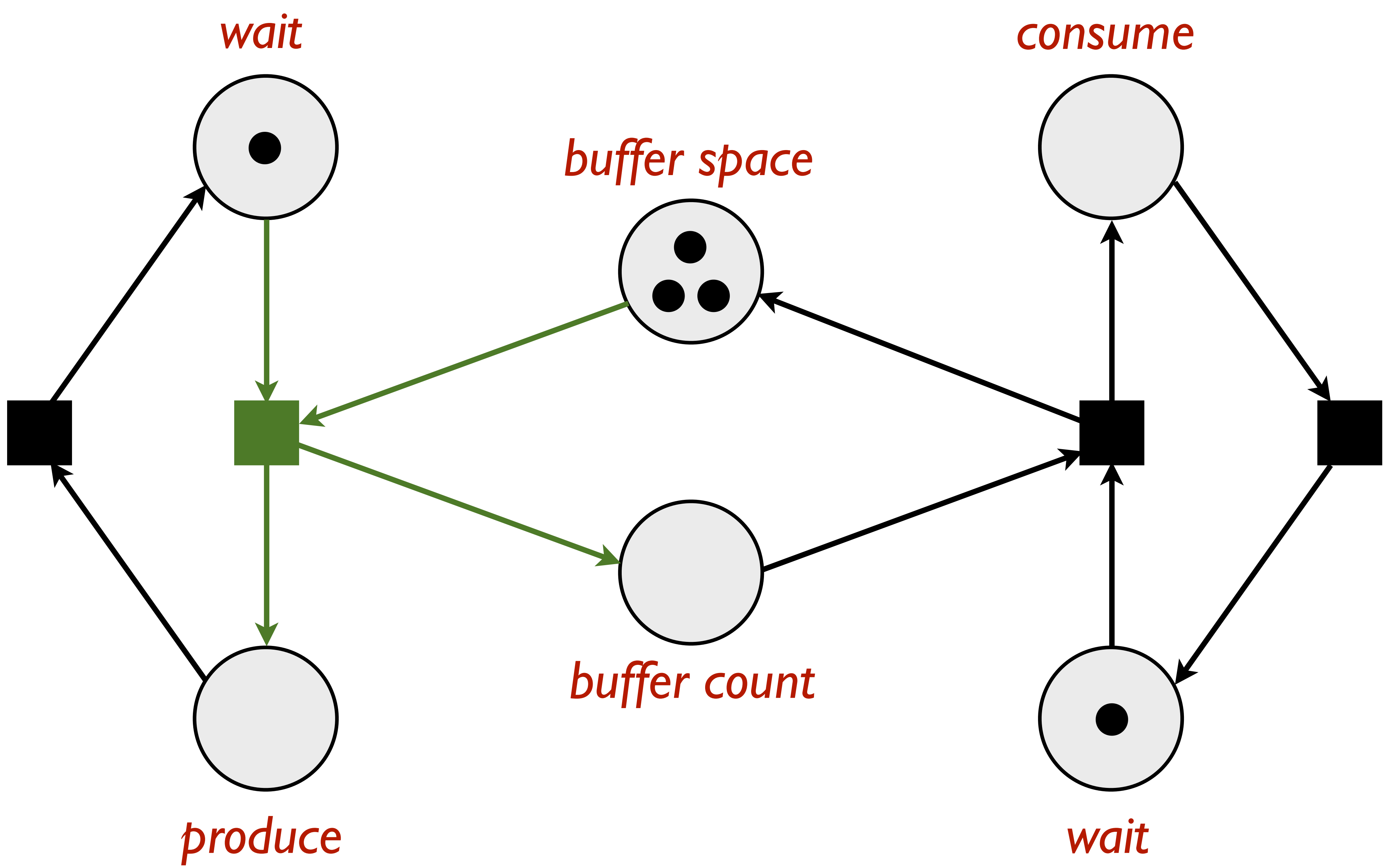

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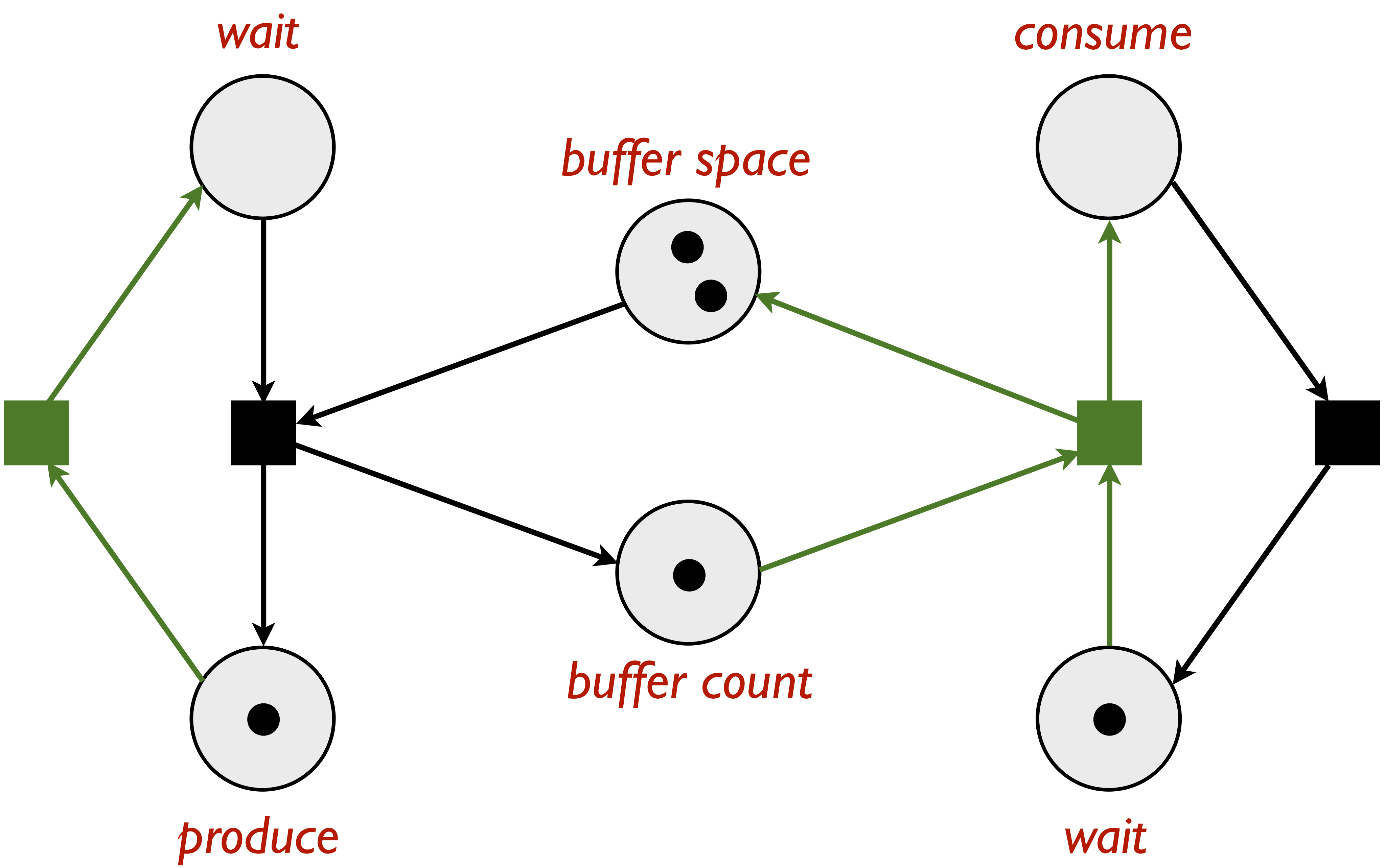

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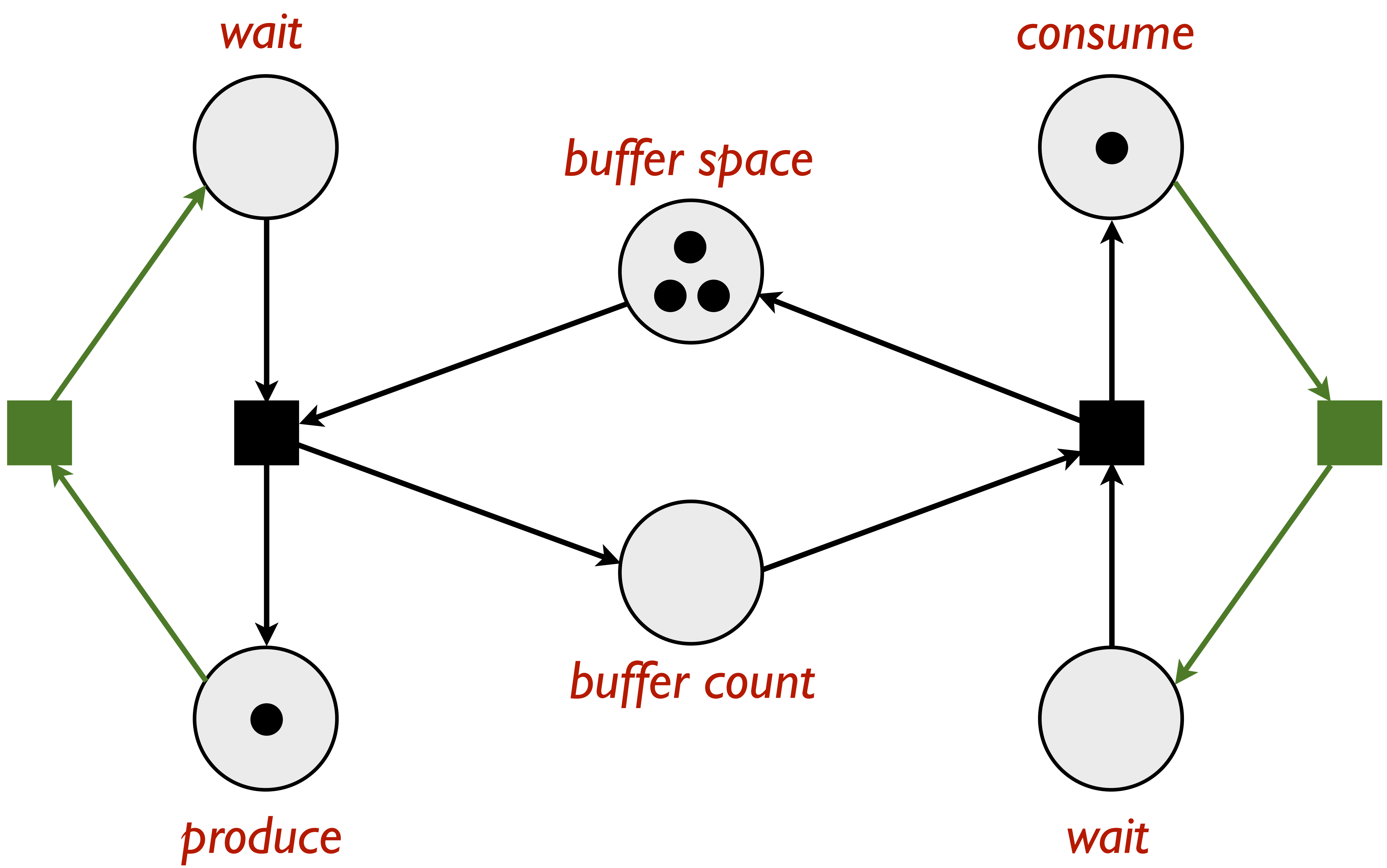
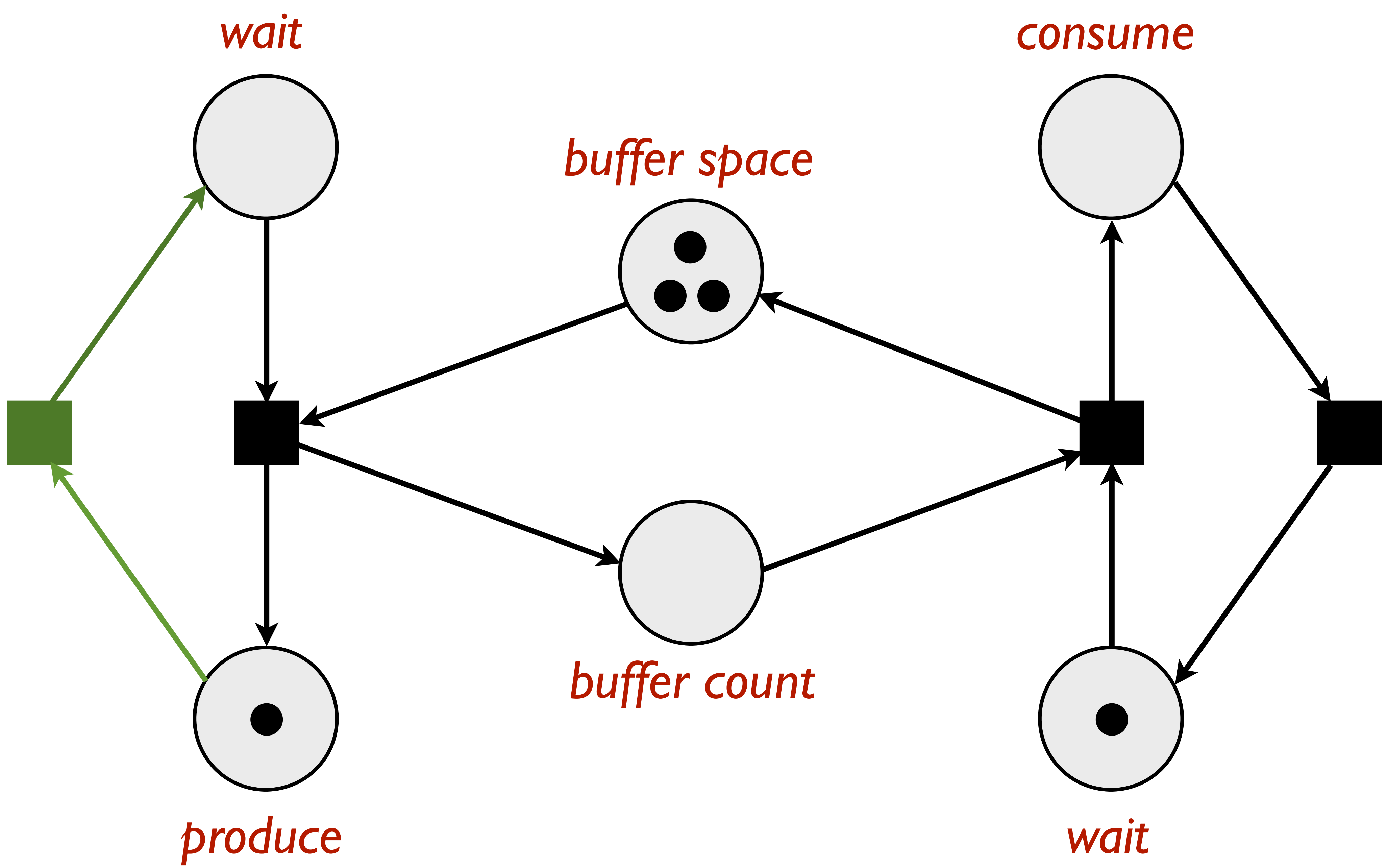
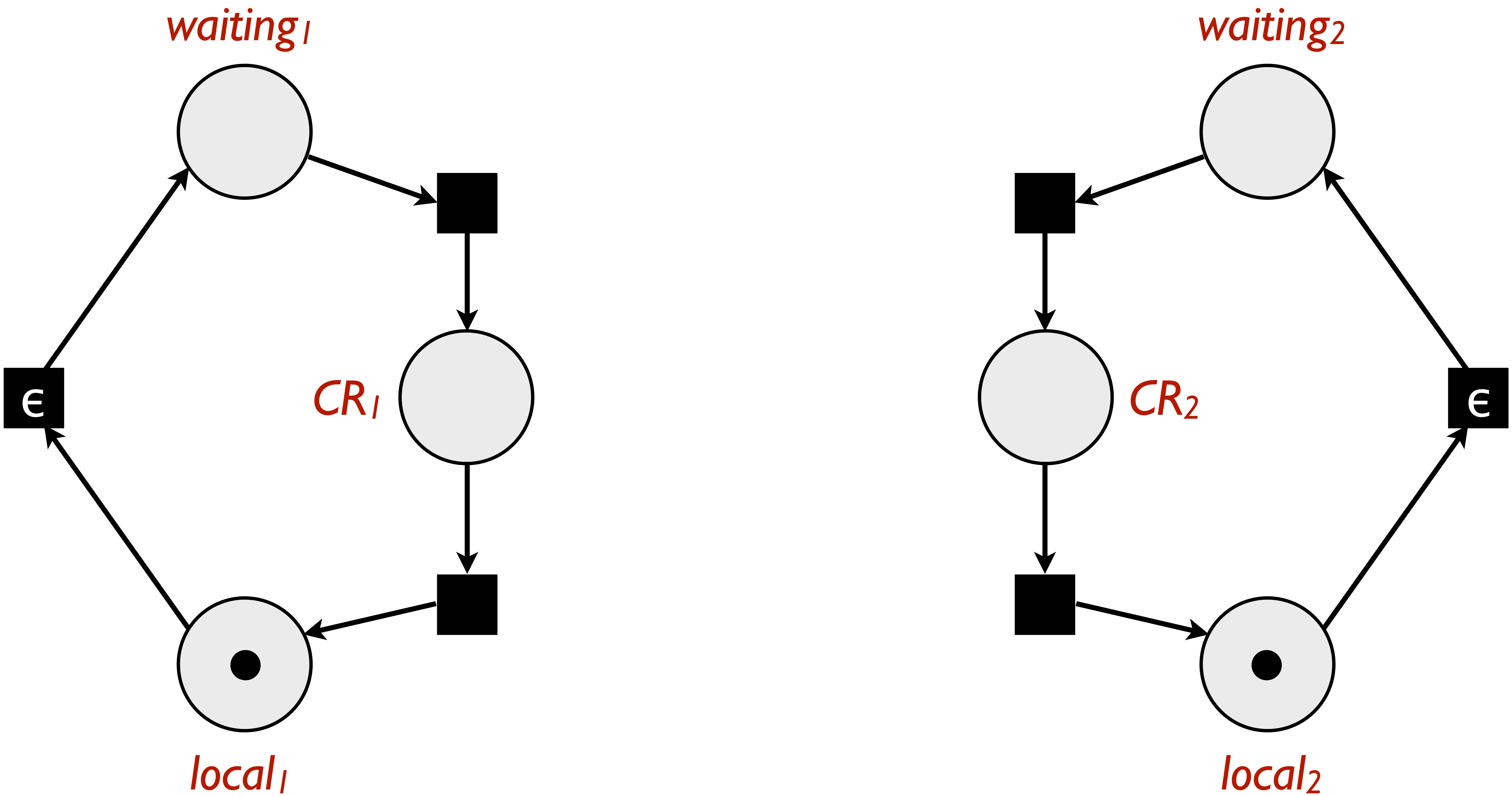


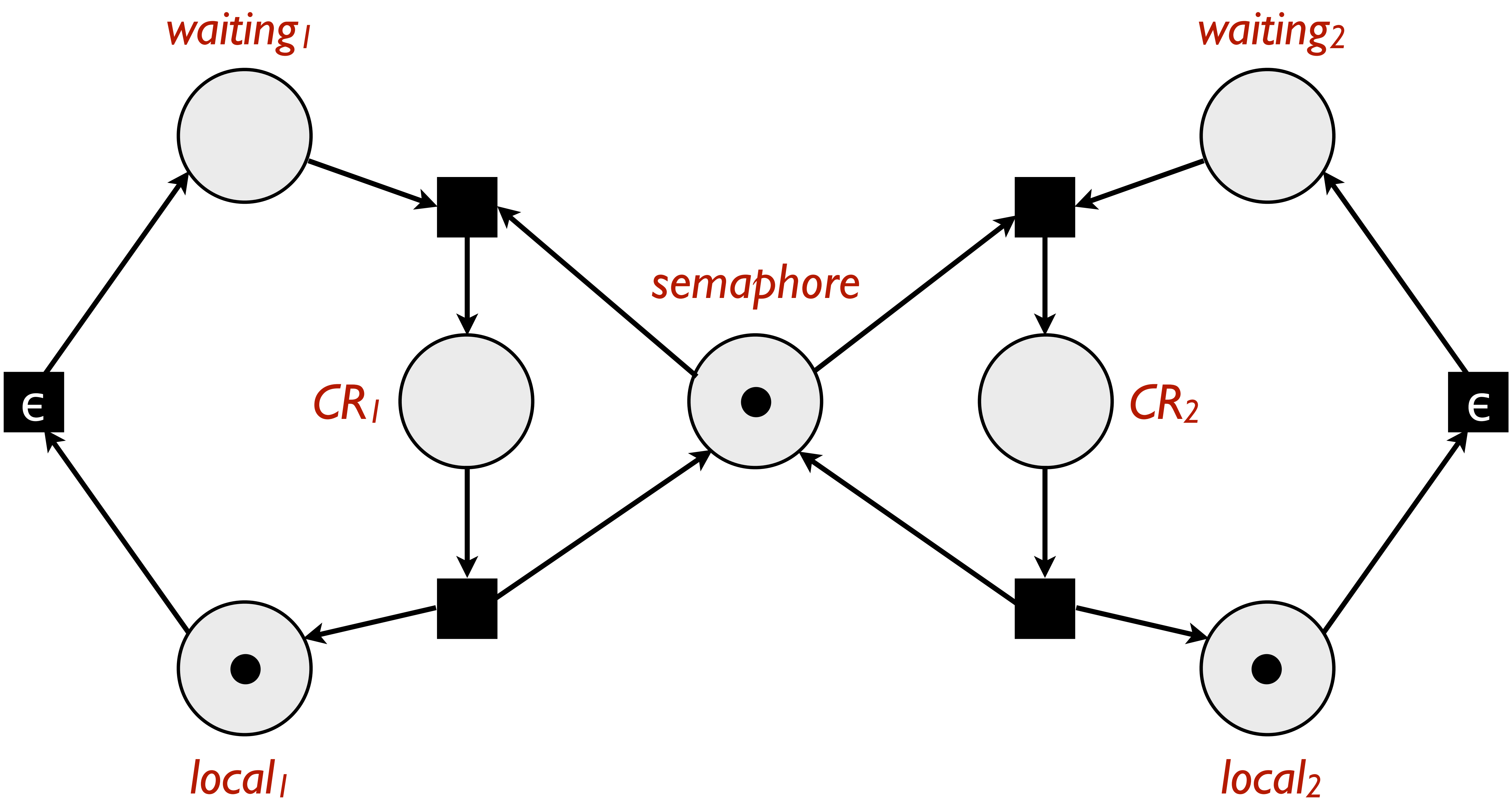
Producer-consumer problem



Mutual exclusion



Mutual exclusion



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Modelling power vs. analysability

- many properties of interest for concurrent systems can be automatically determined for Petri nets
  => *but can be very expensive in the general case*

- properties include:
  => *k-boundedness (i.e. no place ever has more than k tokens)*
  => *liveness*
  => *reachability*

- several tools are available
  => *http://www.informatik.uni-hamburg.de/TGI/PetriNets/tools/quick.html*

Reachability problem

- the problem to decide whether some marking *M* can be derived from the initial marking
- starting point: construct a reachability graph from the initial marking

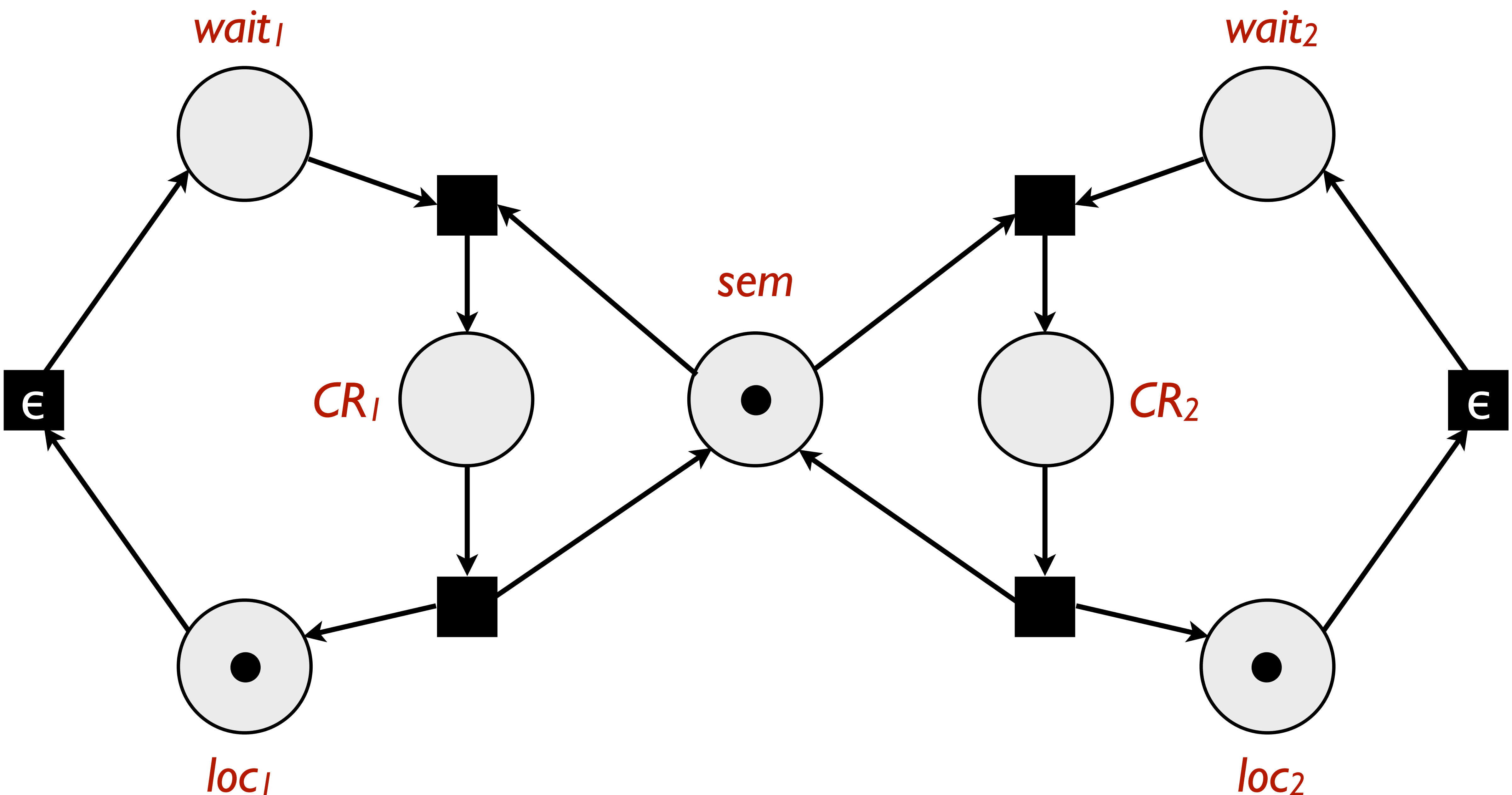
  => *i.e. a transition system completely describing its behaviour*

  => *nodes denote markings*

  => *edges denote occurrences*

- (more sophistication is needed when reachability graphs are not finite)

Reachability graph for our semaphore

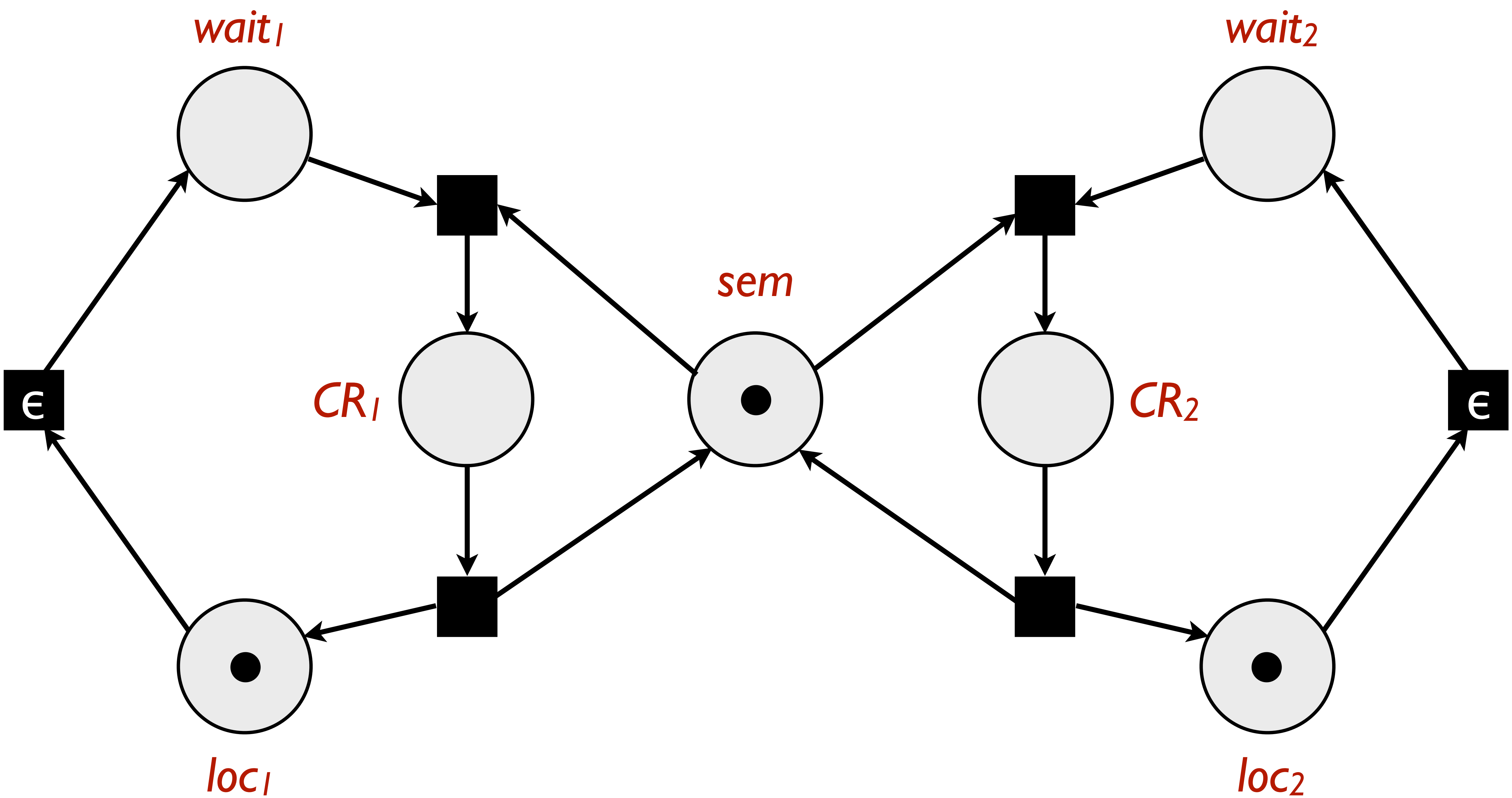


express marking M as a vector:

($M(\text{wait}_1)$ $M(\text{CR}_1)$ $M(\text{loc}_1)$ $M(\mathit{sem})$ $M(\text{wait}_2)$ $M(\text{CR}_2)$ $M(\text{loc}_2)$)

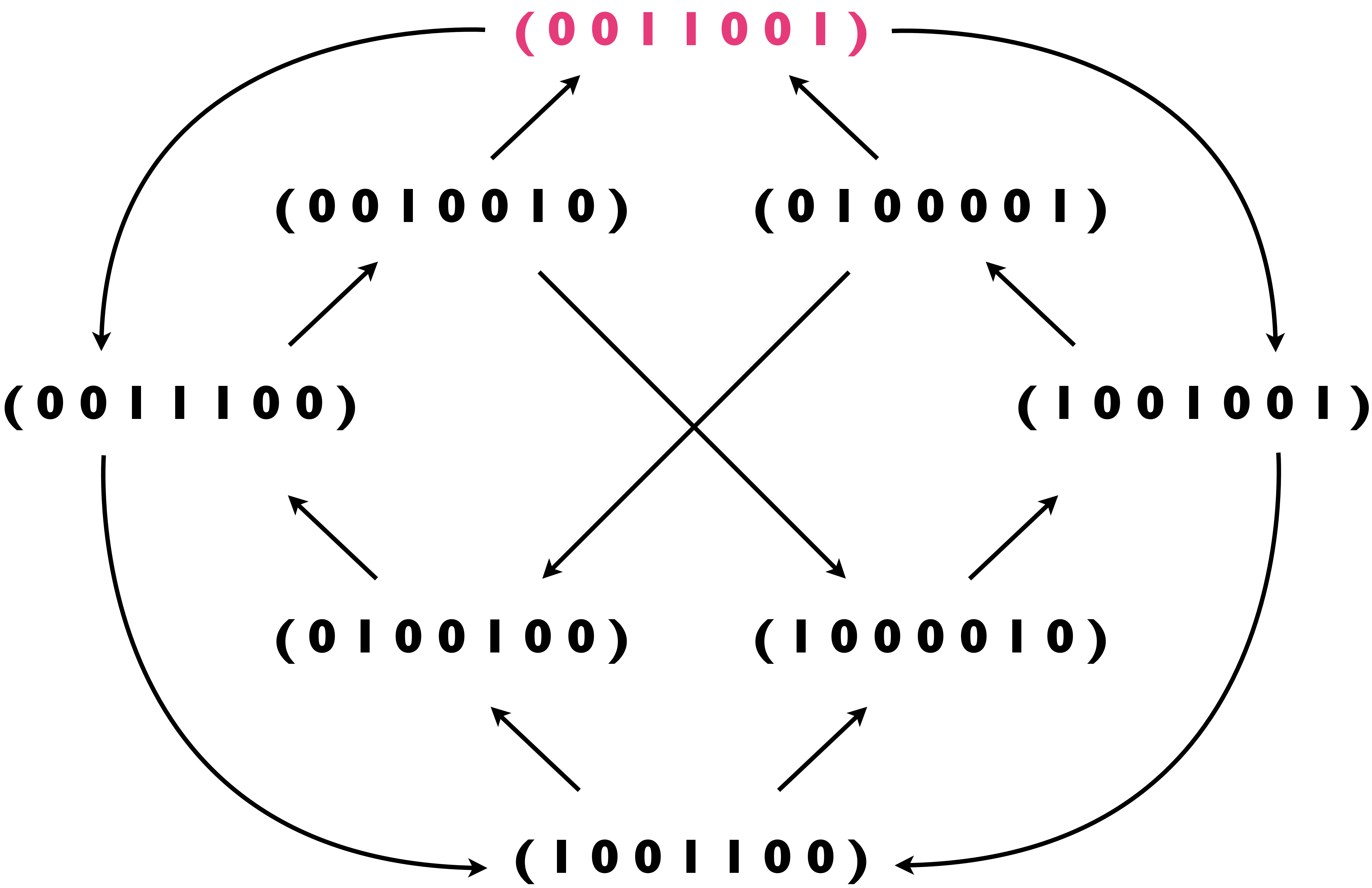
***i.e.* (0 0 1 1 0 0 1)**

Reachability graph for our semaphore



- *prove that **(0 1 0 0 0 1 0)** is unreachable*
- *prove that* $M(\text{CR}_1)+M(\text{CR}_2)+M(\text{sem}) = 1$

Reachability graph for our semaphore



Deciding reachability is expensive

- reachability is an important analysis

- decidable, but expensive in the general case
  => *EXPSPACE-hard*
  => *reachability graph not always finite*

- part II of Reisig (2013) treats the problem with more sophistication than we have

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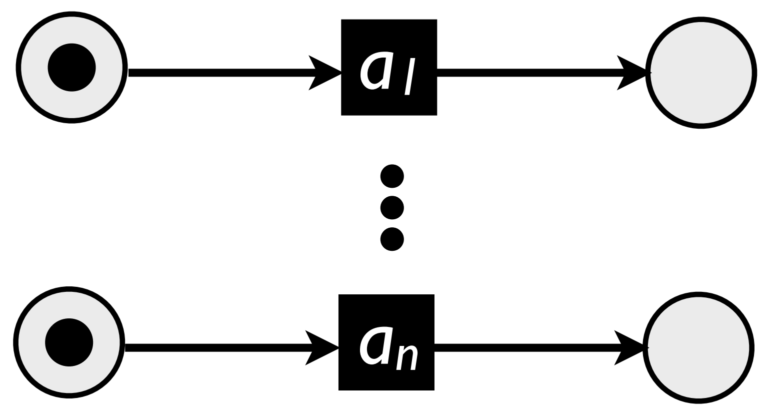
The problem of interleaving semantics

- consider the following Petri net:



- its reachability graph contains 2^n states

  => *state explosion problem*
  => *due to interleaving of occurrences*
  => *unnecessary: ordering of occurrences here immaterial!*

Interleaving vs. true concurrency semantics

- an interleaving semantics imposes a total ordering on sequences of occurrences
  => *completely described by a reachability graph*
  => *nodes denote markings; edges denote occurrences*
  => *state explosion!*

- a true concurrency semantics instead models time as a partial order
  => *two or more occurrences can happen simultaneously*
  => *completely described by a so-called unfolding*

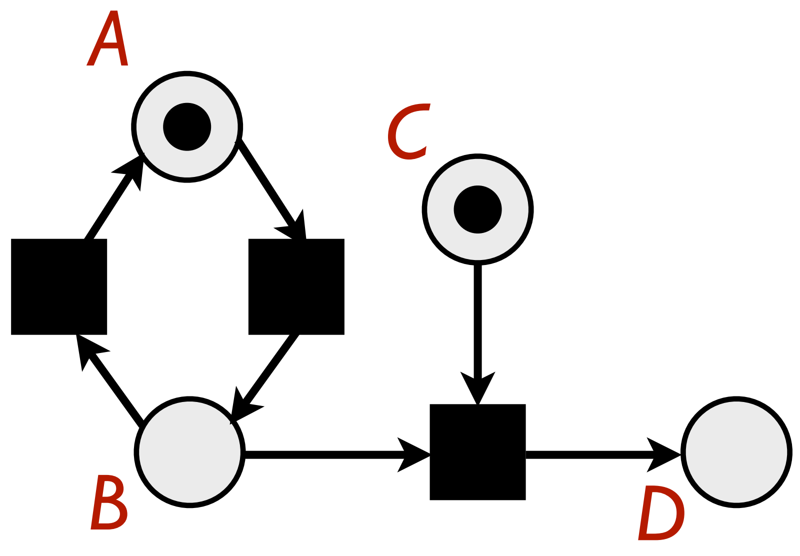
Unfoldings are more compact representations of concurrency

- an unfolding of a Petri net *N* is a Petri net that is more "tree like" - but represents the same behaviour

- explicitly represents concurrency and causal dependence between different behaviours

- *idea:* analyse the unfolding of a Petri net <u>itself</u>, rather than an underlying transition system (as in the interleaving semantics)

Example: an unfolding



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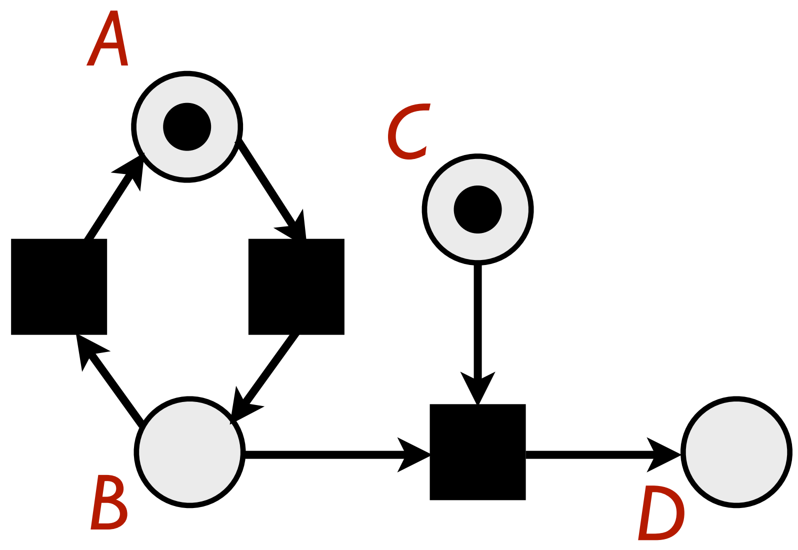

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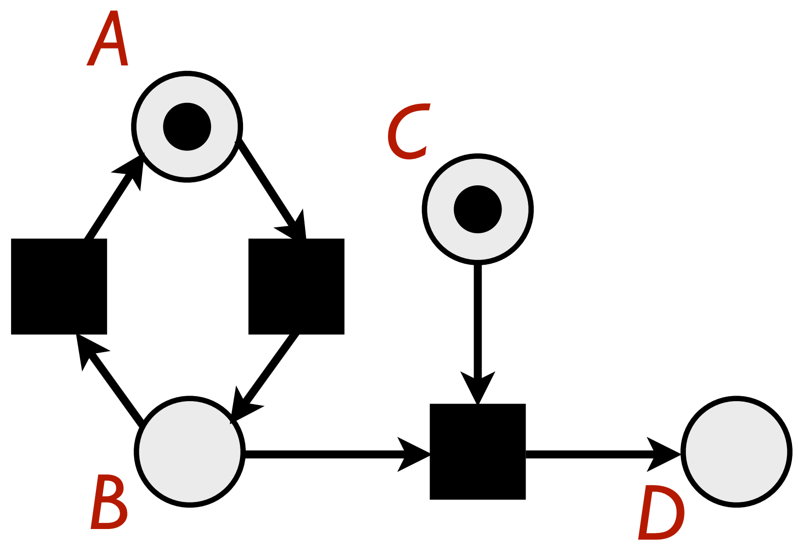

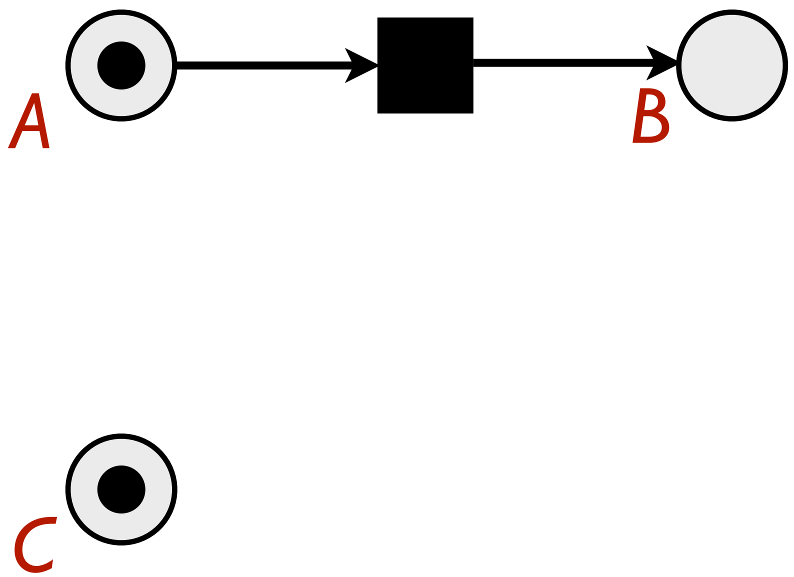

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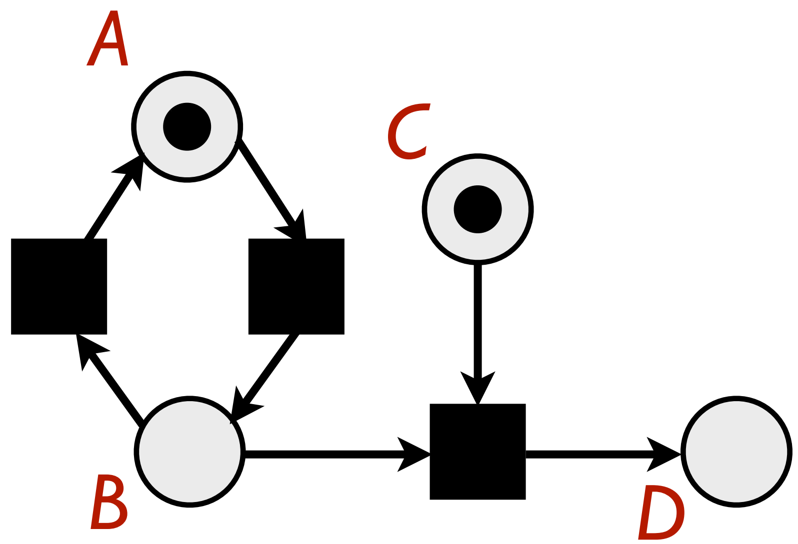

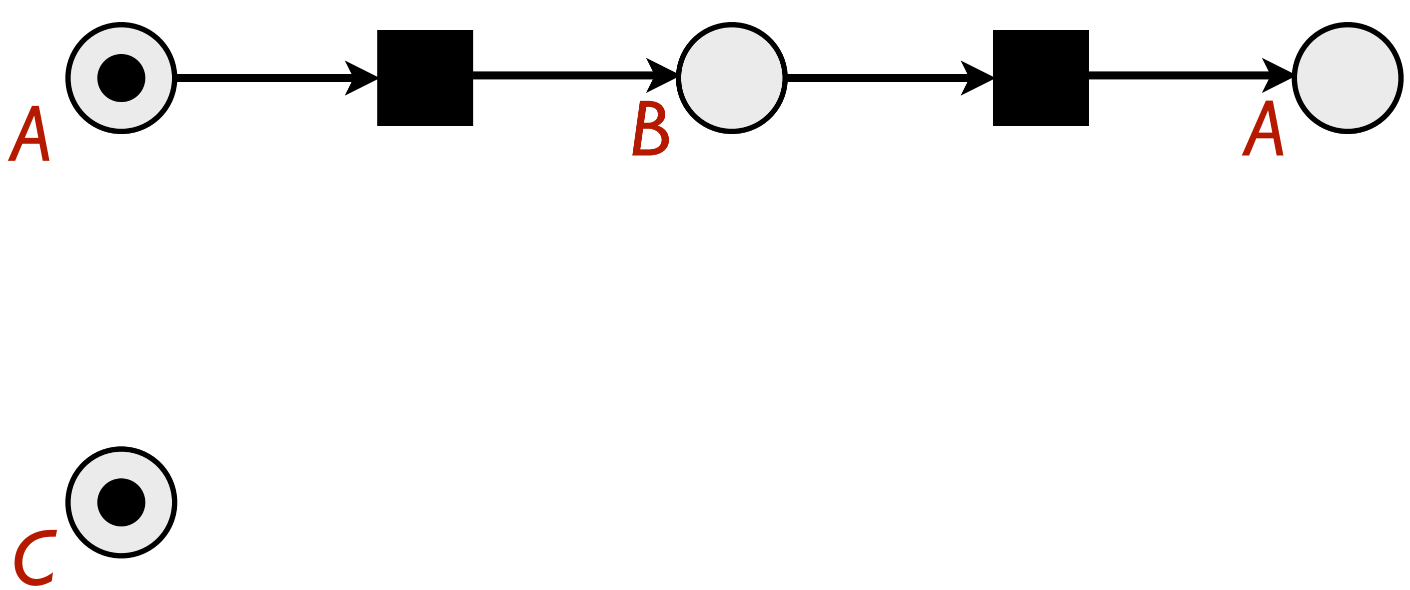

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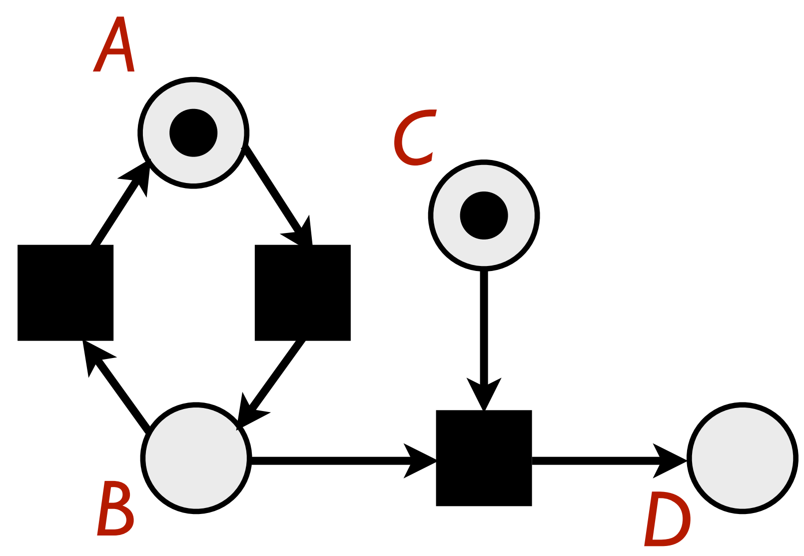

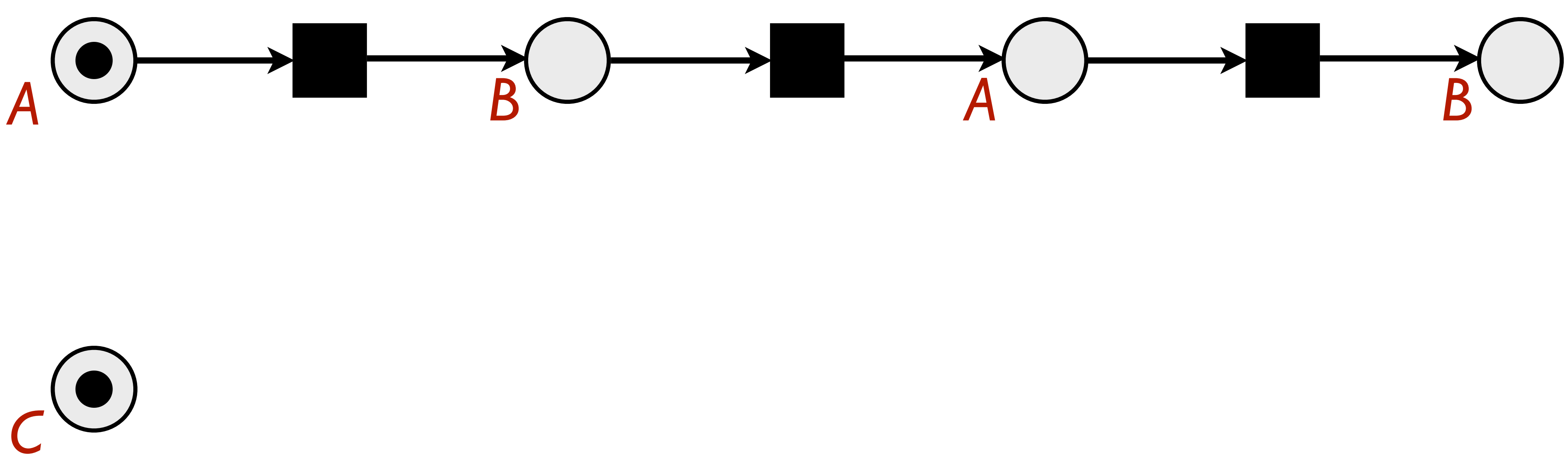

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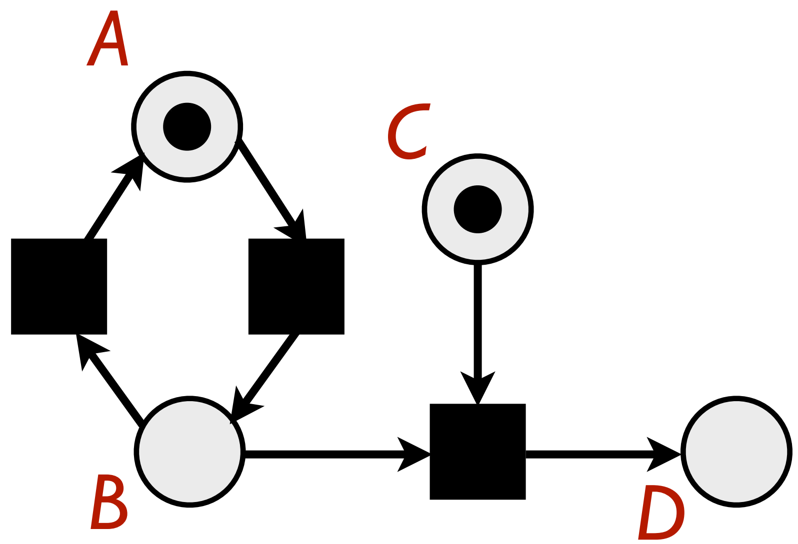

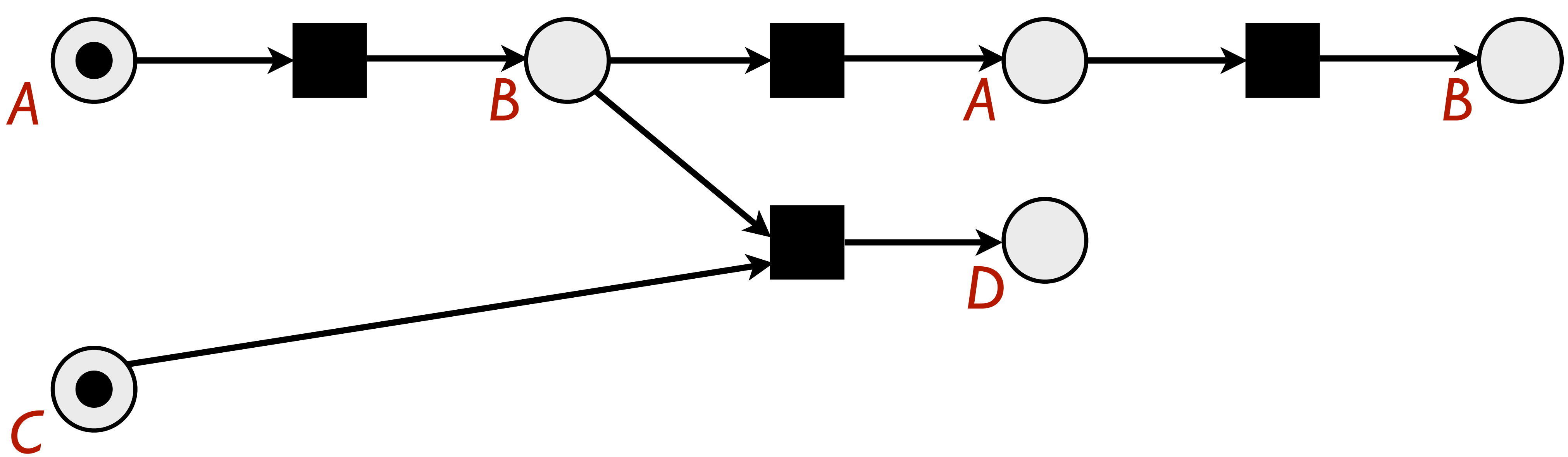

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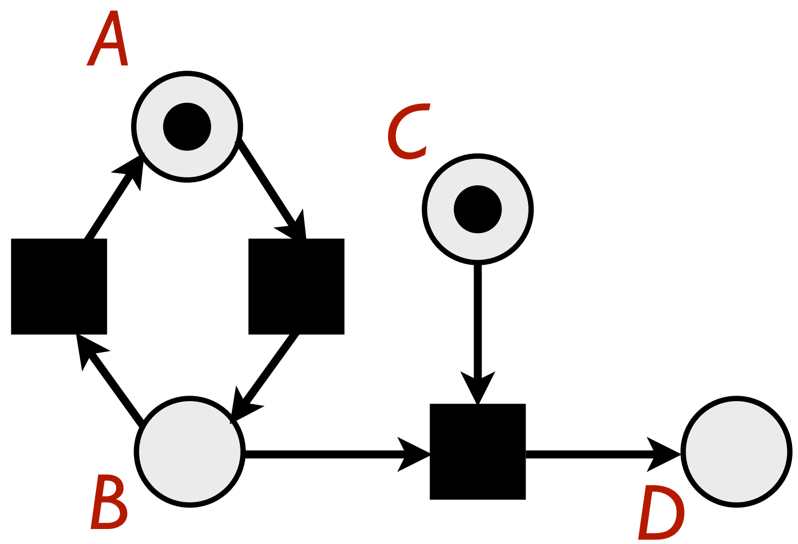

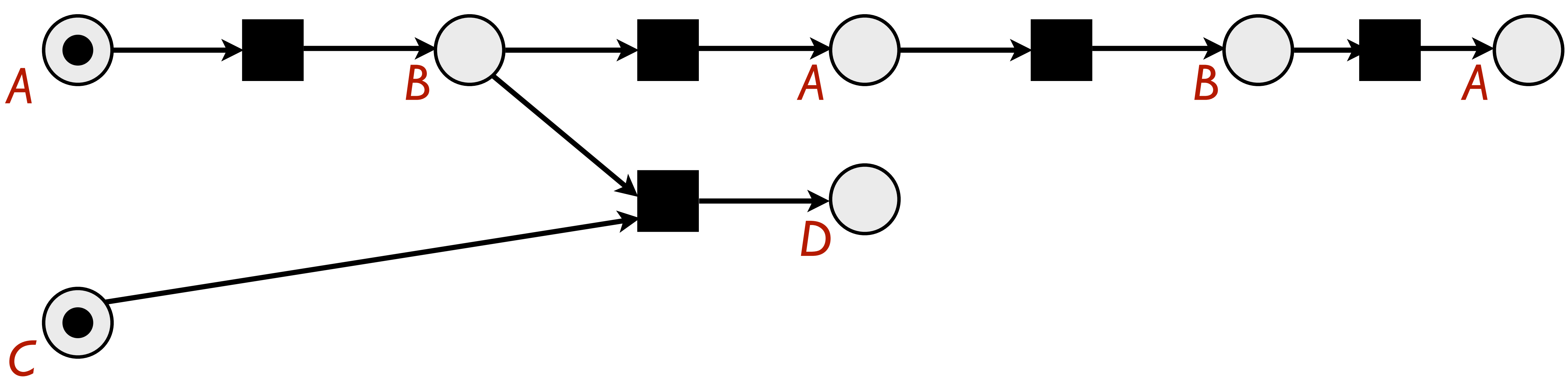

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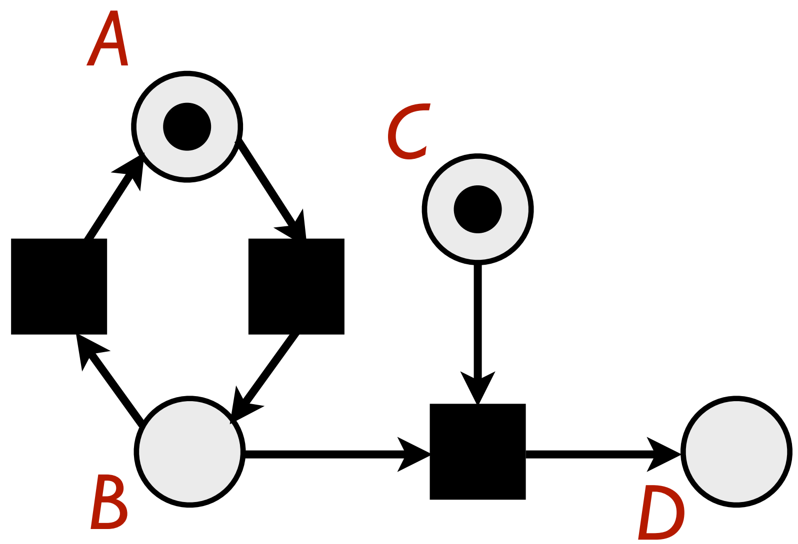
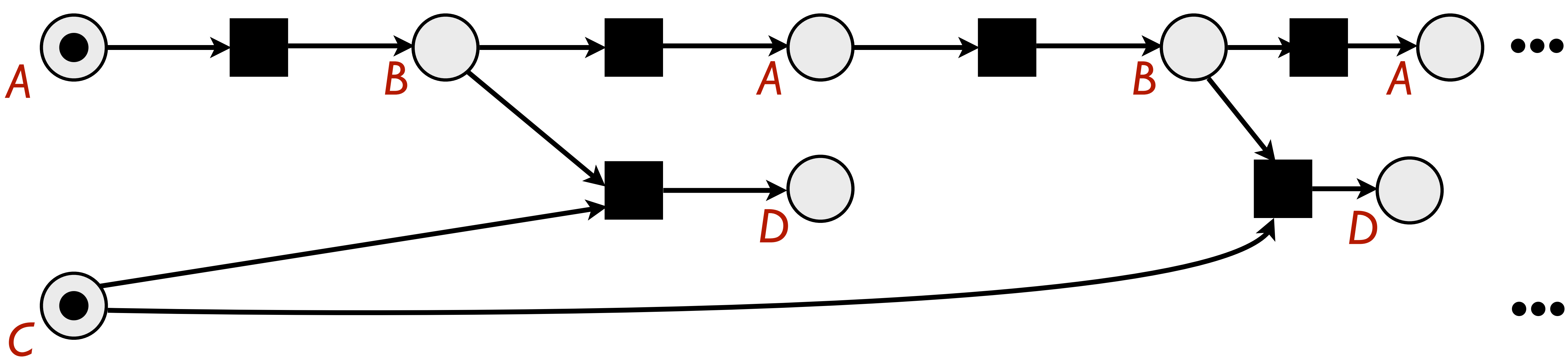




Constructing an unfolding

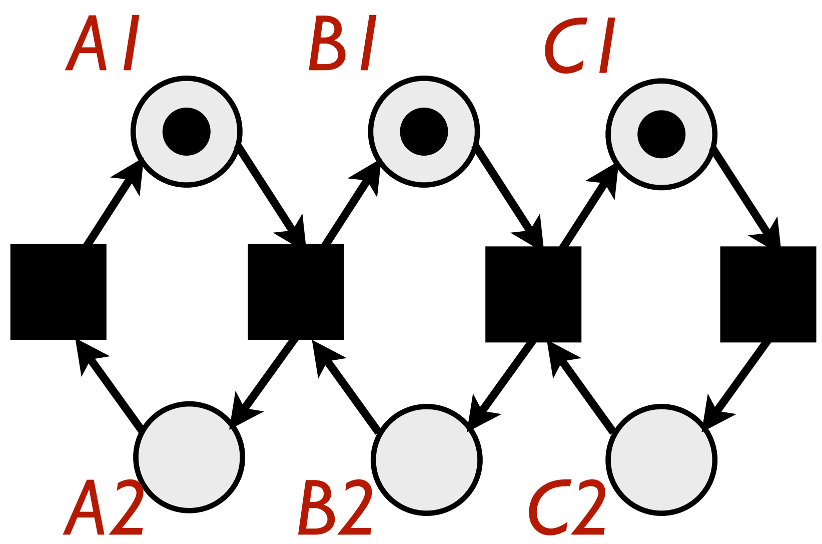
- assumption: Petri nets are 1-bounded
  => *possible to generalise to other Petri net variants*

- steps to construct an unfolding *N'* from a Petri net *N*:

  (1) initialise *N'* with the places in *N* containing tokens in the initial marking
  (2) if a reachable* marking in *N'* enables a transition *t* in *N*, then disjointly add *t* to *N'* and:
  => link it to the corresponding preset
  => disjointly add the postset of *t*
  (3) iterate step 2

  **checking reachability is far easier for the unfolding net class*

Another example



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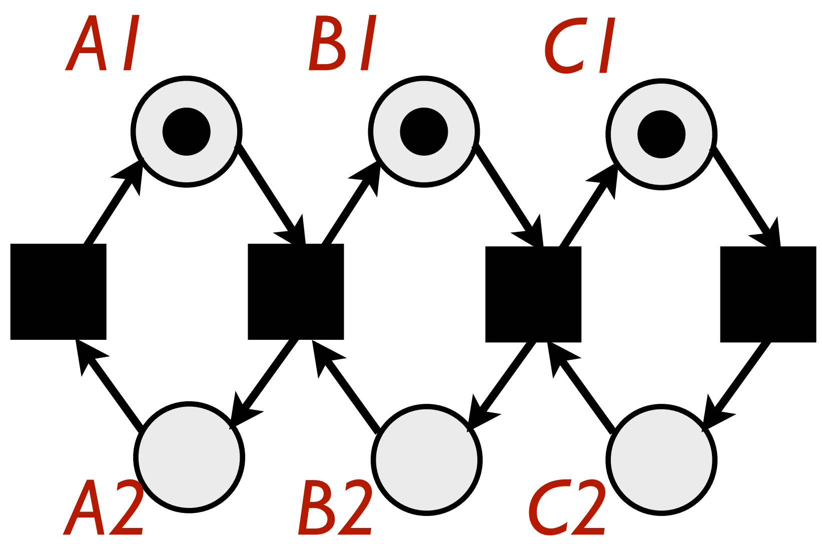

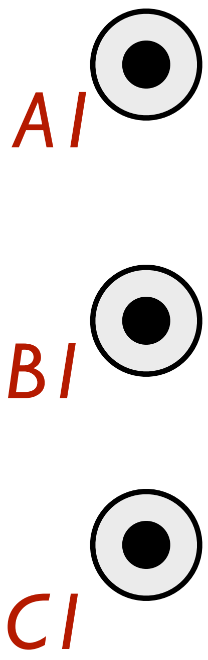

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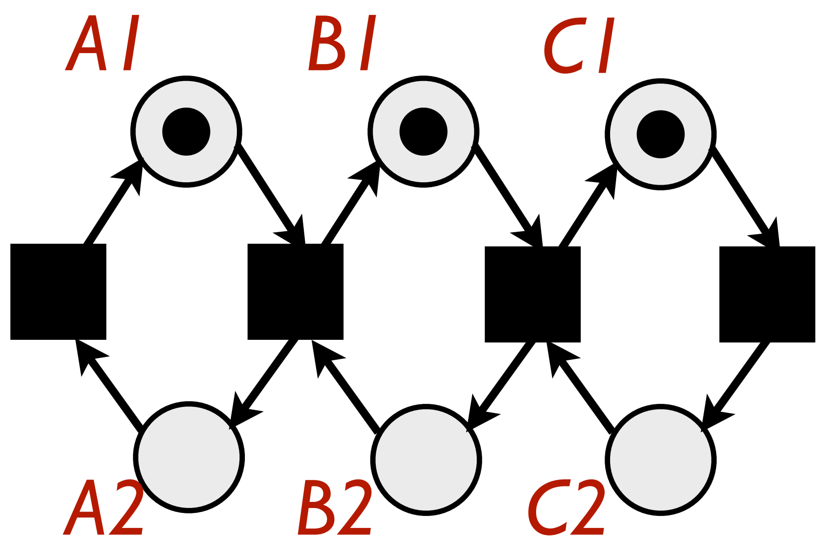
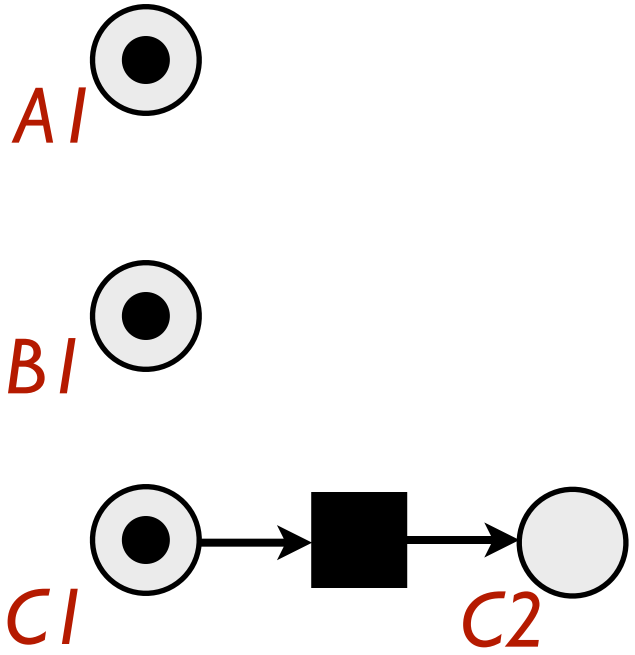
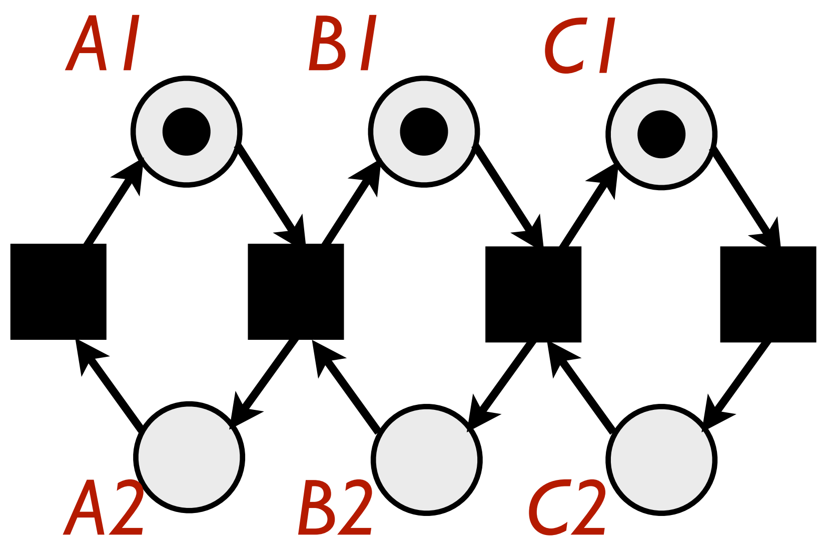
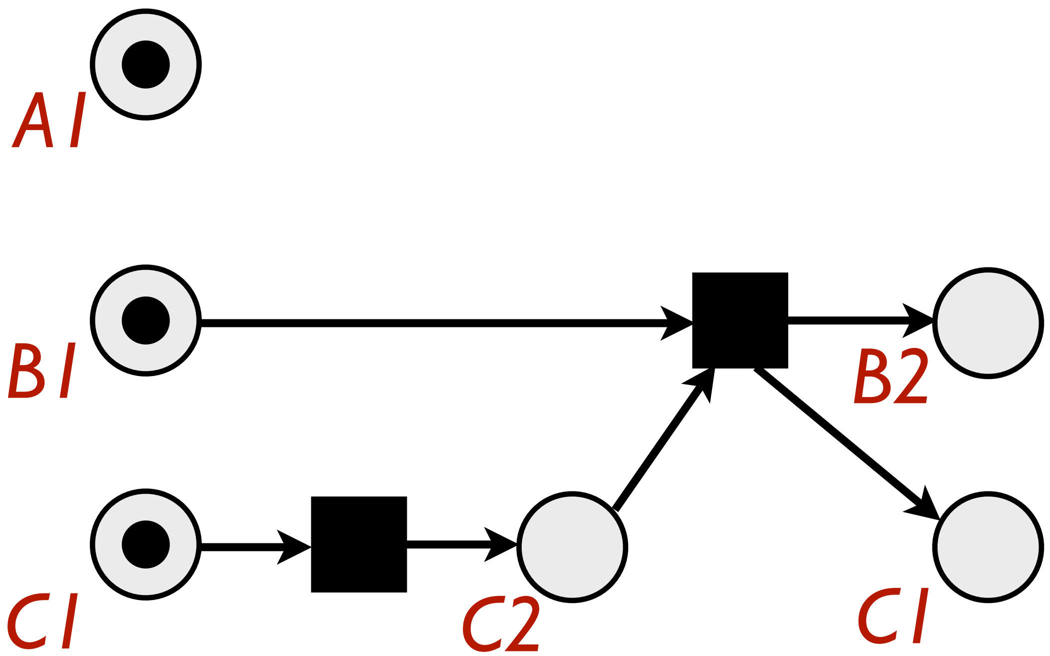


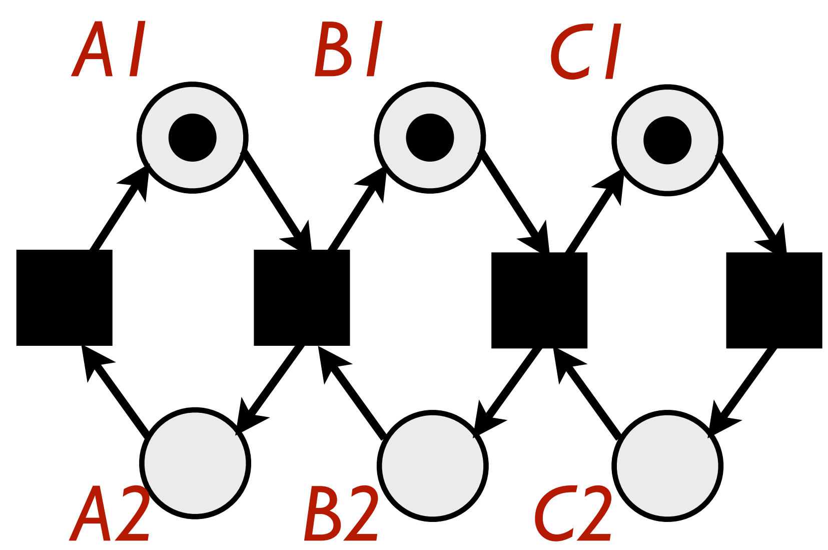


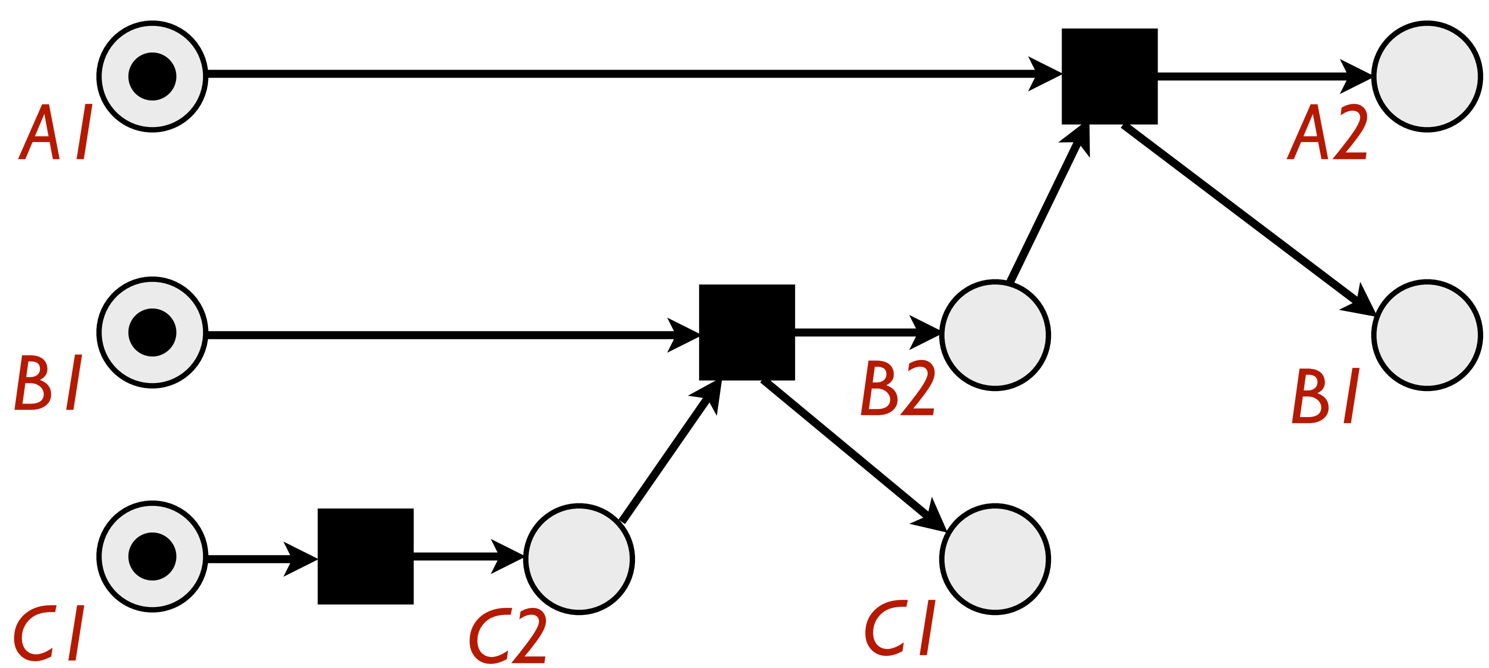
Another example



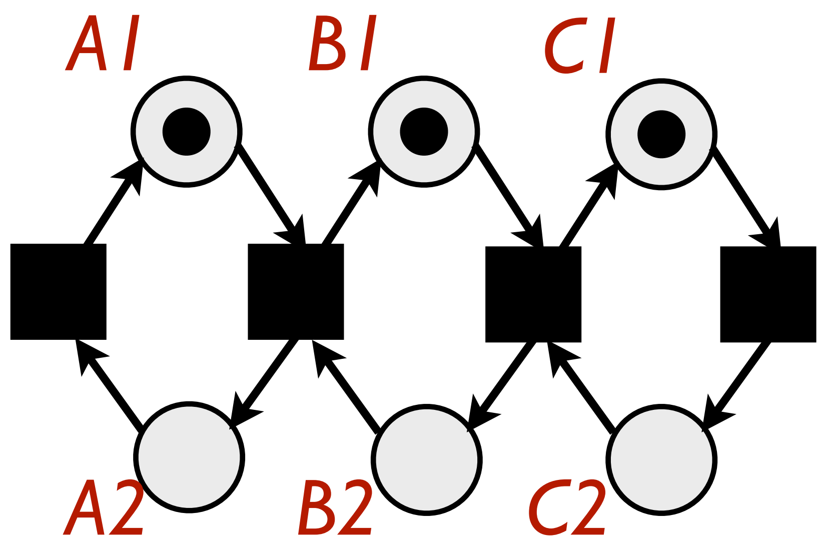


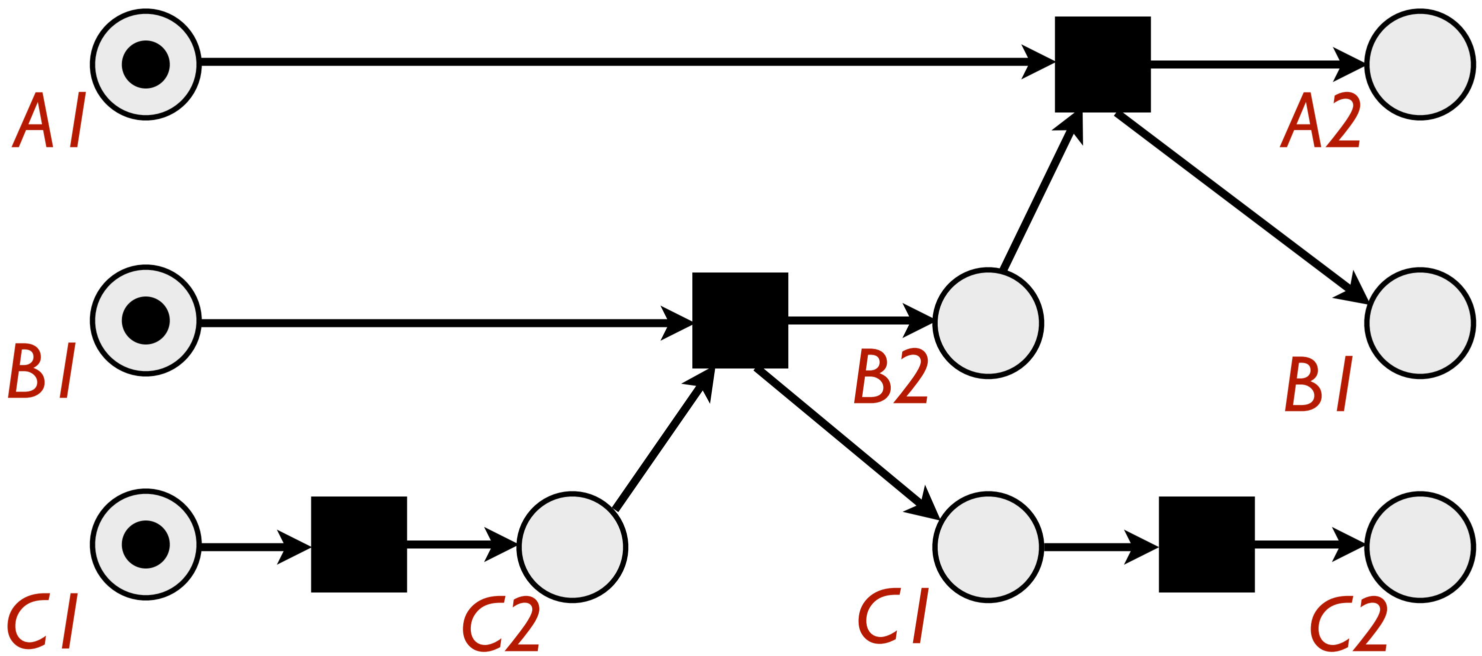
Another example





Another example





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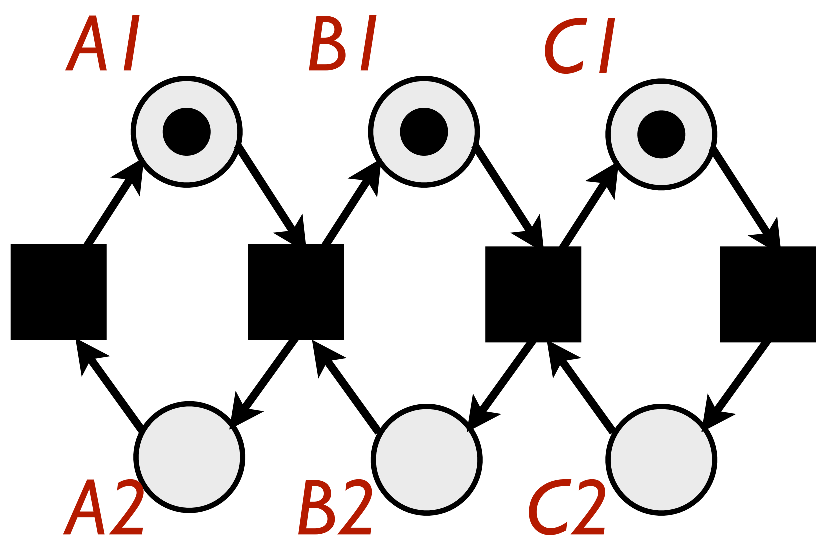

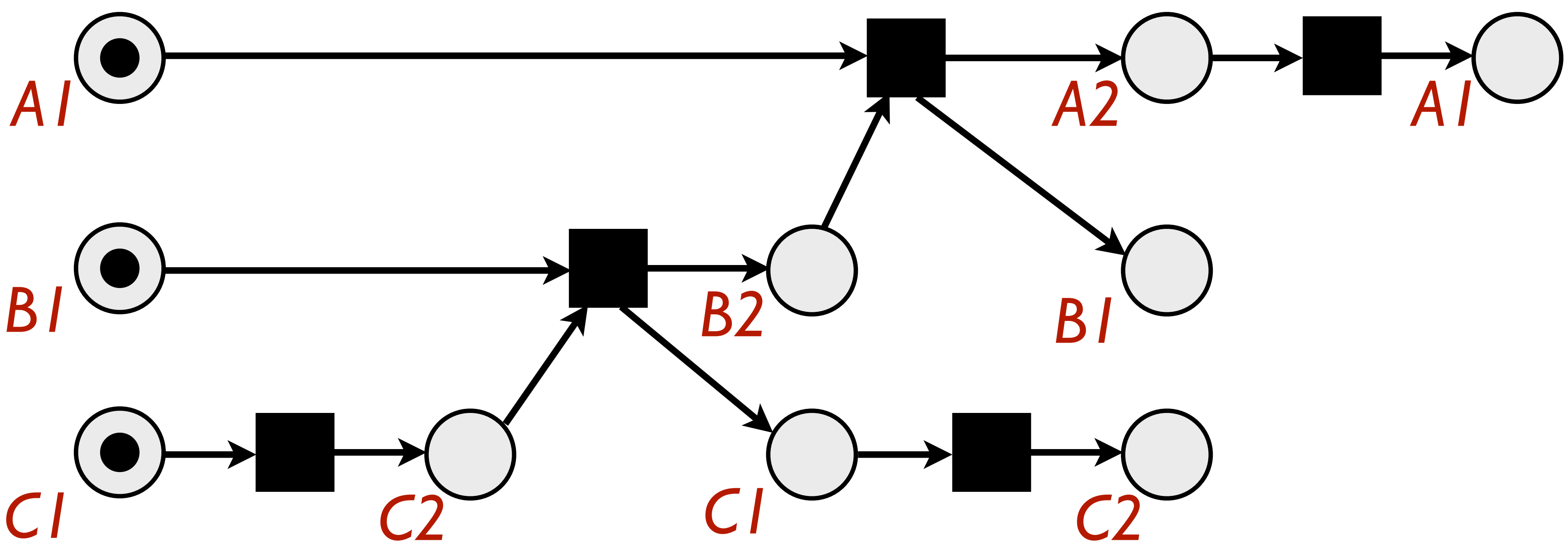

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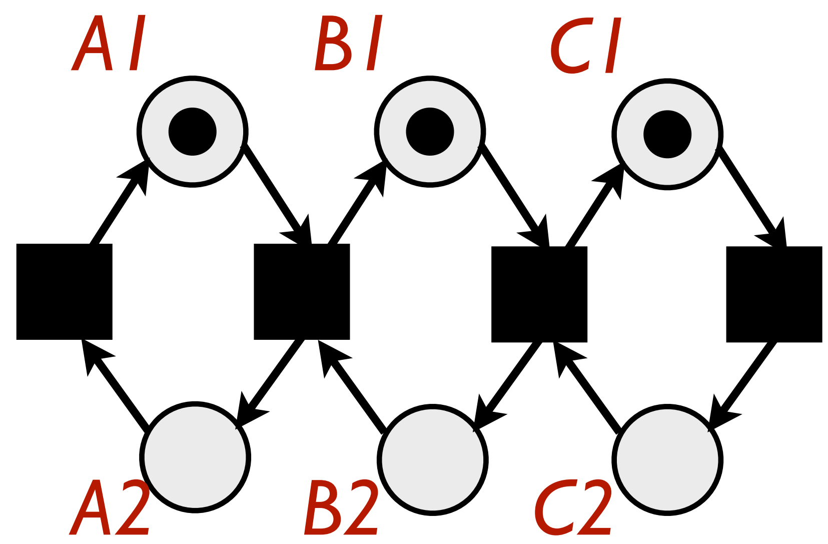
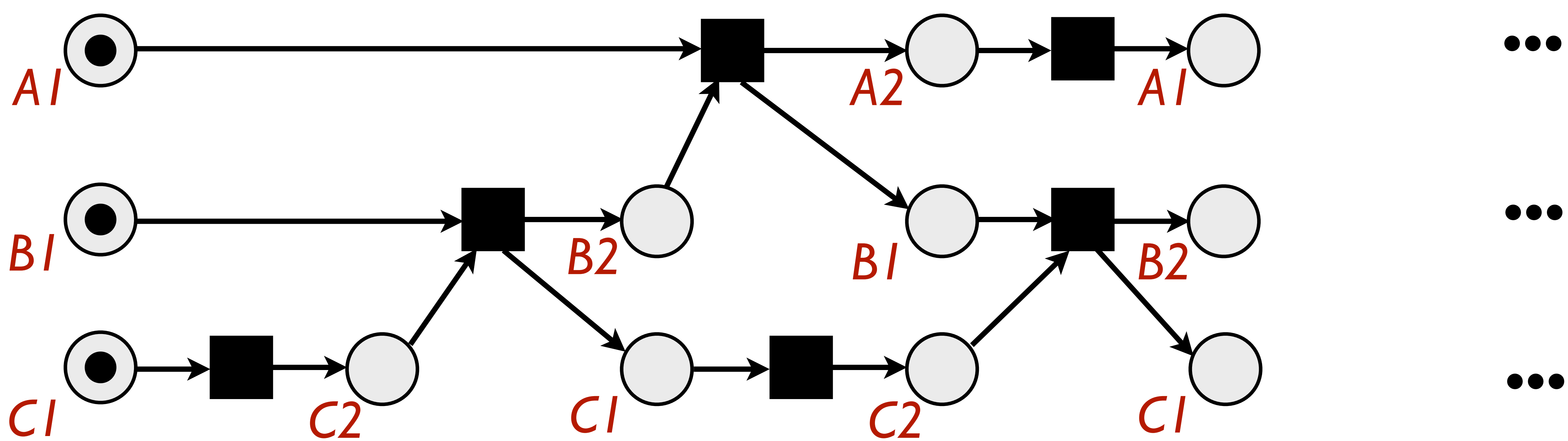
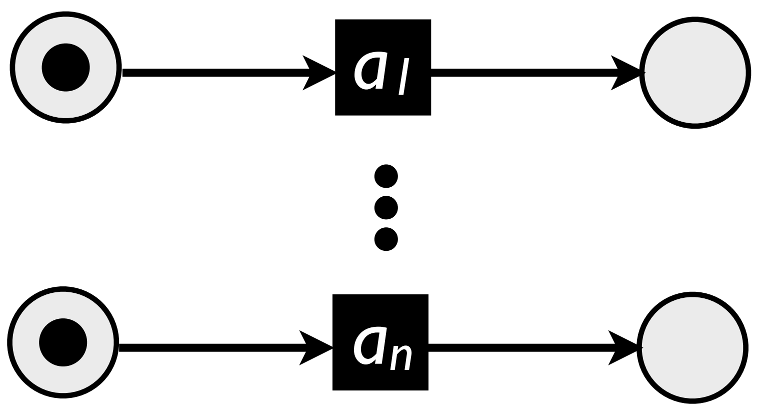




Returning to our small example

- construct an unfolding of the following Petri net:



Returning to our small example

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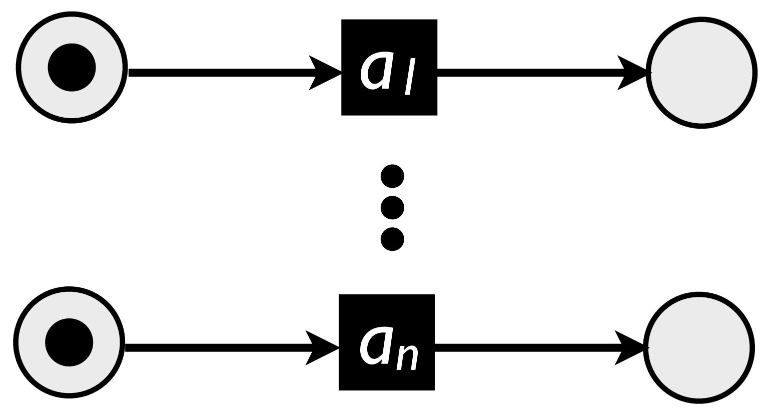
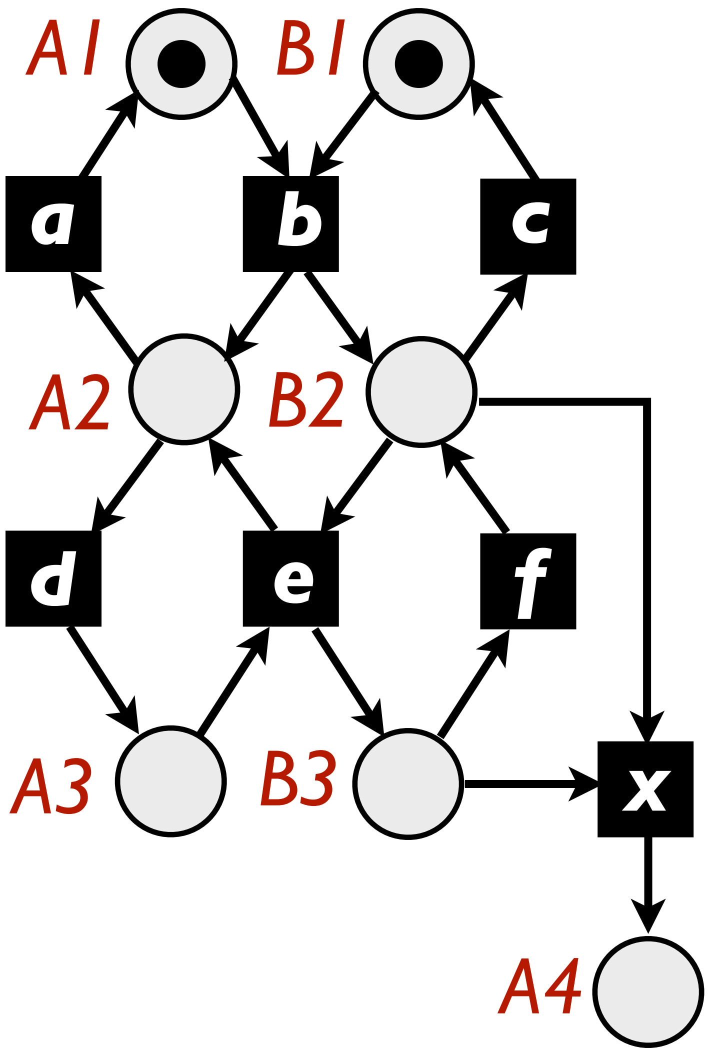


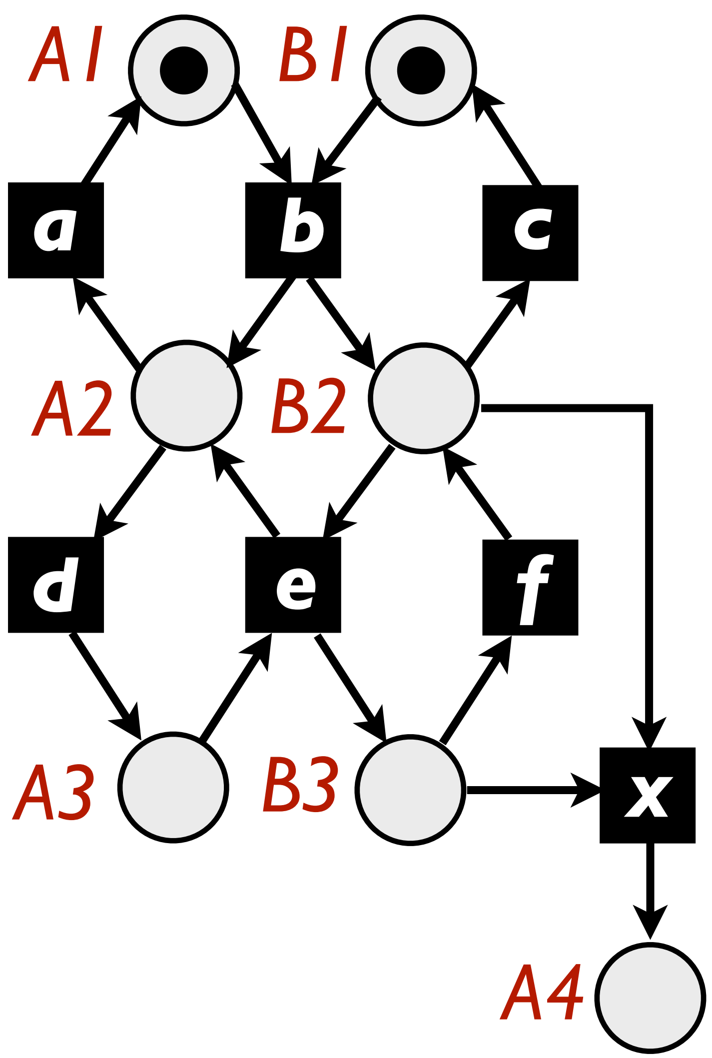
the unfolding is just the Petri net itself!
=> size O(n)
=> whereas interleaving yields 2^n reachable states

Petri net analysis using unfoldings

- suppose we want to know if some transition *t* in a Petri net *N* can occur
- compute an answer by exploring the unfolding of *N* until either:
  => *a transition labelled t is found*
  => *or it can be concluded that no such transition occurs*
- for finite unfoldings, compute and explore the whole structure
- for infinite unfoldings, only a finite prefix is computed and explored



can "x" ever occur?



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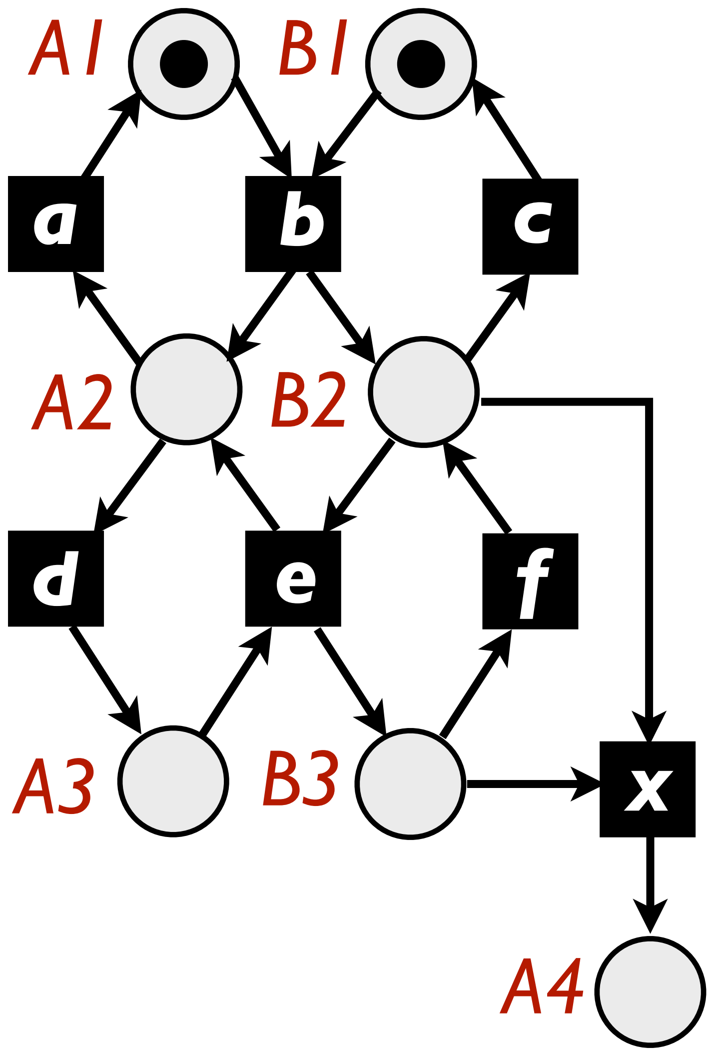

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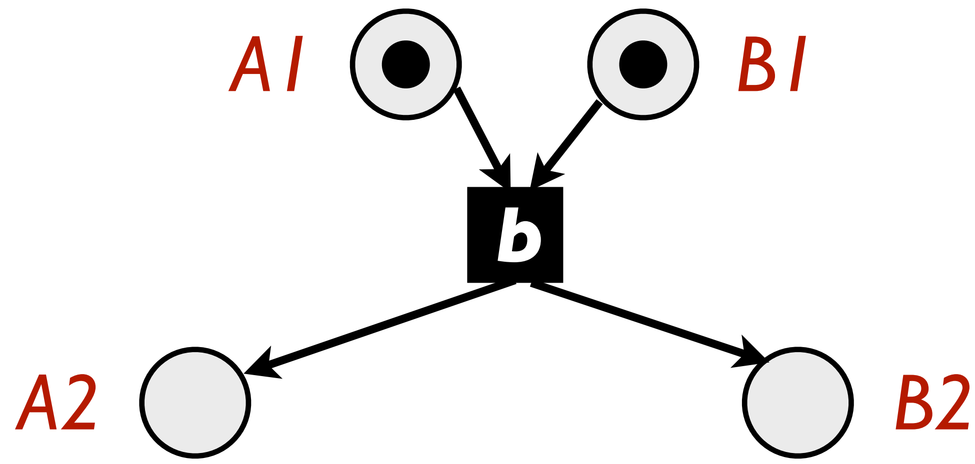

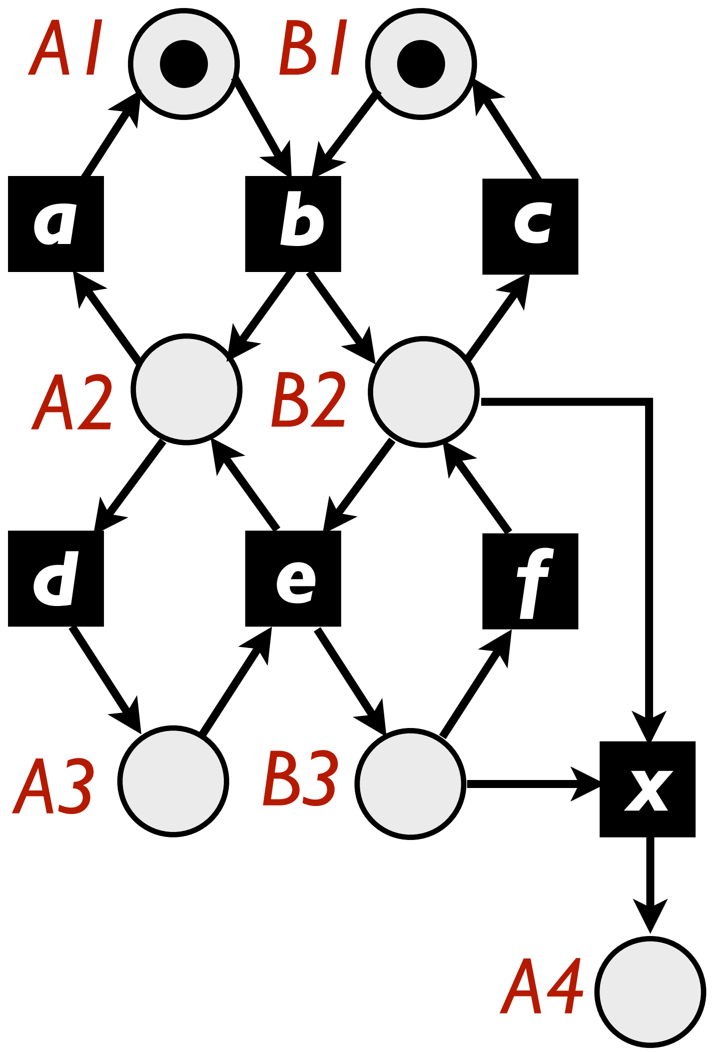

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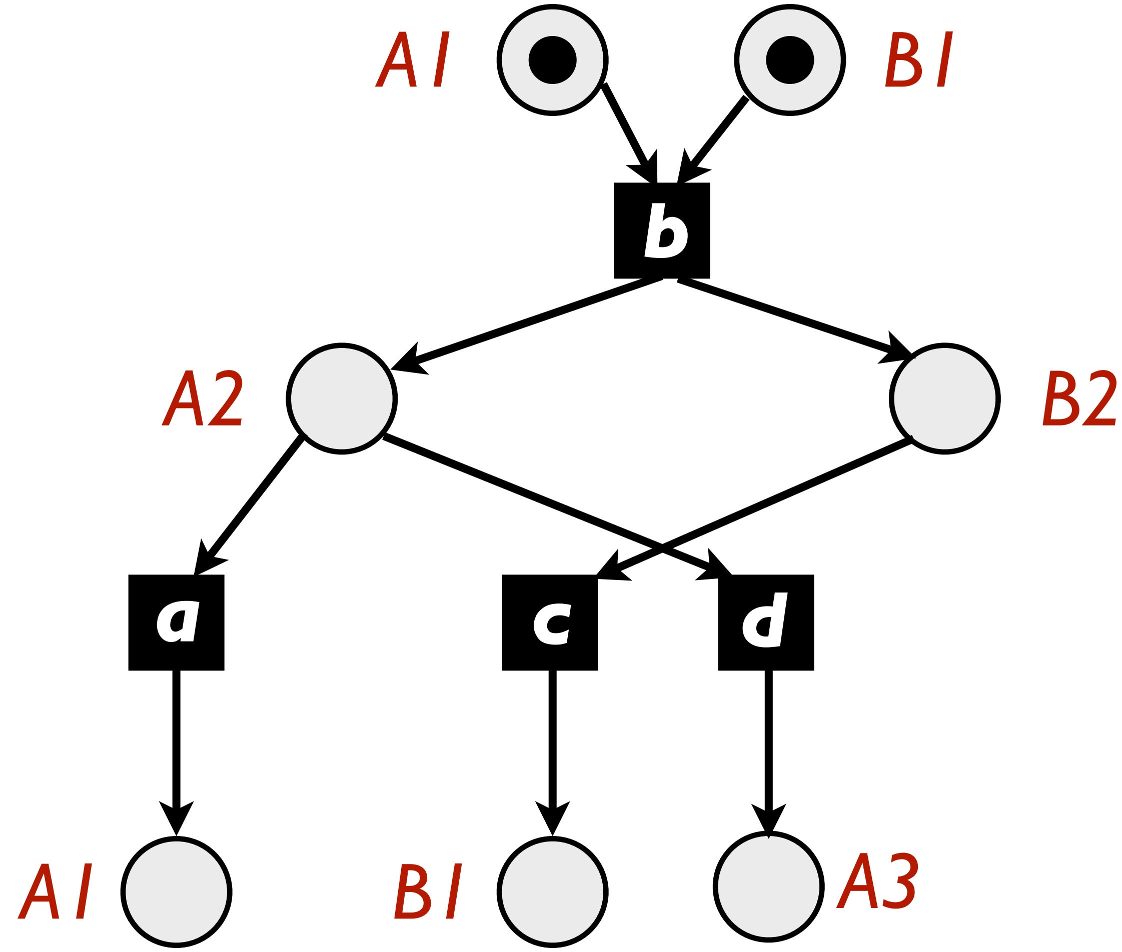

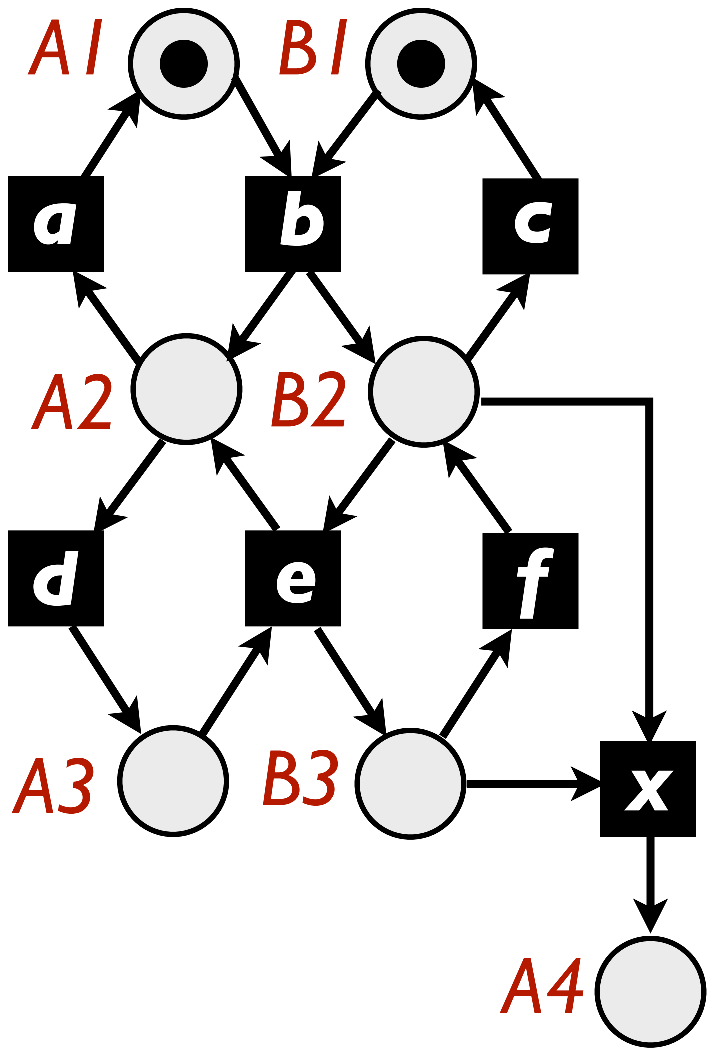

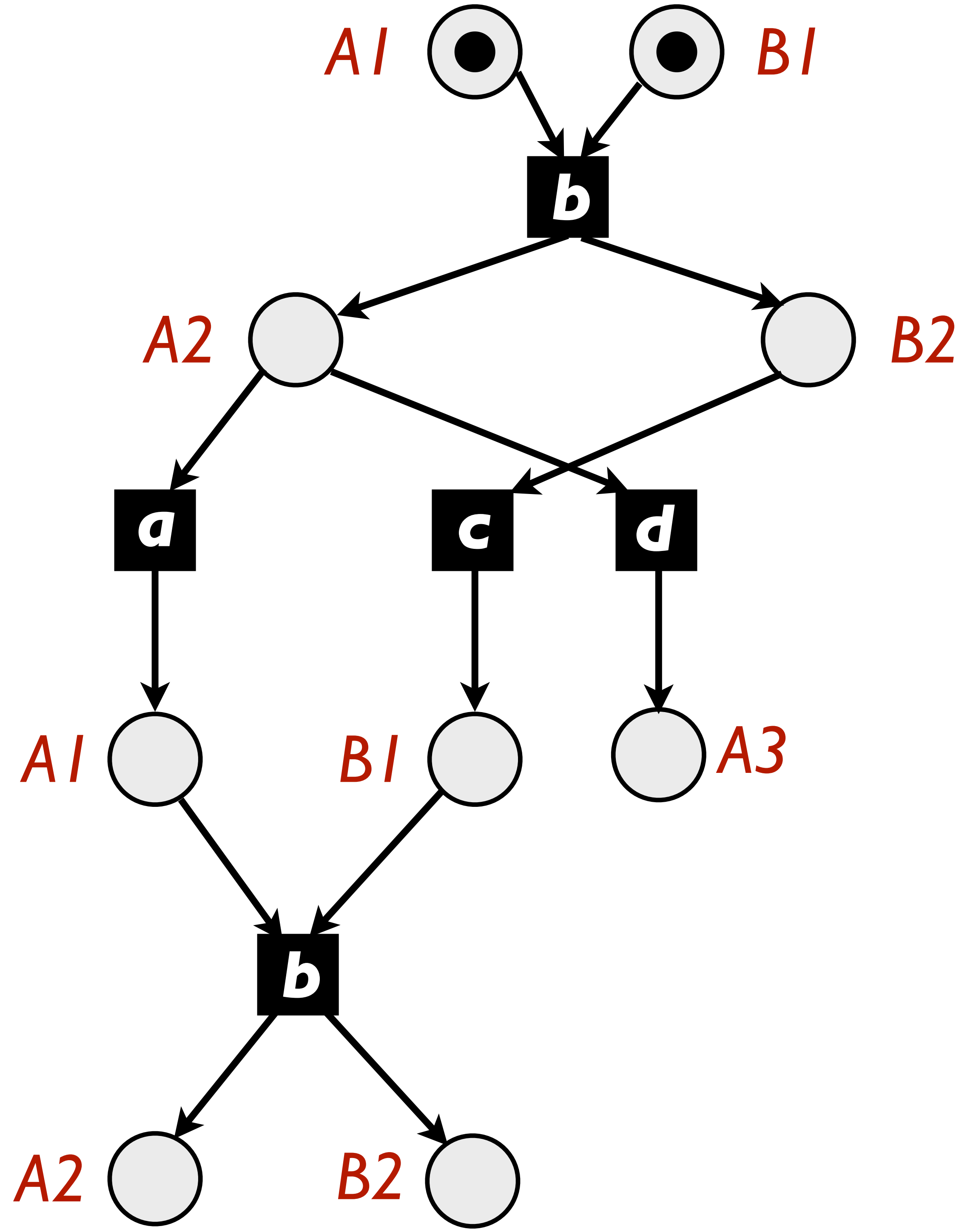

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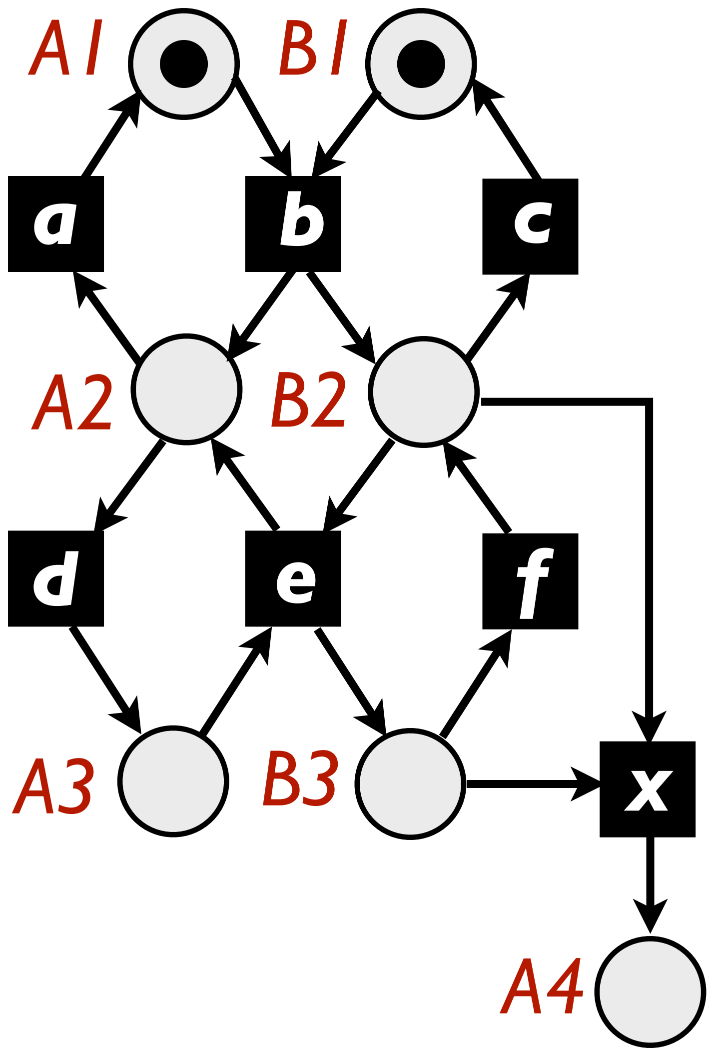

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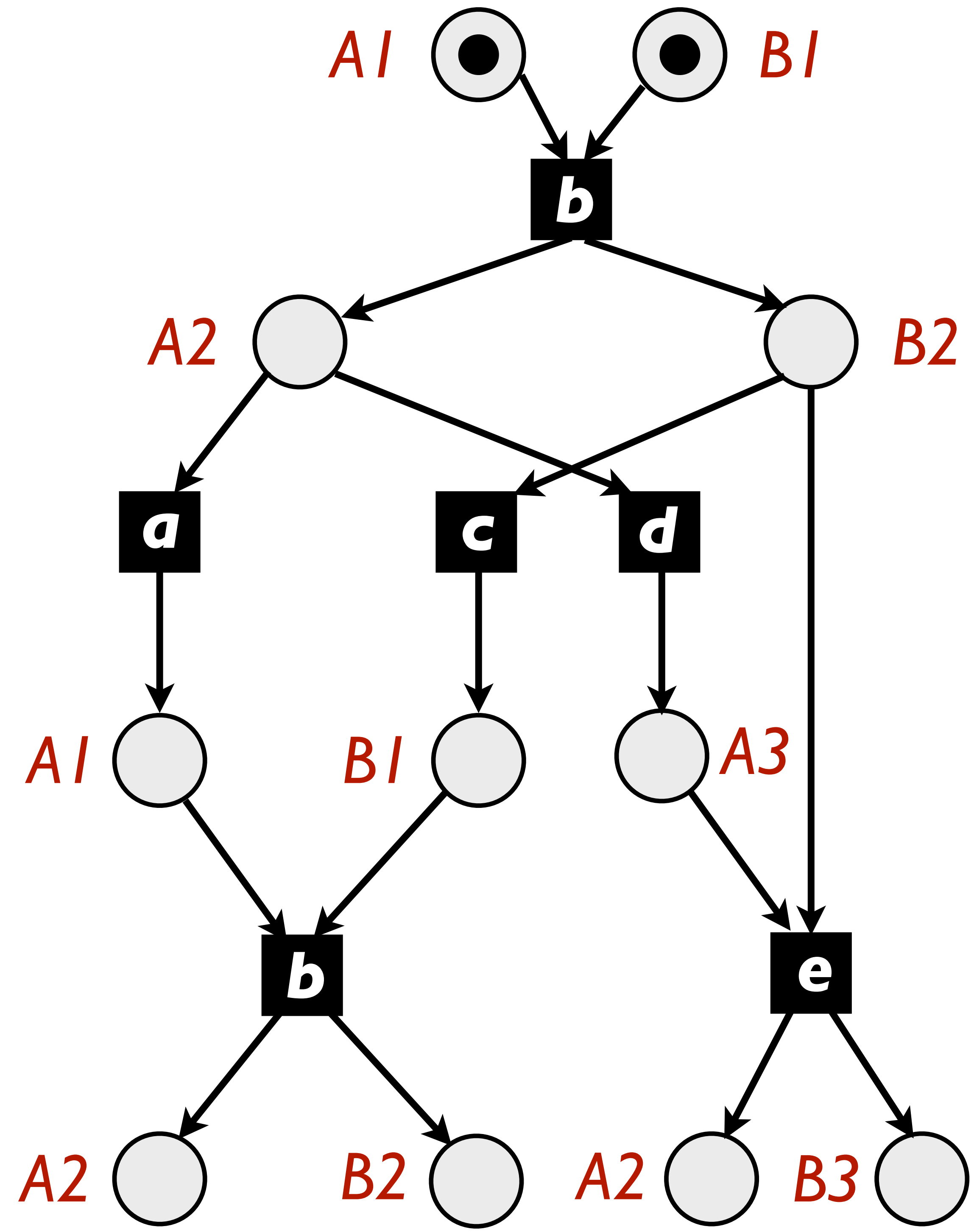

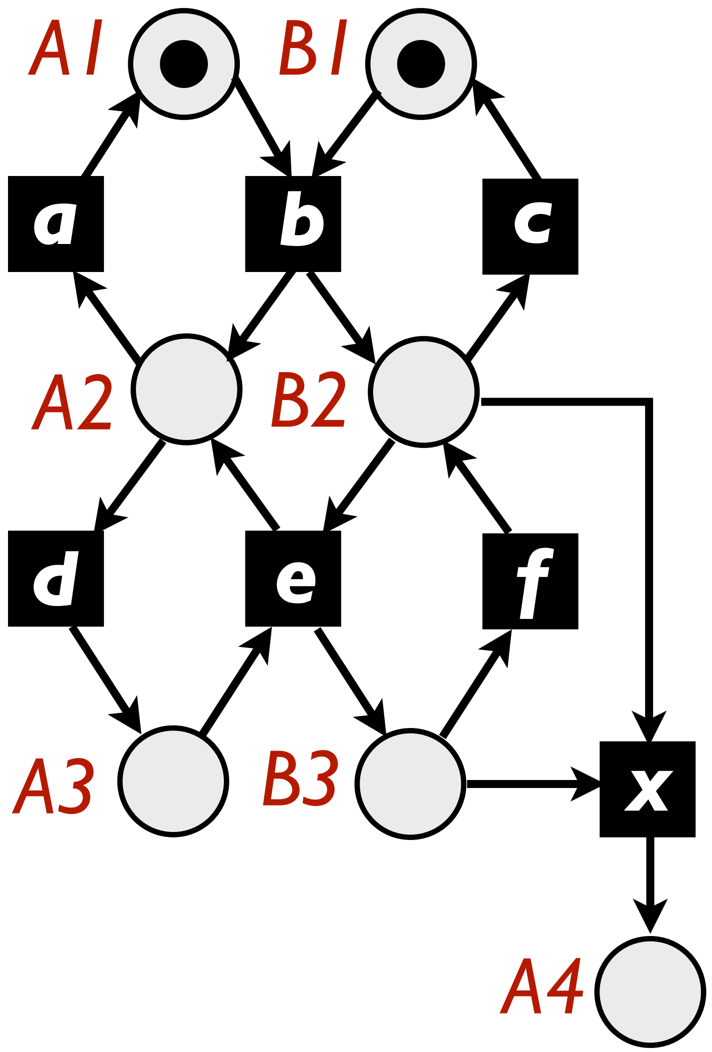

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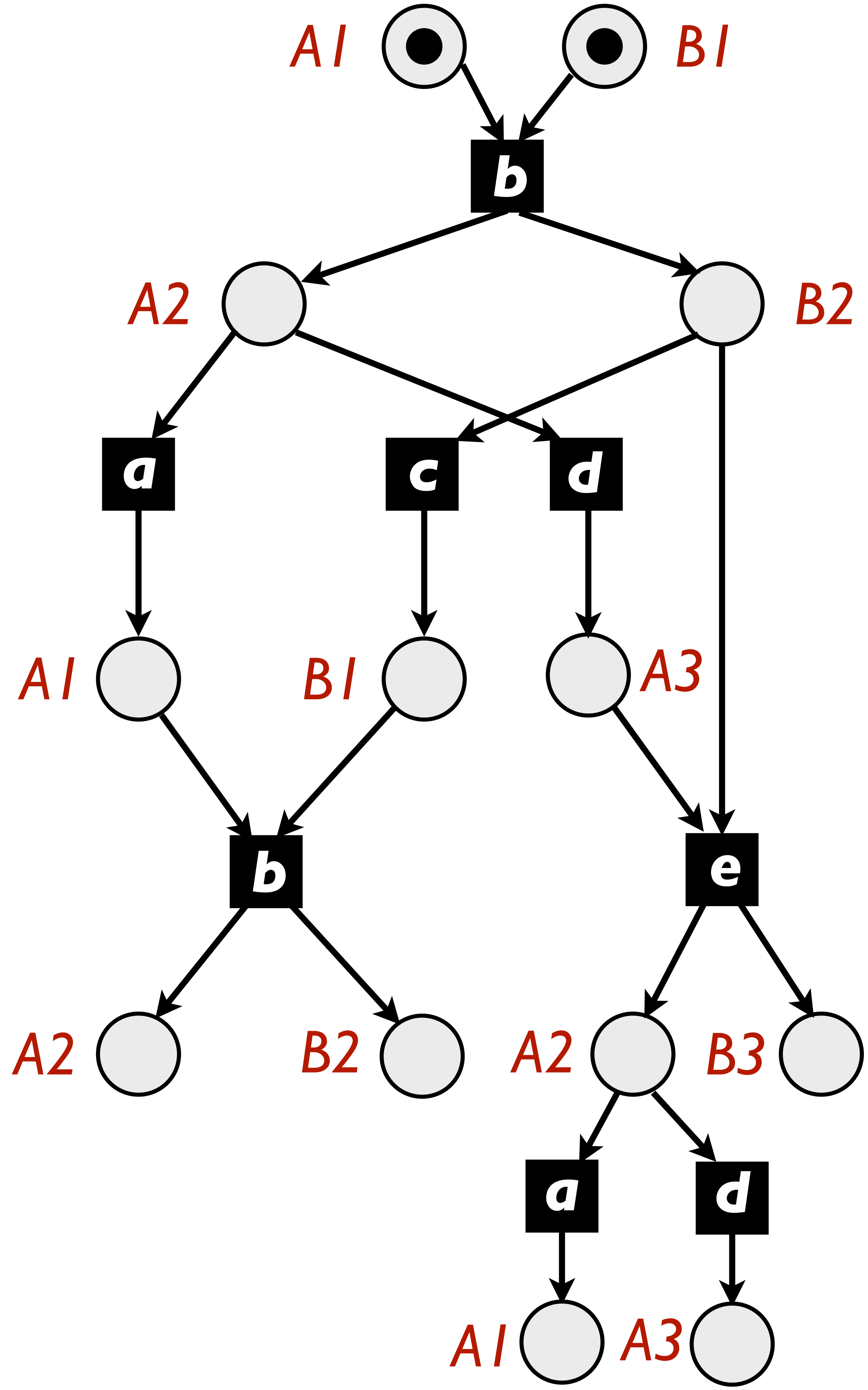

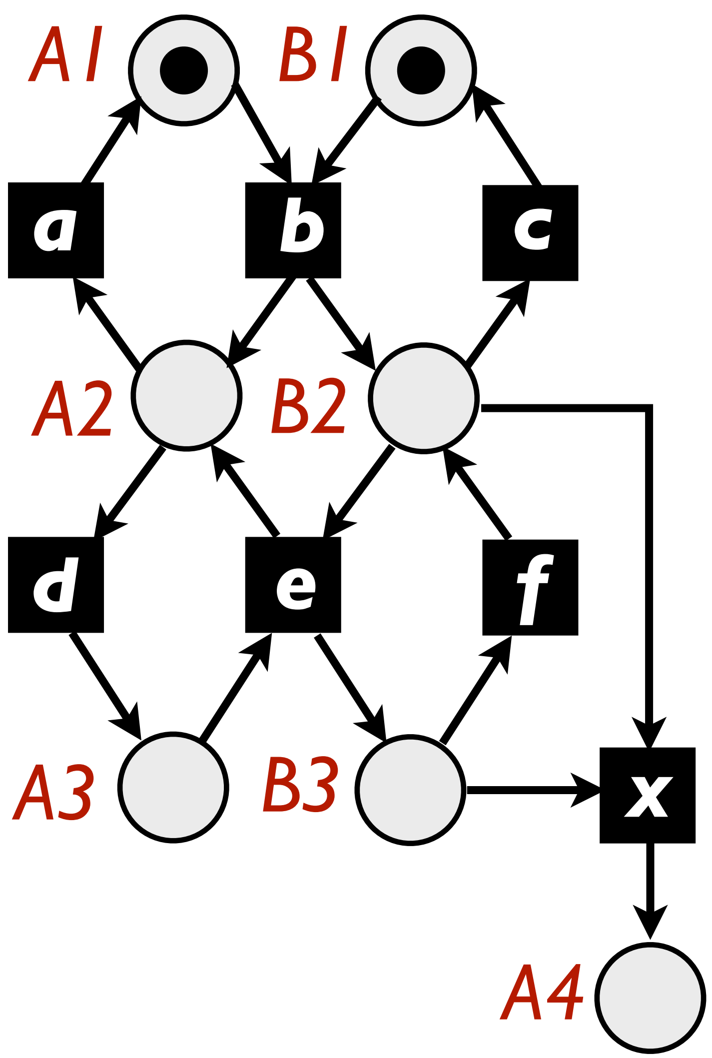

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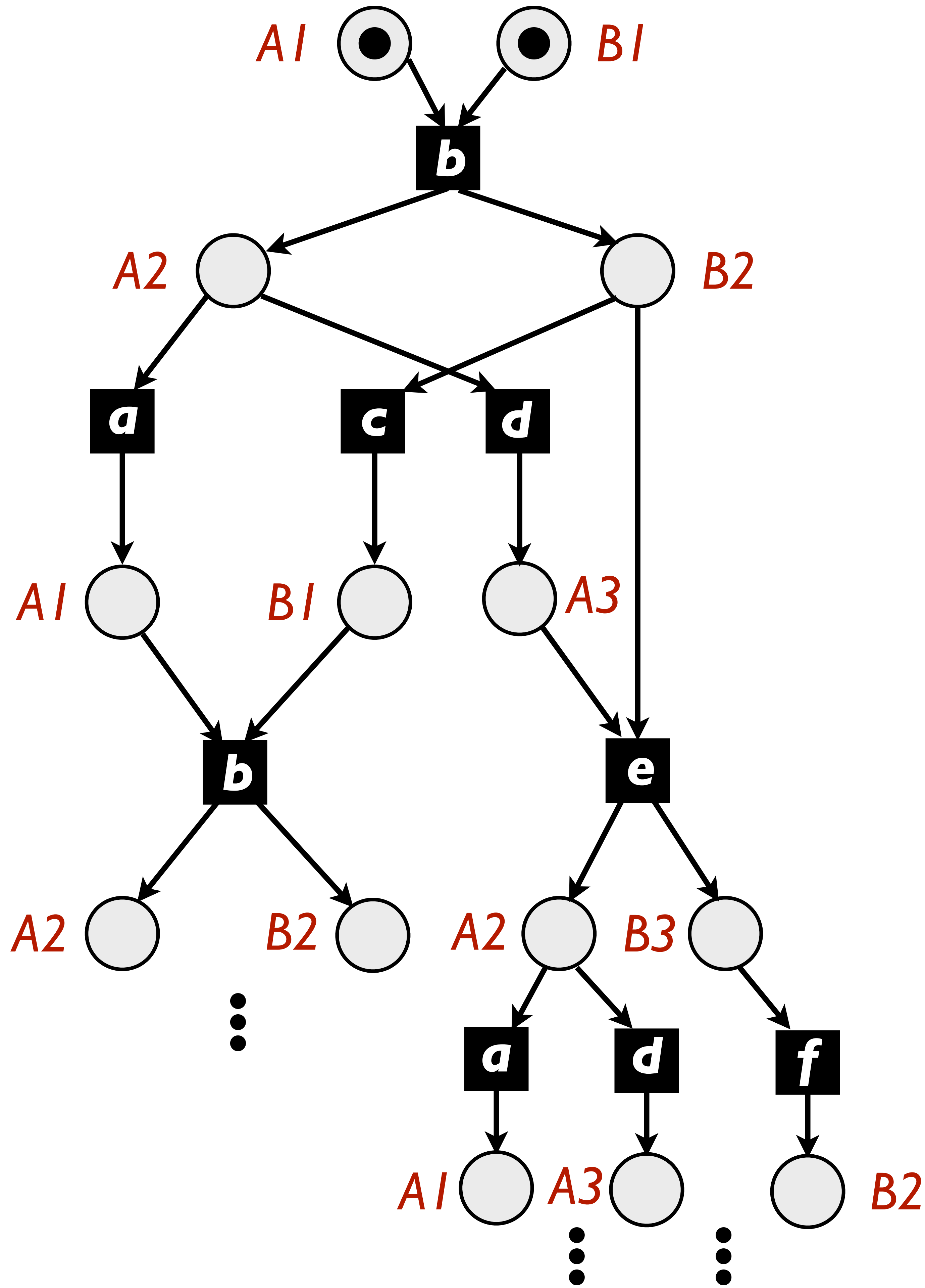
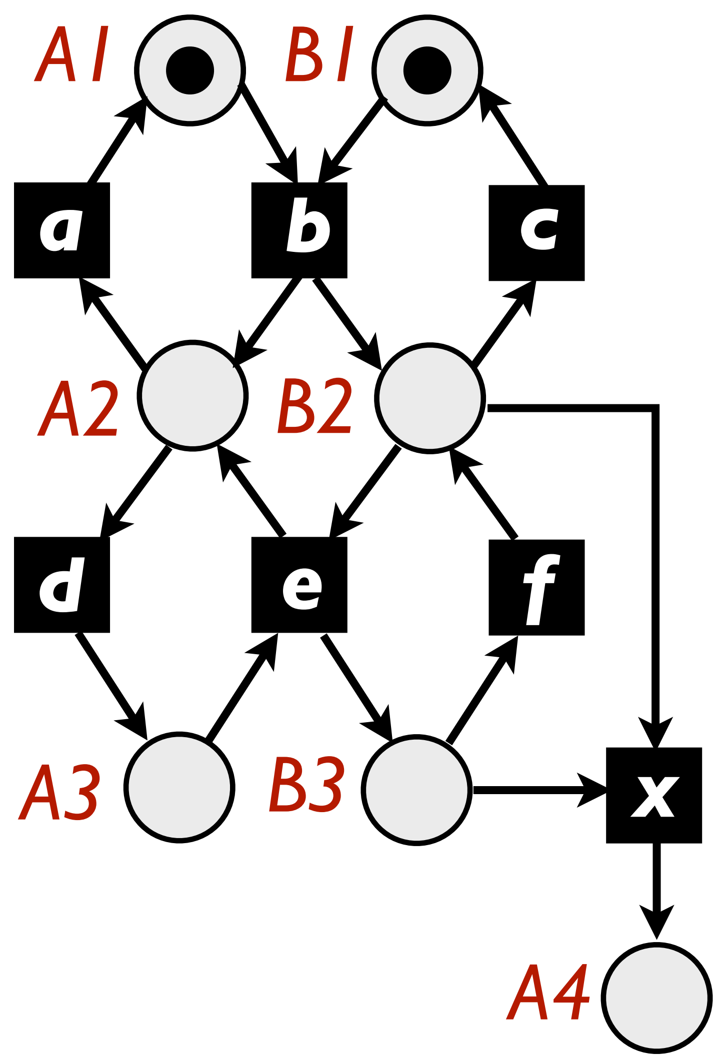
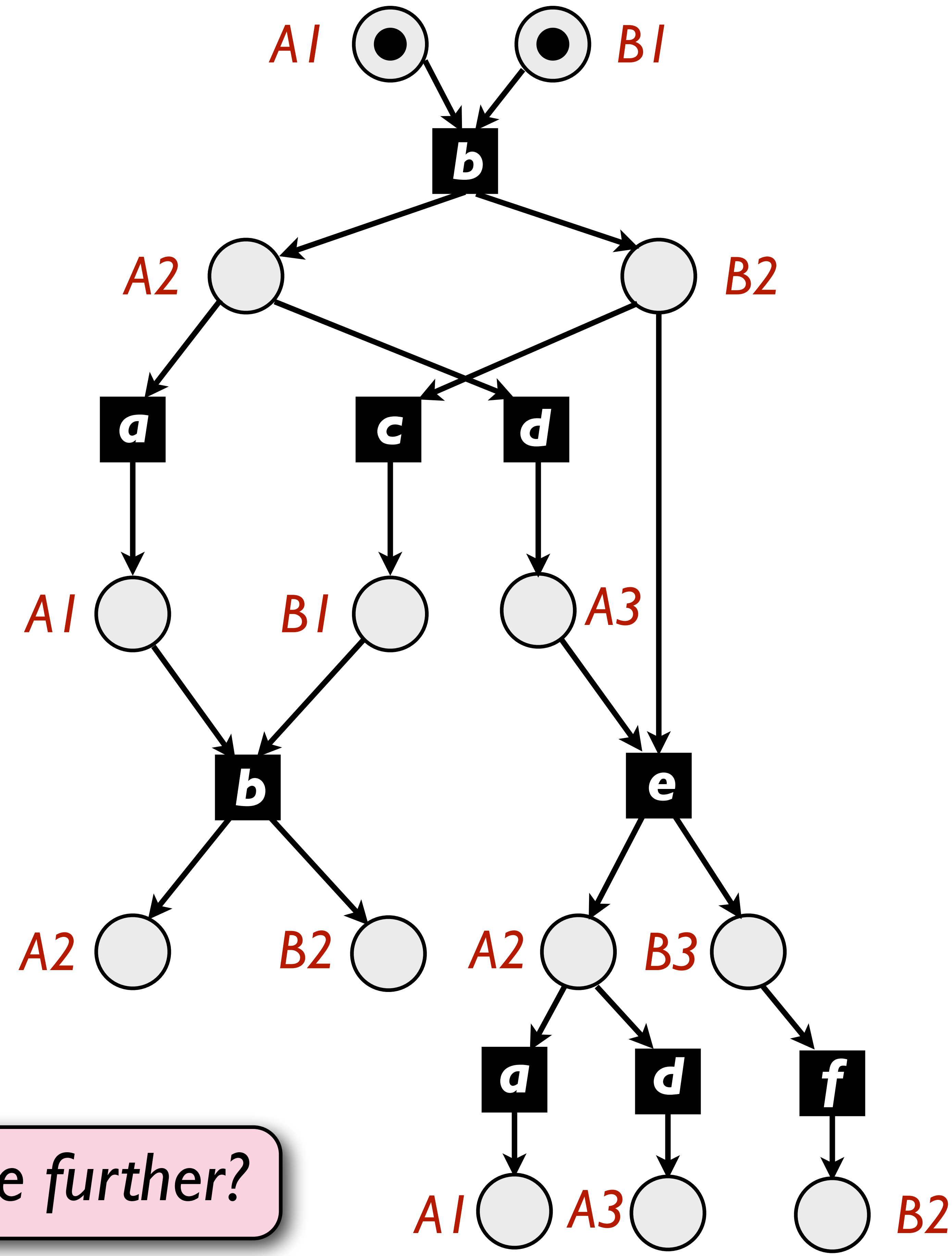






can "x" ever occur?

need we compute further?

Complete finite prefix

- a complete finite prefix is a finite part of an unfolding that is sufficient for deciding certain questions about the original Petri net
  => *e.g. executability, repeated executability, livelock, ...*

- challenge is to determine when to "stop" unfolding without information loss
  => *outside scope of this lecture; see Esparza & Heljanko (2008)*

- previous slide gave a complete finite prefix
  => *no "x" in the prefix; hence "x" can never occur in the original Petri net*

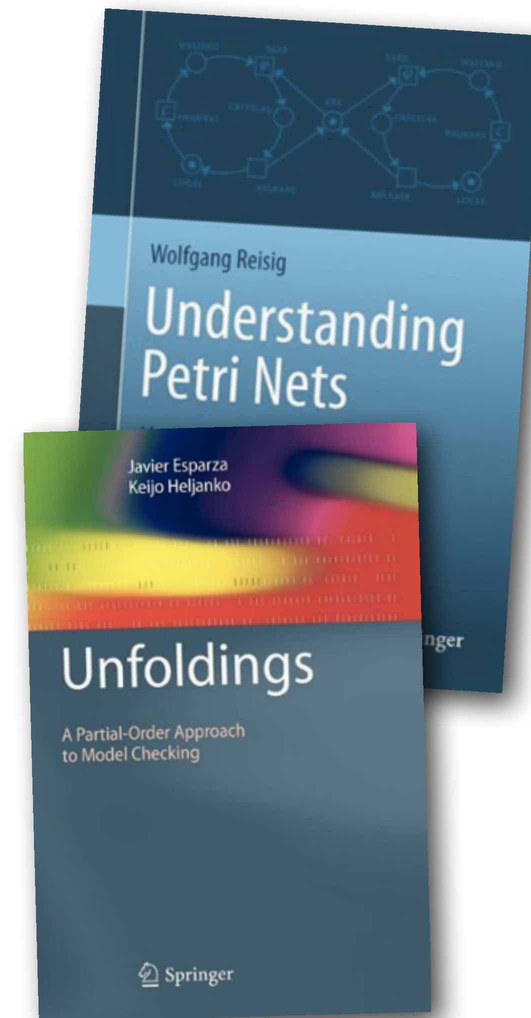
- complete finite prefixes can be exponentially more concise than an interleaving-based representation

Next on the agenda

1. modelling concepts: *cookies for everyone!*
2. synchronisation problems as Petri nets
3. Petri net analyses
4. true concurrency semantics; unfoldings

Main sources for this lecture

- Understanding Petri Nets (2013)
  => *by Wolfgang Reisig*
  => *chapters 1-3*



- Unfoldings (2008)
  => *by Javier Esparza & Keijo Heljanko*
  => *chapters 1-3*

- "A False History of True Concurrency"
  => *http://dx.doi.org/10.1007/978-3-642-16164-3_13*
  => *https://www7.in.tum.de/~esparza/Talks/lmpstrueconc.pdf*

Summary

- Petri nets facilitate a graphical, intuitive means of modelling concurrent and distributed systems
- automatic analyses exist for reachability, boundedness, liveness, ... but are expensive in the general case
- unfoldings (based on true concurrency) may give a more compact representation of concurrency than reachability graphs (based on interleavings)