Software Verification – Exam

ETH Zürich

17 December 2012

Surname, first name: .....

Student number: .....

I confirm with my signature that I was able to take this exam under regular circumstances and that I have read and understood the directions below.

Signature: .....

Directions:

- Exam duration: 1 hour 45 minutes.
- Except for a dictionary you are not allowed to use any supplementary material.
- All solutions can be written directly on the exam sheets. If you need more space for your solution ask the supervisors for a sheet of official paper. You are **not** allowed to use other paper. Please write your student number on **each** additional sheet.
- Only one solution can be handed in per question. Invalid solutions need to be crossed out clearly.
- Please write legibly! We will only correct solutions that we can read.
- Manage your time carefully (take into account the number of points for each question).
- Please **immediately** tell the exam supervisors if you feel disturbed during the exam.

Good luck!

Question	Available points	Your points
1) Axiomatic semantics	18	
2) Separation Logic	15	
3) Data flow analysis	10	
4) Model checking	15	
5) Real time verification	12	
Total	**70**	

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1 Axiomatic semantics (18 points)

Consider the following annotated program, where A is an array indexed from 1 of element type G, n is an integer variable storing A's size, k is another integer variable, v is a variable of type G initialized to some fixed value, *found* is a Boolean variable.

$\{\ n \geq 0\ \}$
1 **from**
2 $k := n$
3 *found* := **False**
4 **until** *found* **or** $k < 1$ **loop**
5 **if** $A[k] = v$ **then**
6 *found* := **True**
7 **else**
8 $k := k - 1$
9 **end**
10 **end**
$\{\ (\mathit{found} \Longrightarrow 1 \leq k \leq n \wedge A[k] = v) \wedge (\ \neg \mathit{found} \Longrightarrow k < 1\ )\}$

1.1 Program semantics (2 points)

Characterize, in plain English, which value of k the program computes from the inputs A, n, and v. In other words: what does the program do?

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1.2 Partial correctness (15 points)

Prove that the triple (precondition, program, postcondition) is a theorem of Hoare's axiomatic system for partial correctness.

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1.3 Termination (1 point)

Find a suitable *variant* function V to prove termination. V must be such that it decreases along all branches of the loop body, and it is nonnegative after every iteration of the loop. You do *not* have to prove termination, just write a suitable variant and informally argue why it is a suitable variant.

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2 Separation Logic (15 points)

A well-formed binary tree t is given by the grammar:

$t \stackrel{\text{def}}{=} n \mid (t_1, t_2)$

So a tree value t can be either a leaf, which is a single number n, or an internal node with a left subtree t_1 and a right subtree t_2.

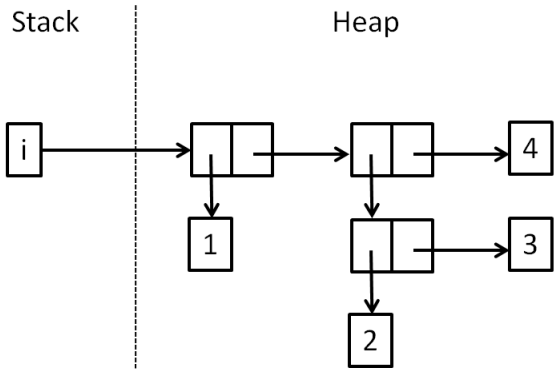
Consider the definition of the recursive predicate *tree* t i which asserts that i is a pointer to a well-formed binary tree t:

$\mathit{tree}\ n\ i \stackrel{\text{def}}{=} i \mapsto n$
$\mathit{tree}\ (t_1, t_2)\ i \stackrel{\text{def}}{=} \exists l, r \cdot i \mapsto l, r * \mathit{tree}\ t_1\ l * \mathit{tree}\ t_2\ r$

With these definitions in mind, answer the following questions.

2.1 Predicate Satisfaction (6 points)

Consider the following program state:



Indicate in the table whether or not a given assertion is satisfied by this state. Indicate satisfaction with a `T` and non-satisfaction with an `F`.

Assertion	`T` or `F`
$\exists z \cdot i \mapsto z$	
$\exists x, y \cdot i \mapsto 1, x * x \mapsto y, 4 * \mathit{true}$	
$\exists j \cdot \mathit{tree}\ (2,3)\ j * \mathit{true}$	
$\exists j, t \cdot i \mapsto j * j \mapsto 1 * \mathit{tree}\ t\ (i+1)$	
$\exists t_1, t_2 \cdot \mathit{tree}\ (t_1, (t_2, 4))\ i$	
$\exists j, k \cdot i \mapsto j * j \mapsto 1 * (i+1) \mapsto k * \mathit{tree}\ ((2,3),4)\ k$	

2.2 Code Verification (9 points)

Give a brief proof outline of the following triple. There must be at least one assertion between every two sub-commands.
$\{\mathit{tree}\ (1, t)\ i\}$
$x := [i];\ \ [i] := 2;\ \ y := [i+1];\ \ \textbf{dispose}\ i;\ \ \textbf{dispose}\ x;\ \ \textbf{dispose}\ (i+1)$
$\{\mathit{tree}\ t\ y\}$

3 Data flow analysis (10 points)

Consider the following program fragment (all variables are of type *INTEGER*):

```
 1  from
 2    x := n
 3  until x ≤ 0 do
 4    x := x − 1
 5    if y > 2 then
 6      y := 1 − y
 7    end
 8    x := y − 4
 9    if z > 0 then
10      x := y + 2
11    end
12    z := x − 1
13  end
14  z := 2 ∗ y
15  print (z)
```

(1) (3 points) Draw the *control flow graph* of the program fragment and label each elementary block.

(2) (5 points) Annotate your control flow graph with the analysis result of a *live variables analysis* of the program fragment.

(3) (2 points) Explain how the live variables analysis can be used for dead code elimination, and apply this technique to the program fragment; it suffices to state which statement(s), if any, would be removed as dead code.

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4 Model Checking (15 points)

Recall the semantics of LTL over finite words with alphabet $\mathcal{P}$. For a word $w = w(1)w(2)\cdots w(n) \in \mathcal{P}^*$ with $n \geq 0$ and a position $1 \leq i \leq n$ the satisfaction relation $\models$ is defined recursively as follows (where $p, q \in \mathcal{P}$).

$w, i \models p$	iff	$p = w(i)$
$w, i \models \neg\phi$	iff	$w, i \not\models \phi$
$w, i \models \phi_1 \wedge \phi_2$	iff	$w, i \models \phi_1$ and $w, i \models \phi_2$
$w, i \models \mathsf{X}\phi$	iff	$i < n$ and $w, i+1 \models \phi$
$w, i \models \phi_1 \,\mathsf{U}\, \phi_2$	iff	there exists $i \leq j \leq n$ such that: $w, j \models \phi_2$ and for all $i \leq k < j$ it is the case that $w, k \models \phi_1$
$w, i \models \Diamond\, \phi$	iff	there exists $i \leq j \leq n$ such that: $w, j \models \phi$
$w, i \models \Box\, \phi$	iff	for all $i \leq j \leq n$ it is the case that: $w, j \models \phi$
$w \models \phi$	iff	$w, 1 \models \phi$

4.1 Automata and LTL formulas (7 points)

Consider the automaton $\mathcal{A}$ (with states A, B, C) in Figure 1, over the alphabet $\{p, q\}$. Notice that A is the initial state and B is final.

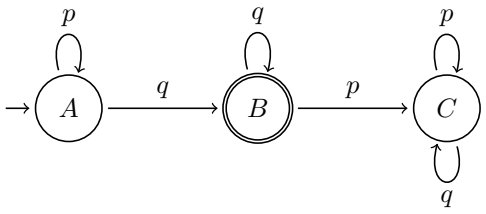

Figure 1: Automaton $\mathcal{A}$ over alphabet $\{p, q\}$.

For each of the following LTL formulas say whether every accepting run of $\mathcal{A}$ satisfies the formula. If it does, argue informally (but precisely) why this is the case; if it does not, provide a counterexample.

(1) $\mathcal{A} \models \Diamond\, p$

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(2) $\mathcal{A} \models \Diamond\, q$

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(3) $\mathcal{A} \models \neg p \ \vee\ \mathsf{X}(\neg q)$

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(4) $\mathcal{A} \models \Box(\, q \Longrightarrow \Box\, q\,)$

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(5) $\mathcal{A} \models \Box(\, p \,\mathsf{U}\, q\,)$

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4.2 Automata-based model checking (8 points)

Show that $\mathcal{A} \models p \Longrightarrow \Diamond\, q$ using automata-based model checking as follows.

Property automaton (4 points). Construct an automaton $\mathcal{F}$ that accepts *precisely* the words that satisfy $\neg(p \Longrightarrow \Diamond\, q)$, that is, the *complement* of the property we want to verify.

Intersection automaton (4 points). Construct the intersection automaton $\mathcal{A} \times \mathcal{F}$ that accepts precisely the words accepted by both $\mathcal{A}$ and $\mathcal{F}$ and show that $\mathcal{A} \times \mathcal{F}$ does not accepts any words.

5 Real Time Verification (12 points)

Recall the semantics of (a subset of) MTL over finite timed words with alphabet $\mathcal{P}$ and time domain $\mathbb{T}$. For a timed word

$$w = [\sigma(1), t(1)][\sigma(2), t(2)] \cdots [\sigma(n), t(n)] \in (\mathcal{P} \times \mathbb{T})^*$$

with $n \geq 0$ and a position $1 \leq i \leq n$ the satisfaction relation $\models$ is defined recursively as follows for $p \in \mathcal{P}$ and J an interval of $\mathbb{T}$ with integer endpoints.

$w, i \models p$	iff	$p = \sigma(i)$
$w, i \models \neg\phi$	iff	$w, i \not\models \phi$
$w, i \models \phi_1 \wedge \phi_2$	iff	$w, i \models \phi_1$ and $w, i \models \phi_2$
$w, i \models \Diamond_J \phi$	iff	there exists $i \leq j \leq n$ such that: $t(j) - t(i) \in J$ and $w, j \models \phi$
$w, i \models \Box_J \phi$	iff	for all $i \leq j \leq n$: if $t(j) - t(i) \in J$ then $w, j \models \phi$
$w \models \phi$	iff	$w, 1 \models \phi$

As time domain $\mathbb{T}$, we will consider either the natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$ or the nonnegative real numbers $\mathbb{R}_{\geq 0}$.

5.1 MTL semantics (4 points)

(1) Are the MTL formulas ψ_1 and ψ_2:

$$\psi_1 \triangleq \Diamond_{[1,1]}\Big(\Diamond_{[1,1]}(p)\Big)$$
$$\psi_2 \triangleq \Diamond_{[2,2]}(p)$$

equivalent over time domain $\mathbb{N}$? If they are, show that their semantics imply each other; if they are not, provide a timed word which is satisfied by one formula and not satisfied by the other.

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(2) Does the answer to the previous question (1) change over the time domain $\mathbb{R}_{\geq 0}$? Explain why it does or does not change.

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5.2 Timed automata and MTL formulas (8 points)

In this section, we take $\mathbb{R}_{\geq 0}$ as the time domain $\mathbb{T}$. Consider the timed automaton $\mathcal{T}$ (with locations A, B, C) in Figure 2, over the alphabet $\{p, q\}$ and with clocks x and y. Notice that A is the initial location and C is final.

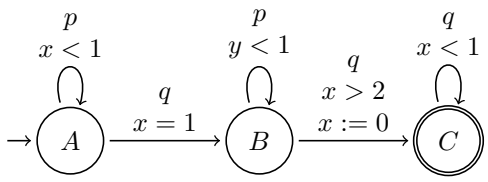

Figure 2: Timed automaton $\mathcal{T}$ over alphabet $\{p, q\}$ with clocks x and y.

For each of the following MTL formulas say whether every accepting run of $\mathcal{T}$ satisfies the formula. If it does, argue informally (but precisely) why this is the case; if it does not, provide a counterexample.

(1) $\mathcal{T} \models \Diamond_{(0,2)}\, q$

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(2) $\mathcal{T} \models \Box_{[0,\infty)}\Big(\Diamond_{(1,\infty)}\, p\Big)$

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(3) $\mathcal{T} \models \Box_{[0,\infty)}\Big(q \Longrightarrow \Box_{[0,\infty)}(\neg p)\Big)$

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(4) $\mathcal{T} \models \Diamond_{(2,\infty)}(q) \ \wedge \ \Box_{(3,\infty)}(\neg q)$

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