Software Verification – Exam

ETH Zürich

15 December 2014

Surname, first name: ....

Student number: ....

I confirm with my signature that I was able to take this exam under regular circumstances and that I have read and understood the directions below.

Signature: ....

Directions:

- Exam duration: 1 hour 45 minutes.
- Except for a dictionary you are not allowed to use any supplementary material.
- All solutions can be written directly on the exam sheets. If you need more space for your solution ask the supervisors for a sheet of official paper. You are **not** allowed to use other paper. Please write your student number on **each** additional sheet.
- Only one solution can be handed in per question. Invalid solutions need to be crossed out clearly.
- Please write legibly! We will only correct solutions that we can read.
- Manage your time carefully (take into account the number of points for each question).
- Please **immediately** tell the exam supervisors if you feel disturbed during the exam.

Good luck!

Question	Available points	Your points
1) Axiomatic semantics	8	
2) Separation logic	12	
3) Data Flow Analysis	9	
4) Model checking	9	
5) Software model checking	11	
Total	**49**	

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1 Axiomatic semantics (8 points)

Consider the following annotated program, where a and b are distinct arrays (indexed from 0) both of length n, k is an integer variable, and x **mod** y denotes the remainder of the integer division of x by y. Note that the **else** branch is empty.

```
  $\{\ n > 0 \ \wedge\ \forall\ i : 0 \leq i < n \Longrightarrow a[i] = b[i] = 0\ \}$
from
  $k := 0$
until $k = n$ loop
  if $k$ mod $3 = 0$ then
    $b[k] := a[k] + 1$
  else
  end
  $k := k + 1$
end
  $\{\ \forall\ i : 0 \leq i < n \ \wedge\ i \textbf{ mod } 3 = 0 \Longrightarrow b[i] = 1\ \}$
```

Prove that the above triple (precondition, program, postcondition) is a theorem of Hoare's axiomatic system for partial correctness. In other words, prove the program correct with respect to the given specification.

Solution:

```
{ $n > 0 \land \forall\ i : 0 \le i < n \implies a[i] = b[i] = 0$ }
from
{ $n \ge 0$ }
{ $0 \le 0 \le n \land$
  $\forall\ i : 0 \le i < 0 \land i \textbf{ mod } 3 = 0 \implies b[i] = 1 \land$
  $\forall\ i : 0 \le i < n \implies a[i] = 0$ }
  $k := 0$
{ $0 \le k \le n \land$
  $\forall\ i : 0 \le i < k \land i \textbf{ mod } 3 = 0 \implies b[i] = 1 \land$
  $\forall\ i : 0 \le i < n \implies a[i] = 0$ }
until $k = n$ loop
{ $0 \le k < n \land$
  $\forall\ i : 0 \le i < k \land i \textbf{ mod } 3 = 0 \implies b[i] = 1 \land$
  $\forall\ i : 0 \le i < n \implies a[i] = 0$ }
  if $k \textbf{ mod } 3 = 0$ then
  { $-1 \le k < n \land k \textbf{ mod } 3 = 0 \land$
    $\forall\ i : 0 \le i < k \land i \textbf{ mod } 3 = 0 \implies b[i] = 1 \land$
    $k \textbf{ mod } 3 = 0 \implies a[k] + 1 = 1 \land$
    $\forall\ i : 0 \le i < n \implies a[i] = 0$ }
    $b[k] := a[k] + 1$
  { $-1 \le k < n \land$
    $\forall\ i : 0 \le i < k \land i \textbf{ mod } 3 = 0 \implies b[i] = 1 \land$
    $k \textbf{ mod } 3 = 0 \implies b[k] = 1 \land$
    $\forall\ i : 0 \le i < n \implies a[i] = 0$ }
  else
  { $-1 \le k < n \land k \textbf{ mod } 3 \ne 0 \land$
    $\forall\ i : 0 \le i < k \land i \textbf{ mod } 3 = 0 \implies b[i] = 1 \land$
    $k \textbf{ mod } 3 = 0 \implies b[k] = 1 \land$
    $\forall\ i : 0 \le i < n \implies a[i] = 0$ }
  end
{ $-1 \le k < n \land$
  $\forall\ i : 0 \le i < k \land i \textbf{ mod } 3 = 0 \implies b[i] = 1 \land$
  $k \textbf{ mod } 3 = 0 \implies b[k] = 1 \land$
  $\forall\ i : 0 \le i < n \implies a[i] = 0$ }
{ $0 \le k + 1 \le n \land$
  $\forall\ i : 0 \le i < k + 1 \land i \textbf{ mod } 3 = 0 \implies b[i] = 1 \land$
  $\forall\ i : 0 \le i < n \implies a[i] = 0$ }
  $k := k + 1$
{ $0 \le k \le n \land$
  $\forall\ i : 0 \le i < k \land i \textbf{ mod } 3 = 0 \implies b[i] = 1 \land$
  $\forall\ i : 0 \le i < n \implies a[i] = 0$ }
end
{ $k = n \land$
  $\forall\ i : 0 \le i < n \land i \textbf{ mod } 3 = 0 \implies b[i] = 1 \land$
  $\forall\ i : 0 \le i < n \implies a[i] = 0$ }
{ $\forall\ i : 0 \le i < n \land i \textbf{ mod } 3 = 0 \implies b[i] = 1$ }
```

2 Separation Logic (12 points)

2.1 Linked List Proof (7 points)

We can assert that a heap contains a linked list by using the following inductively defined predicate:

$$\begin{aligned}\mathrm{list}([\,], i) &\iff \mathrm{emp} \land i = nil \\ \mathrm{list}(a :: as, i) &\iff \exists j.\ i \mapsto a, j * \mathrm{list}(as, j)\end{aligned}$$

where *nil* is a constant used to terminate the list.

Give a proof outline for the following triple using the list predicate and the proof rules of separation logic:

$\{\mathrm{list}(a :: as, x)\}$

```
y := cons(0,0);
temp1 := [x];
temp2 := [x+1];
[y] := temp1;
[y+1] := nil;
dispose(x);
dispose(x+1);
x := temp2;
```

$\{\mathrm{list}(as, x) * \mathrm{list}(a :: [\,], y)\}$

Sample solution:

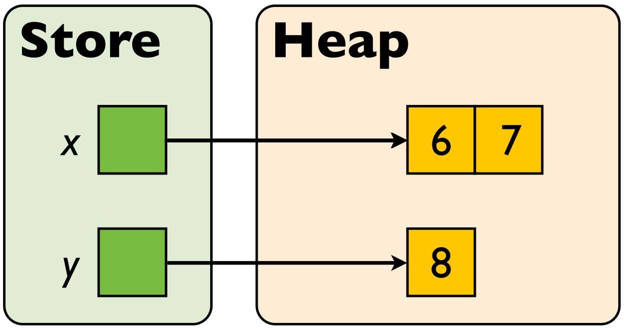
```
{list(a::as,x)}
     y := cons(0,0);
{list(a::as,x) * y|->0,0}
{EX j. x|->a,j * list(as,j) * y|->0,0}
     {x|->a,j * list(as,j) * y|->0,0}
          temp1 := [x];
     {x|->a,j * list(as,j) * y|->0,0 /\ temp1 = a}
          temp2 := [x+1];
     {x|->a,j * list(as,j) * y|->0,0 /\ temp1 = a /\ temp2 = j}
          [y] := temp1;
     {x|->a,j * list(as,j) * y|->temp1,0 /\ temp1 = a /\ temp2 = j}
          [y+1] := nil;
     {x|->a,j * list(as,j) * y|->temp1,nil /\ temp1 = a /\ temp2 = j}
          dispose(x);
     {x+1|->j * list(as,j) * y|->temp1,nil /\ temp1 = a /\ temp2 = j}
          dispose(x+1);
     {list(as,j) * y|->temp1,nil /\ temp1 = a /\ temp2 = j}
{EX j. list(as,j) * y|->temp1,nil /\ temp1 = a /\ temp2 = j}
{list(as,temp2) * y|->temp1,nil /\ temp1 = a}
     x := temp2;
{list(as,temp2) * y|->temp1,nil /\ temp1 = a /\ x = temp2}
{list(as,x) * y|->temp1,nil /\ temp1 = a}
{list(as,x) * y|->a,nil}
{list(as,x) * y|->a,nil * list([],nil)}
{list(as,x) * list(a::[],y)}
```

2.2 Satisfaction of Assertions (5 points)

For each separation logic assertion below, draw a program state (i.e. store and heap) that satisfies it.

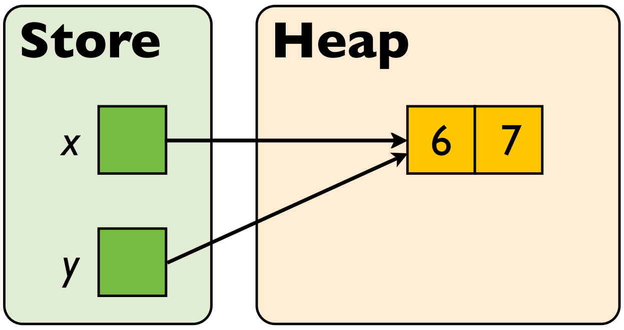
(a). $(x \mapsto 6, 7) * \neg(x \mapsto 6, 7)$

Sample solution:



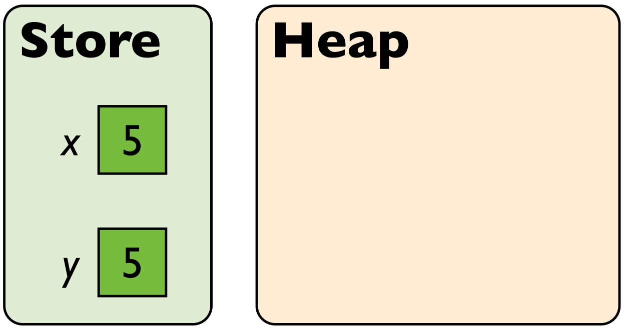
(b). $(x \mapsto 6, 7) \wedge (y \mapsto 6, 7)$

Sample solution:



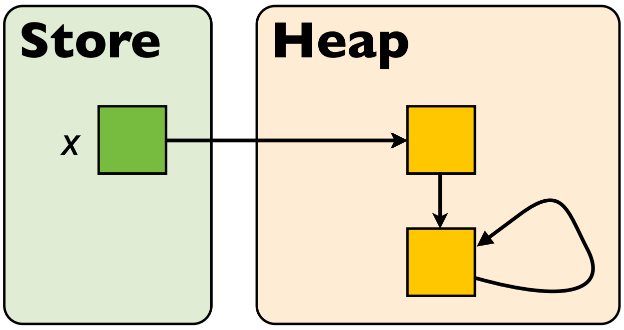
(c). $\mathrm{emp} \Rightarrow x = y$

Sample solution:



(d). $\exists i.\ x \mapsto i * i \mapsto i$

Sample solution:



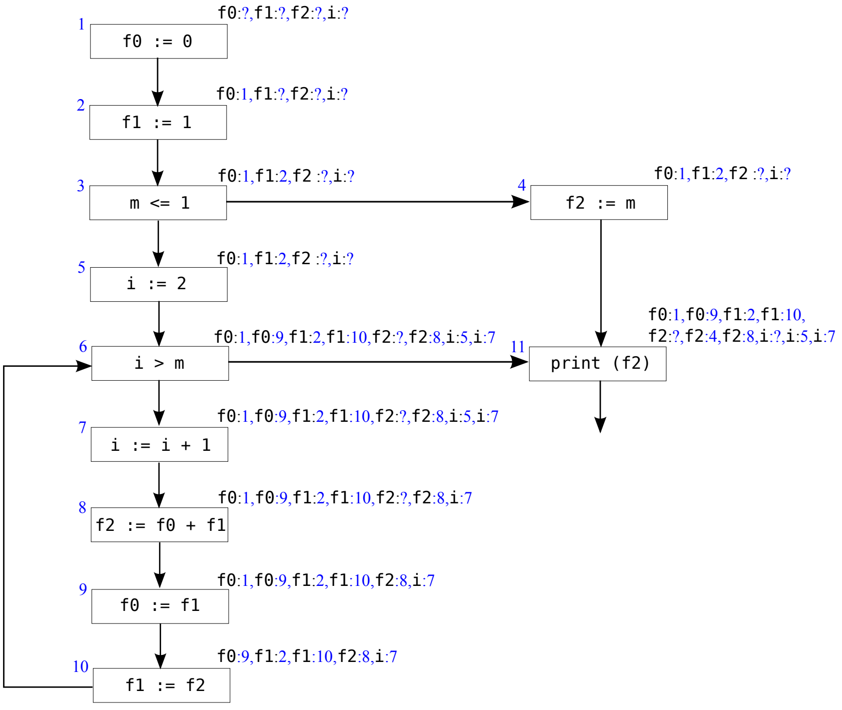
3 Data Flow Analysis (9 points)

Consider the following program fragment, which computes the value of the mth Fibonacci number for $m \geq 1$.

```
f0 := 0
f1 := 1
if m ≤ 1 then
  f2 := m
else
  from
    i := 2
  until i > m loop
    i := i + 1
    f2 := f0 + f1
    f0 := f1
    f1 := f2
  end
end
print (f2)
```

(a). Draw the control flow graph of the program fragment and label each elementary block. **(2 points)**

(b). Annotate your control flow graph with the analysis result of a Reaching Definitions analysis of the program fragment, considering the variables *f0*, *f1*, *f2*, *i*. **(5 points)**



(c). Copy Propagation, an application of the Reaching Definitions analysis, is defined as follows:

> A use of a variable x at a program point ℓ' can be replaced by y if $[x := y]^{\ell}$ is the only definition of x that reaches ℓ' and y is not modified between ℓ and ℓ'.

Is there a possibility for copy propagation in the program fragment? In justifying your answer, use your analysis result. **(2 points)**

There is no possibility for copy propagation in the program fragment. The variables *f0*, *f1*, *f2*, i are used only in blocks 6-11, and in none of these blocks copy propagation can be applied: for blocks 6-9 and 11, the analysis result shows that each of the used variables in the blocks could be reached by more than one definition; and for block 10, while it is reached by only one definition, $[f2 := f0 + f1]^{8}$, that definition is not a copy statement.

4 Model Checking (9 points)

Recall the semantics of LTL over finite words with alphabet $\mathcal{P}$. For a word $w = w(1)w(2)\cdots w(n) \in \mathcal{P}^*$ with $n \geq 0$ and a position $1 \leq i \leq n$ the satisfaction relation $\models$ is defined recursively as follows (where $p, q \in \mathcal{P}$).

$$
\begin{array}{lll}
w, i \models p & \text{iff} & p = w(i) \\
w, i \models \neg\phi & \text{iff} & w, i \not\models \phi \\
w, i \models \phi_1 \land \phi_2 & \text{iff} & w, i \models \phi_1 \;\text{ and }\; w, i \models \phi_2 \\
w, i \models \mathsf{X}\phi & \text{iff} & i < n \text{ and } w, i+1 \models \phi \\
w, i \models \phi_1 \,\mathsf{U}\, \phi_2 & \text{iff} & \text{there exists } i \leq j \leq n \text{ such that: } w, j \models \phi_2 \\
 & & \text{and for all } i \leq k < j \text{ it is the case that } w, k \models \phi_1 \\
w, i \models \Diamond\, \phi & \text{iff} & \text{there exists } i \leq j \leq n \text{ such that: } w, j \models \phi \\
w, i \models \Box\, \phi & \text{iff} & \text{for all } i \leq j \leq n \text{ it is the case that: } w, j \models \phi \\
w \models \phi & \text{iff} & w, 1 \models \phi
\end{array}
$$

4.1 Automata and LTL formulas (5 points)

Consider the automaton $\mathcal{A}$ (with states A, B, C, D) in Figure 1, over the alphabet $\{p, q\}$. Notice that A is the initial state, A and D are final states, and the automaton is nondeterministic.

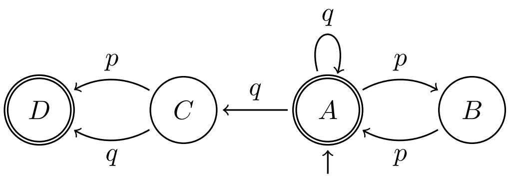


Figure 1: Automaton $\mathcal{A}$ over alphabet $\{p, q\}$.

For each of the following LTL formulas say whether every accepting run of $\mathcal{A}$ satisfies the formula. If it does, argue informally (but precisely) why this is the case; if it does not, provide a counterexample.

(1) $\mathcal{A} \models \Box(\Diamond\, q)$

No: the word $w = p\,p$ is accepted by $\mathcal{A}$, but no letter q appears in it; hence $\Diamond\, q$ does not hold.

(2) $\mathcal{A} \models \Box(p \implies \mathsf{X}\, p)$

No: the word $w = q\,p$ is accepted by $\mathcal{A}$, but $w, 2 \not\models \mathsf{X}\, p$ because there are no letters after the unique p in w.

(3) $\mathcal{A} \models p \implies \mathsf{X}\, p$

Yes: every word w accepted by $\mathcal{A}$ such that p holds in the first position drives $\mathcal{A}$ into state B, from where a p must follow for w to be accepted.

(4) $\mathcal{A} \models \Diamond(\Box\, q)$

No: the empty word ϵ is accepted by $\mathcal{A}$, but ϵ does not satisfy formula $\Diamond(\Box\, q)$, which requires a valid position in the word (ϵ has none, that is

$n = 0$). Another counterexample is the word $w = pp$, where q does not appear.

(5) $\mathcal{A} \vDash \square(p \implies (p \mathsf{U} q))$

No: the word $w = pp$ is accepted by $\mathcal{A}$, but $w, 2 \nvDash p \mathsf{U} q$ because no q occurs eventually.

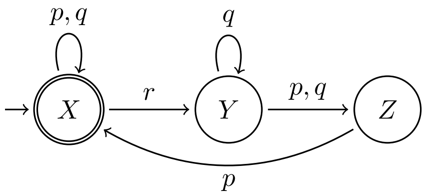
4.2 Automata-based model checking (4 points)

Consider the LTL formula:

$$\phi \quad \triangleq \quad \square(r \implies \mathsf{X}(q \mathsf{U} (\neg r \wedge \mathsf{X} p)))$$

Property automaton. Construct an automaton $\mathcal{F}$ that accepts *precisely* the words that satisfy ϕ.

Solution:
An edge with two letters denote a double transition (one for each of the letters).



5 Software model checking (11 points)

Consider the following code snippet C, where x and y are integer variables, and z is a Boolean variable.

```
assume $z =$ True end
if $x > y$ and $x \leq 0$ then
    $z :=$ True
else
    $z :=$ False
end
assert $y \leq 0$ or not $z$ end
```

Recall that:

- *Pred* (*exp*) denotes the weakest under-approximation of the expression *exp* that is expressible as a Boolean combination of the given predicates.
- The predicate abstraction of an assume instruction **assume** *exp* **end** is **assume not** *Pred* (**not** *exp*) **end** followed by a parallel conditional assignment updating the predicates with respect to the original **assume**.
- The predicate abstraction of an assert instruction **assert** *exp* **end** simply is **assert** *Pred* (*exp*) **end**, provided that *exp* can be approximated exactly by means of the available predicates (which is the case in this exercise).

5.1 Boolean abstractions (8 points)

Build the Boolean abstraction A of the code snippet C above with respect to the predicates:

$$\begin{array}{lcl} p & \equiv & x > y \\ q & \equiv & y > 0 \\ r & \equiv & z = \textbf{True} \end{array}$$

Solution:
After the usual simplifications, the predicate abstraction A is:

```
assume $r$ end
if ? then
    assume $p$ and not $q$ end
    $r :=$ True
else
    $r :=$ False
end
assert not $q$ or not $r$ end
```

5.2 Traces (3 points)

Trace classification: Using the table below, classify every possible **initial** abstract state I into the following categories (note that, due to nondeterminism, a state may belong to more than one category):

- *invalid*: I may lead to a trace that is infeasible in A;
- *spurious counterexample*: I may lead to an error trace in A that is infeasible in C (that is, it is invalid in C);
- *error*: I may lead to an error trace in A that is feasible in C (that is, it is also an error in C);
- *valid*: I may lead to a valid (without errors) trace in A.

Solution:

INITIAL STATE I	CLASSIFICATION (invalid, spurious, error, valid)
$\{p,\ q,\ r\}$	valid or invalid
{**not** p, q, r}	valid or invalid
{p, **not** q, r}	valid
{**not** p, **not** q, r}	valid or invalid
{p, q, **not** r}	invalid
{**not** p, q, **not** r}	invalid
{p, **not** q, **not** r}	invalid
{**not** p, **not** q, **not** r}	invalid

Every initial state I with the component "**not** r" is *invalid* because of the initial **assume** in A. The remaining states are *valid* since they can lead to the error-free abstract traces listed below. Furthermore, the initial states $\{p,\ q,\ r\}$, {**not** p, q, r}, and {**not** p, **not** q, r} are also invalid because the nondeterministic conditional **if** ? may lead to a violation **assume** p **and not** q **end**, that is to an infeasible trace.

$\{p,\ q,\ r\}$
  [**not** (p **and not** q)]
$\{p,\ q,\ r\}$
  r := **False**
{p, q, **not** r}
  assert not q **or not** r **end**

{**not** p, q, r}
  [**not** (p **and not** q)]
{**not** p, q, r}
  r := **False**
{**not** p, q, **not** r}
  assert not q **or not** r **end**

```
{p, not q, r}
    [p and not q]
{p, not q, r}
    r := True
{p, not q, r}
    assert not q or not r end

{not p, not q, r}
    [not (p and not q)]
{not p, not q, r}
    r := False
{not p, not q, not r}
    assert not q or not r end
```

Implications for the concrete program: Based on the classification, what do you conclude about the correctness of the concrete program C? Please justify your answer.

Solution:
Since A determines a set of abstract traces that are an over-approximation of C's set of concrete traces, we can conclude that C is correct because A is.