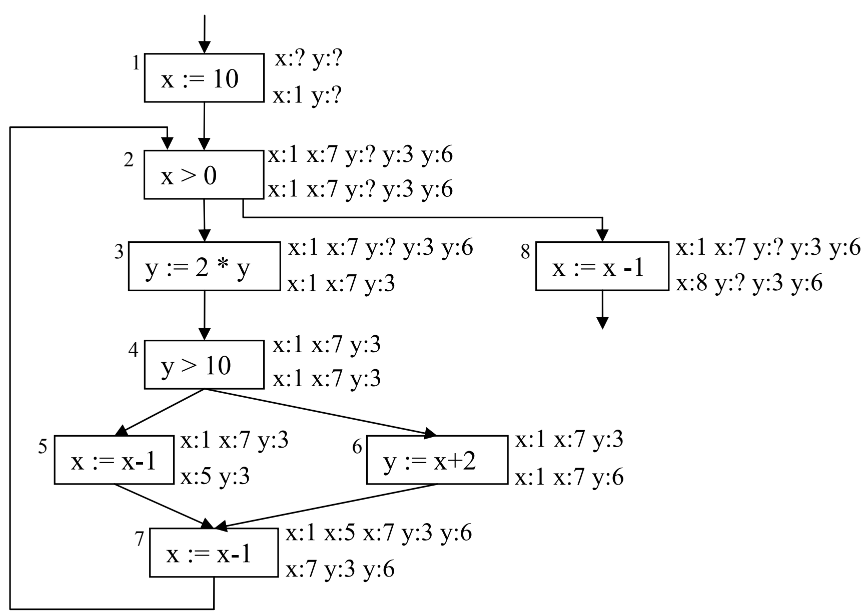
Problem Sheet 3: Data Flow Analysis
Sample Solutions

Chris Poskitt*
ETH Zürich

Starred exercises (∗) are more challenging than the others.

1 Reaching Definitions Analysis

i-ii. The control flow graph and the results of the reaching definitions analysis are given in the diagram below:



iii. We give the use-definition information for `x` and `y` in the table below (you could also annotate the diagram above with additional arrows).

*Solutions adapted from an earlier version of the course, when Stephan van Staden was the teaching assistant.

Program Block	x	y
1	$\emptyset$	$\emptyset$
2	$\{1,7\}$	$\emptyset$
3	$\emptyset$	$\{?,3,6\}$
4	$\emptyset$	$\{3\}$
5	$\{1,7\}$	$\emptyset$
6	$\{1,7\}$	$\emptyset$
7	$\{1,5,7\}$	$\emptyset$
8	$\{1,7\}$	$\emptyset$

2 Live Variables Analysis

i. Below we identify the blocks of the program:

```
[x := y]^1
[x := x-1]^2
[x := 4]^3
while [y < x]^4 do
    [y := y+x]^5
end
[y := 0]^6
```

ii. The system of equations for a live variable analysis are as follows:

$$
\begin{aligned}
\mathrm{LV_{entry}}(1) &= (\mathrm{LV_{exit}}(1) - \{\mathtt{x}\}) \cup \{\mathtt{y}\} \\
\mathrm{LV_{entry}}(2) &= (\mathrm{LV_{exit}}(2) - \{\mathtt{x}\}) \cup \{\mathtt{x}\} \\
\mathrm{LV_{entry}}(3) &= \mathrm{LV_{exit}}(3) - \{\mathtt{x}\} \\
\mathrm{LV_{entry}}(4) &= \mathrm{LV_{exit}}(4) \cup \{\mathtt{x}, \mathtt{y}\} \\
\mathrm{LV_{entry}}(5) &= (\mathrm{LV_{exit}}(5) - \{\mathtt{y}\}) \cup \{\mathtt{x}, \mathtt{y}\} \\
\mathrm{LV_{entry}}(6) &= \mathrm{LV_{exit}}(6) - \{\mathtt{y}\}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{LV_{exit}}(1) &= \mathrm{LV_{entry}}(2) \\
\mathrm{LV_{exit}}(2) &= \mathrm{LV_{entry}}(3) \\
\mathrm{LV_{exit}}(3) &= \mathrm{LV_{entry}}(4) \\
\mathrm{LV_{exit}}(4) &= \mathrm{LV_{entry}}(5) \cup \mathrm{LV_{entry}}(6) \\
\mathrm{LV_{exit}}(5) &= \mathrm{LV_{entry}}(4) \\
\mathrm{LV_{exit}}(6) &= \emptyset
\end{aligned}
$$

iii. We begin the iteration by initialising every set to $\emptyset$. Then, we iteratively update the sets by applying the equation system above. (For simplicity, the columns omit sets when a particular iteration does not update the previous value.)

LV Sets	Iterations $\longrightarrow$				Final Values
$LV_{entry}(1)$	$\emptyset$	{y}			{y}
$LV_{entry}(2)$	$\emptyset$	{x}		{x, y}	{x, y}
$LV_{entry}(3)$	$\emptyset$		{y}		{y}
$LV_{entry}(4)$	$\emptyset$	{x, y}			{x, y}
$LV_{entry}(5)$	$\emptyset$	{x, y}			{x, y}
$LV_{entry}(6)$	$\emptyset$				$\emptyset$
$LV_{exit}(1)$	$\emptyset$	{x}		{x, y}	{x, y}
$LV_{exit}(2)$	$\emptyset$		{y}		{y}
$LV_{exit}(3)$	$\emptyset$	{x, y}			{x, y}
$LV_{exit}(4)$	$\emptyset$	{x, y}			{x, y}
$LV_{exit}(5)$	$\emptyset$	{x, y}			{x, y}
$LV_{exit}(6)$	$\emptyset$				$\emptyset$

iv. We eliminate blocks b of the form `[x := ...]`^{b} if `x` is not an element of $LV_{exit}(b)$:

```
[x := y]^1
[x := 4]^3
while [y < x]^4 do
    [y := y+x]^5
end
```

v. (∗) The program is not yet free of dead variables: `x` in block 1 is still dead. We strengthen the definition of LV_{entry}:

$$LV_{entry}(b) = \begin{cases} (LV_{exit}(b) - kill_{LV}(b)) \cup gen_{LV}(b) & \text{if } kill_{LV}(b) \subseteq LV_{exit}(b) \\ LV_{exit}(b) & \text{otherwise} \end{cases}$$

The rationale is this: if a block assigns to a variable that is not live afterwards, then it must be eliminated, and should not influence the analysis by adding the variables it reads to the live variable set.

Performing a chaotic iteration with this new equation yields the following results:

LV Sets	Final Values
$LV_{entry}(1)$	{y}
$LV_{entry}(2)$	{y}
$LV_{entry}(3)$	{y}
$LV_{entry}(4)$	{x, y}
$LV_{entry}(5)$	{x, y}
$LV_{entry}(6)$	$\emptyset$
$LV_{exit}(1)$	{y}
$LV_{exit}(2)$	{y}
$LV_{exit}(3)$	{x, y}
$LV_{exit}(4)$	{x, y}
$LV_{exit}(5)$	{x, y}
$LV_{exit}(6)$	$\emptyset$

with which we can eliminate all of the dead code in the program:

```
[x := 4]^3
while [y < x]^4 do
    [y := y+x]^5
end
```