Problem Sheet 5: Model Checking
Sample Solutions

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1 Evaluating LTL Formulae on Automata

i. Yes: whenever `start` occurs, `stop` must occur eventually since it is the only means of getting to the accepting state.

ii. No: a counterexample is `pull push`.

iii. Yes: the formula asserts that from every position in a word (if there are any), eventually either `turn_off` or `push` will occur. One of these events must occur to return to the accepting state.

iv. No: the empty word is a counterexample ($\Diamond\, p$ demands the existence of a future position in the word for which p holds — the empty word cannot possibly satisfy it as it has no positions).

v. Yes: if the word is empty, then it will satisfy the first disjunct ("always false" holds simply because there are no positions in the empty word to check against); if the word is non-empty, the final position in the word must be `turn_off` or `push`, and hence the second disjunct will be satisfied.

vi. No: a counterexample is the empty word; or `turn_on turn_off`.

2 Equivalence of LTL Formulae

i.

	$w, i \models \text{true} \ \mathsf{U}\ F$	
iff	for some $i \leq j \leq n$ we have $w, j \models F$	
	and for all $i \leq k < j$ we have $w, k \models \text{true}$	**[definition of until]**
iff	for some $i \leq j \leq n$ we have $w, j \models F$	**[semantics of true]**

ii.

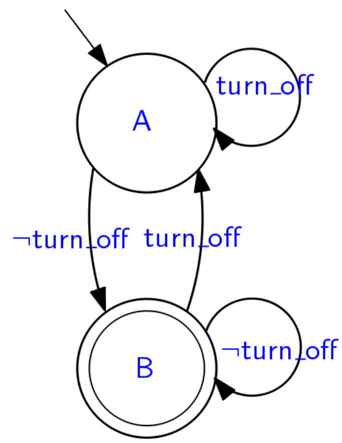
	$w, i \models \neg\Diamond\neg F$	
iff	$w, i \not\models \Diamond\neg F$	**[definition of not]**
iff	it is *not* the case that for some $i \leq j \leq n$ we have $w, j \models \neg F$	**[semantics of eventually]**
iff	for all $i \leq j \leq n$ it is not the case that $w, j \models \neg F$	**[semantics of quantifiers]**
iff	for all $i \leq j \leq n$ it is not the case that $w, j \not\models F$	**[semantics of negation]**
iff	for all $i \leq j \leq n$, $w, j \models F$	**[simplify double negation]**

iii.

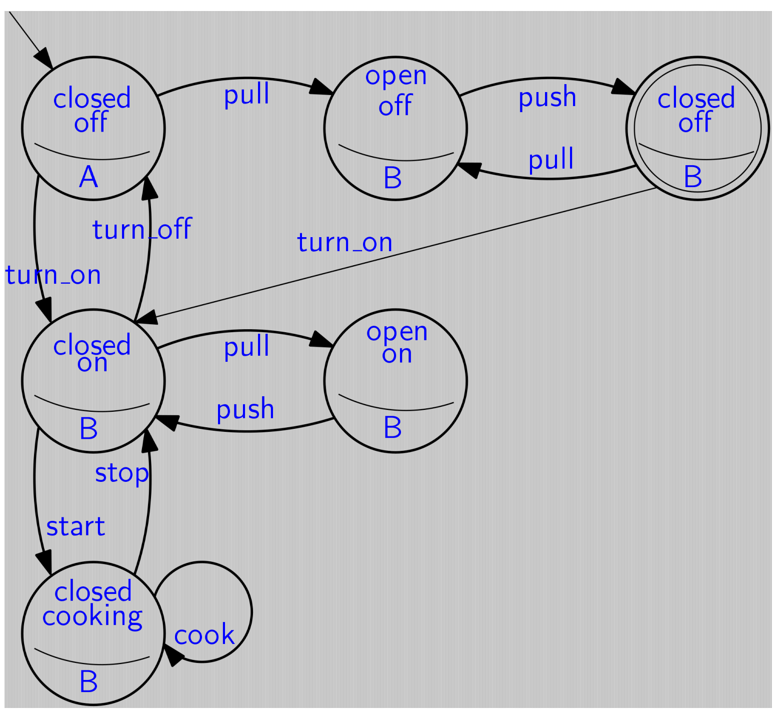
	$w, i \models \Diamond\Diamond p$	
iff	for some $i \leq j \leq n$ we have $w, j \models \Diamond p$	**[semantics of eventually]**
iff	for some $i \leq j \leq h \leq n$ we have $w, h \models p$	**[sem. eventually; merging intervals]**
iff	for some $i \leq h \leq n$ we have $w, h \models p$	**[a fortiori]**
iff	$w, i \models \Diamond\ p$	**[semantics of eventually]**

3 Automata-Based Model Checking

i. The automaton we build from the temporal formula is the following.



ii. The intersection automaton is the following:



iii. Any accepting run is a counterexample to the LTL formula being a property of the microwave oven automaton. There are several, for example: `pull push`, `pull push pull push`, ...