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Software Verification

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Lecture 6: Program Analysis

Program Analysis

An Informal Overview

Applications of program analysis

Two important application fields of program analysis:

- **Program optimizations**
  - Program analysis provides techniques for transforming programs during compilation to avoid redundant computations
- **Verification**
  - Program analysis can provide warnings about possible unintended program behavior (e.g. buffer overflows) or prove programs free from such behavior

Program analysis is a static technique, i.e. analyses are performed without running the program.

How can this work?

We are interested to have questions such as the following answered by an analysis:

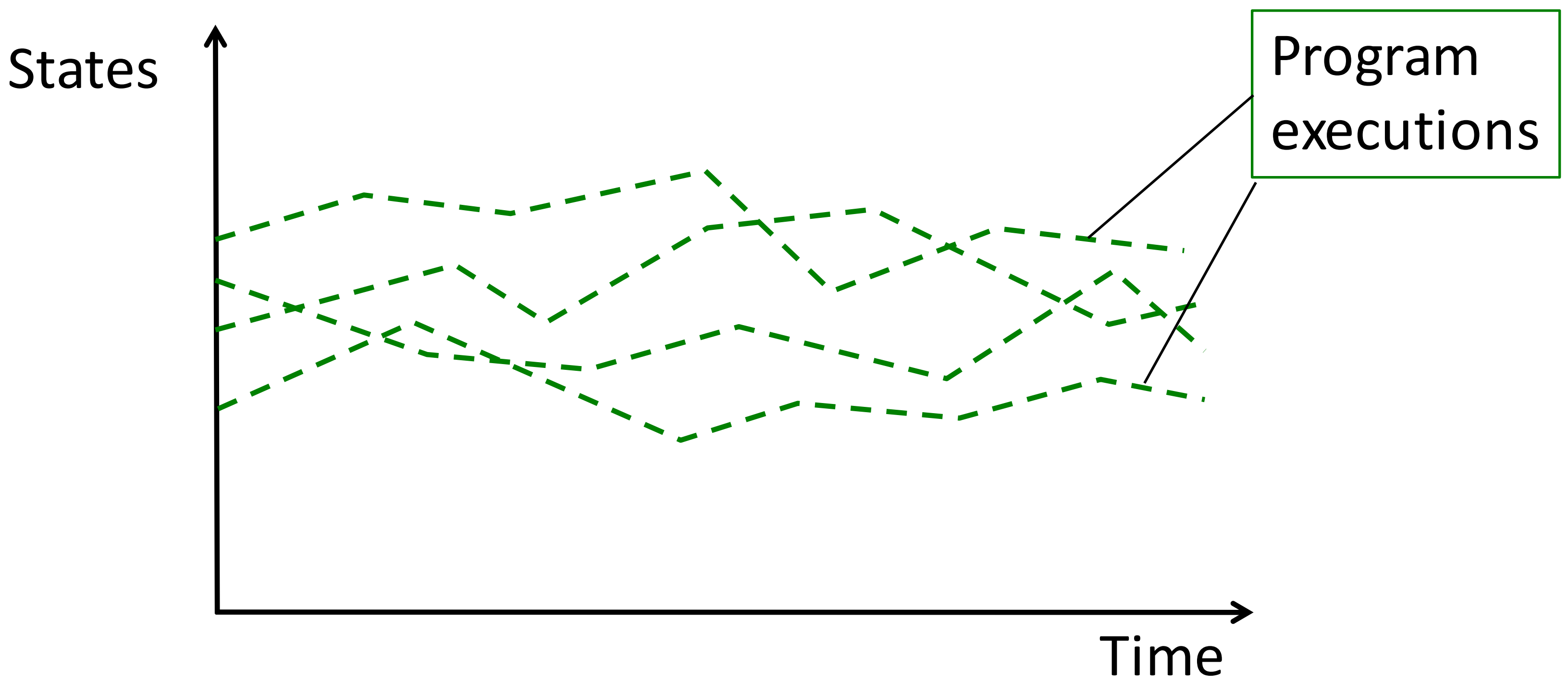
- Will the value of variable x be read in the future?
- Can buffer b overflow in line i of the program?
- Can void dereferencing occur during execution? etc.

From computability theory (Rice's theorem) we know however: "All **non-trivial** questions about the behavior of **Turing-complete** programs are **undecidable**." So, how can this work?

Key idea: We can settle for approximative answers, as explained on the following slides.

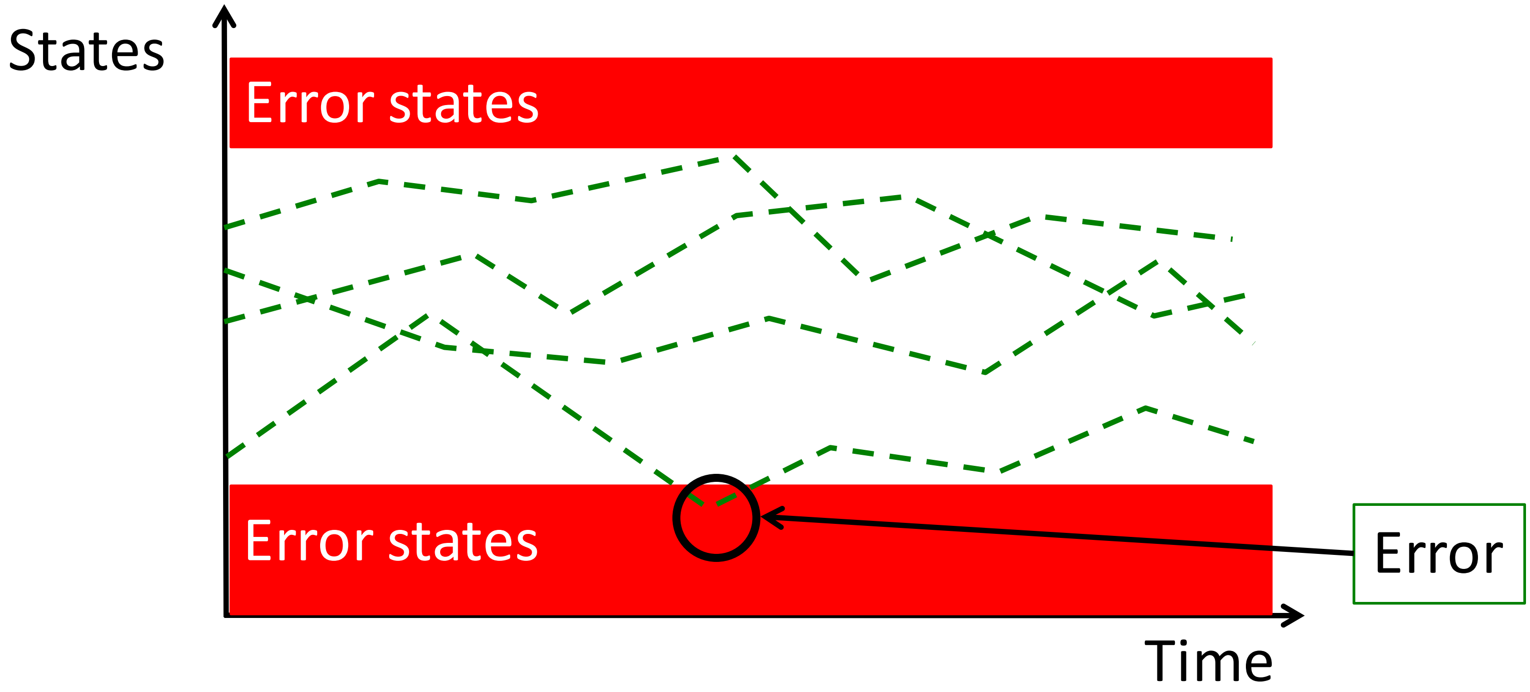
The approach

Assume we depict the set of all possible concrete executions of a program as trajectories through the state space:



Safety properties

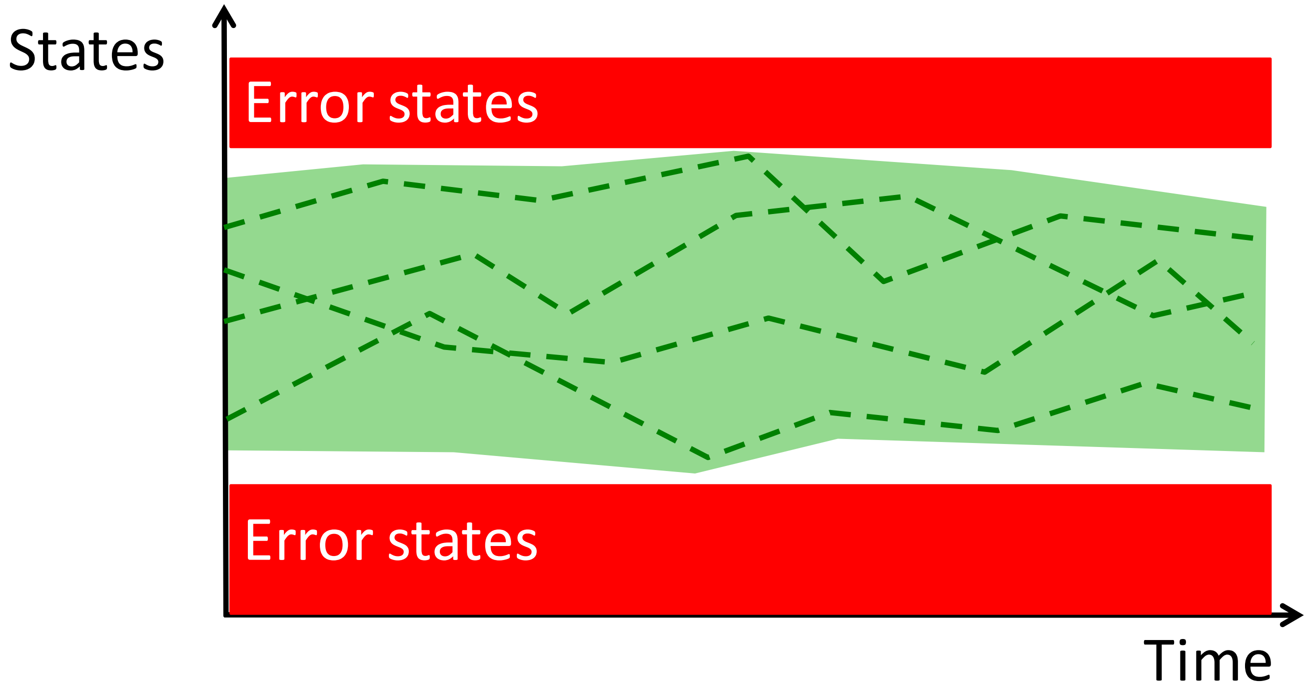
Many program analysis questions can be stated as safety properties, which express that no possible execution can enter an error state (e.g. a state where “buffer b overflows”).



Approximations

As mentioned, proving that a non-trivial safety property holds is undecidable for the set of concrete executions.

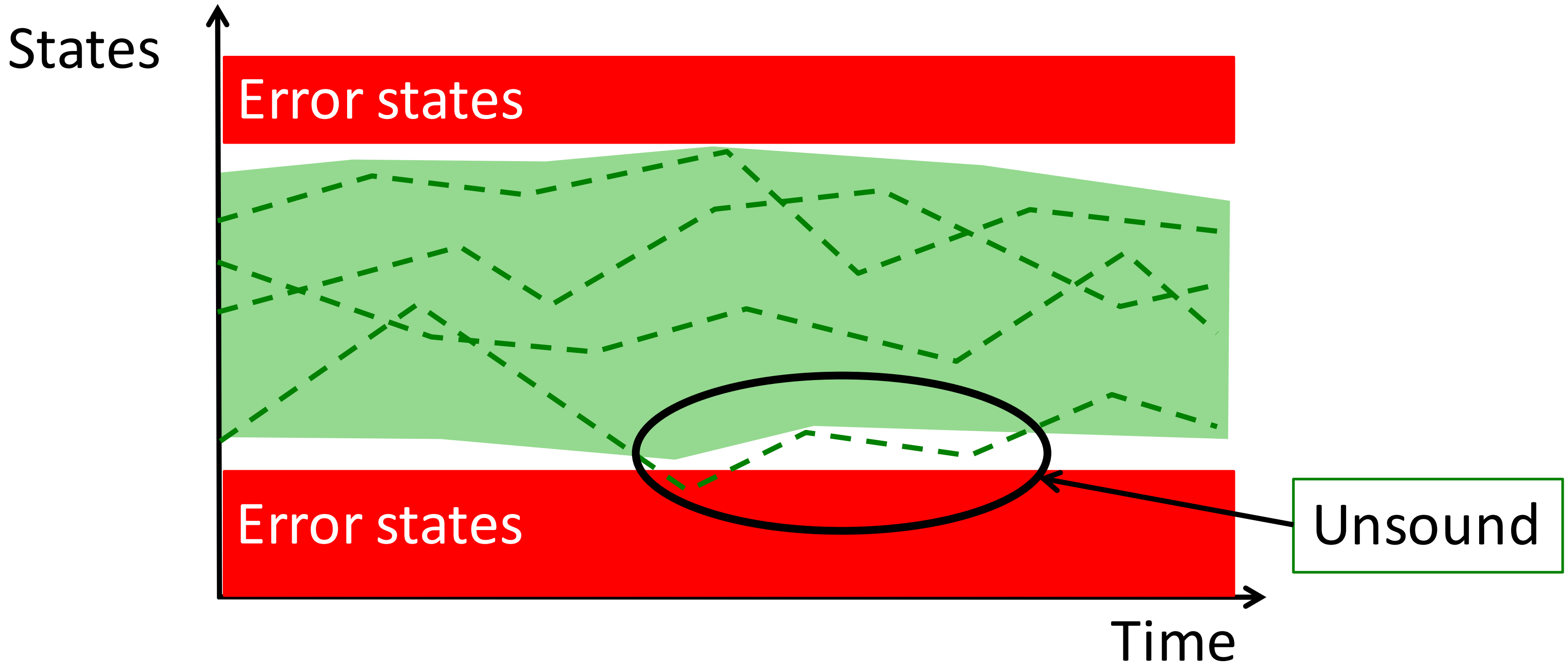
Instead we compute an abstraction of the behavior which over-approximates all concrete executions:



Soundness of the analysis

We want our analysis to be sound so that all possible program executions are captured.

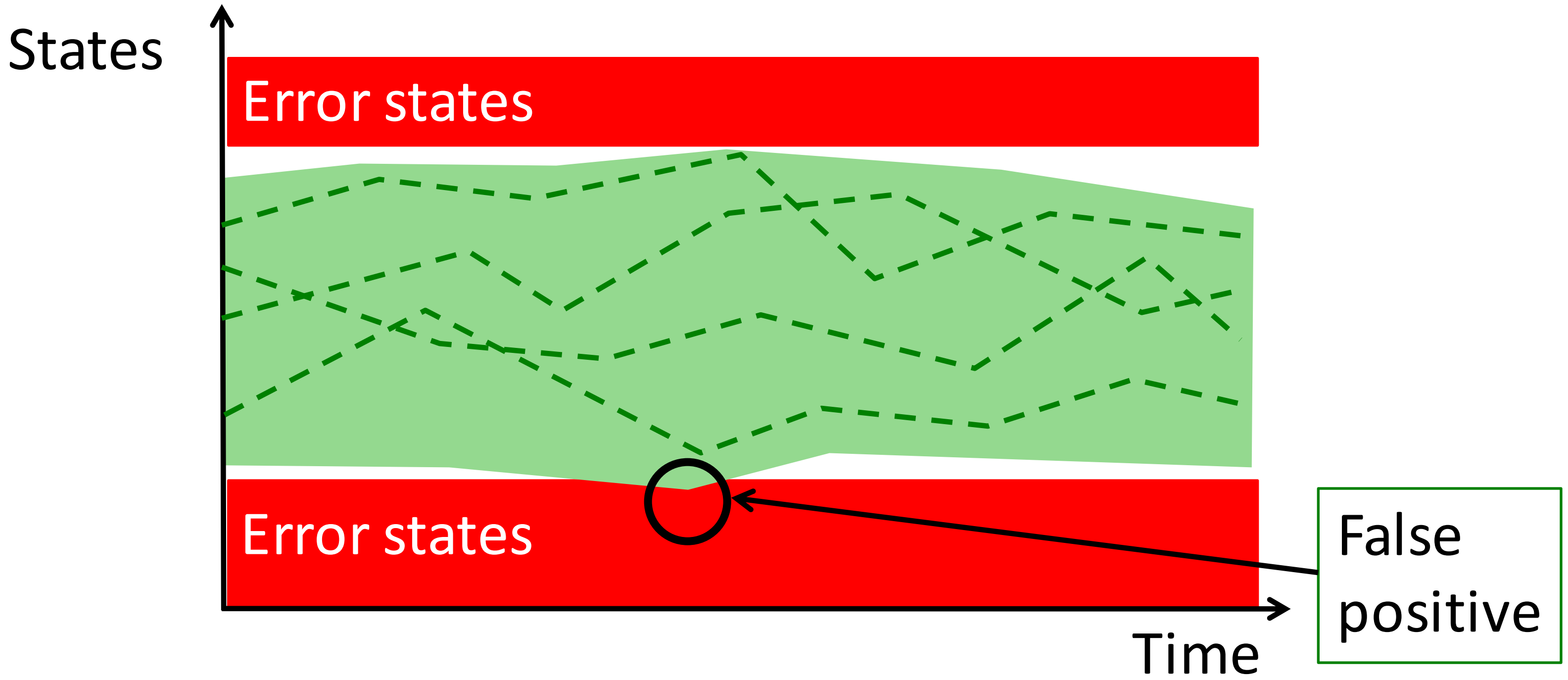
Example of an unsound analysis:



Precision

We also want our analysis to be as precise as possible. Otherwise, if there are too many false alarms, the analysis will be unusable.

Errors reported by the analysis which cannot occur in a concrete execution are called false positives.



Precision vs. efficiency

While we want our analysis to be precise, we often have to trade off precision with efficiency:

- While a very precise analysis might still be computable, it might need to run for too long to be practical.
- Imprecise analyses leave us with a large number of warnings, and manual checking has to show whether a particular warning is an error or a false positive.

Defining new program analyses is thus an art that tries to balance precision and efficiency.

Types of program analyses

Several types of program analyses have been established:

- Data flow analysis
- Control flow analysis
- Abstract interpretation
- Type systems

In this lecture we will focus on data flow analysis, and in the next on abstract interpretation.

Summary

Program analysis provides a set of static techniques for computing sound abstractions of the run-time behavior of a program.

Data Flow Analysis

Preliminaries

Data flow analysis

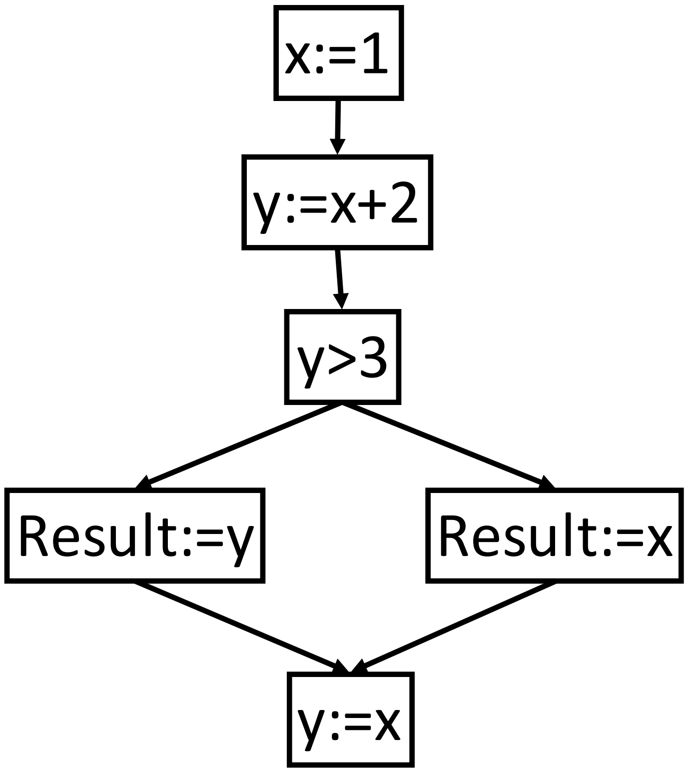
Data flow analysis is a technique to derive information about the possible program values produced at a specific program point.

Data flow analyses take as an input the control flow graph of a program, and proceed by examining how data values are changed when being propagated along its edges (hence the name "data flow").

Control flow graph

Control flow graph: graph representation of all possible execution paths of a program.

```
x := 1
y := x + 2
if (y > 3) then
        Result := y
else
        Result := x
end
y := x
```

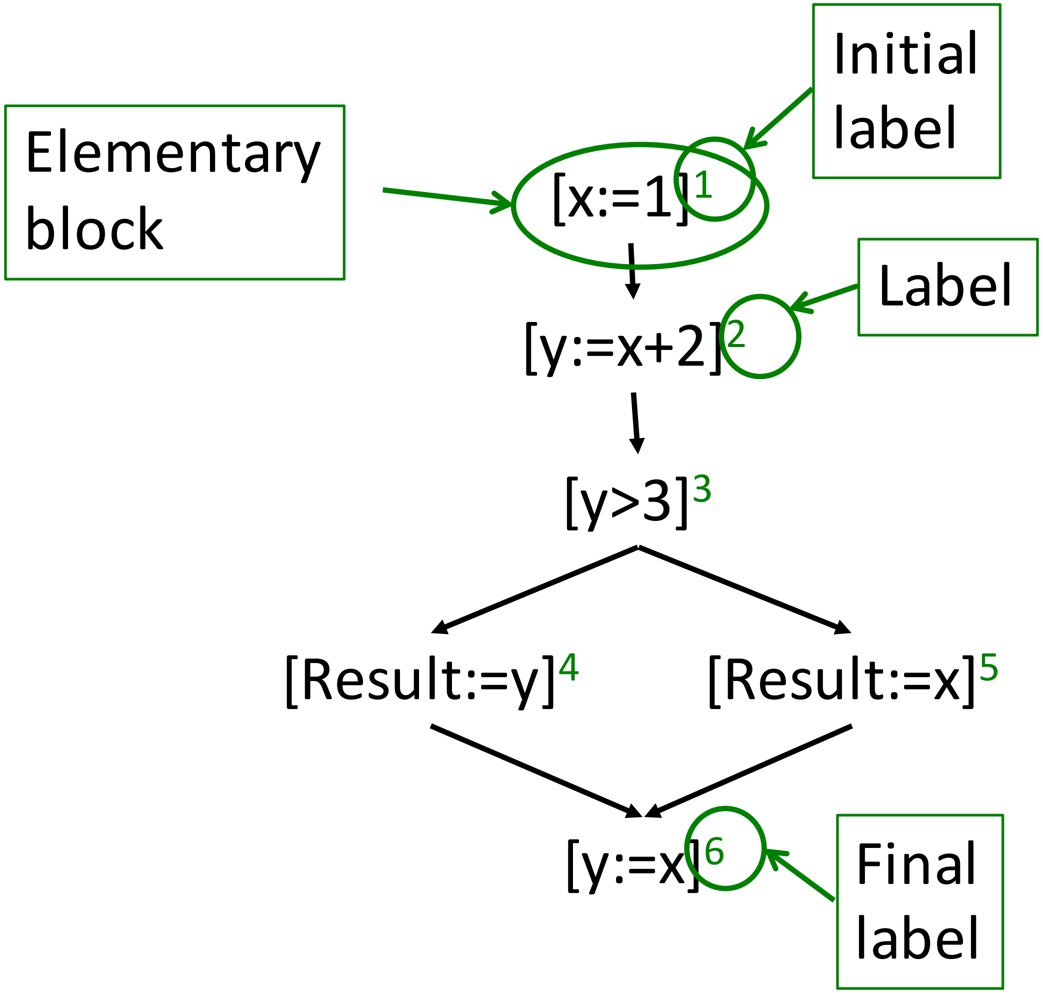


Labels

In order to be able to refer to specific program points, program analyses introduce labels into the program.

The labeled program fragments are called elementary blocks.

```
[x := 1]1
[y := x + 2]2
if [y > 3]3 then
        [Result := y]4
else
        [Result := x]5
end
[y := x]6
```



Data Flow Analysis

Live Variables Analysis

Live variables analysis

We present a first example of a data flow analysis: live variables (LV) analysis.

➢ A variable is live at the exit from a block if there is some path from the block to a use of the variable that does not redefine the variable.

➢ The aim of the live variables analysis is to determine

> "For each program point, which variables *may* be live at the exit from the point."

Example:

$[x := 2]^1$; $[y := 4]^2$; $[x := 1]^3$;

if $[y > x]^4$ **then** $[z := y]^5$ **else** $[z := 2 * z]^6$ **end**; $[x := z]^7$

Is the variable x live at the exit from block 1?

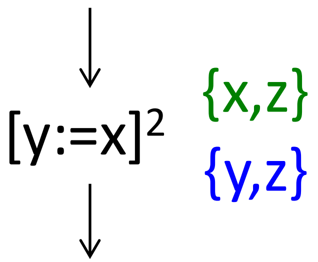
Live variables analysis

Analysis idea:

- Record sets of *possibly live* variables
- Distinguish entry and exit of blocks
- Work backwards

(LV1) Blocks:

$LV_{entry} = (LV_{exit} \setminus \text{“assigned”}) \cup \text{“used”}$

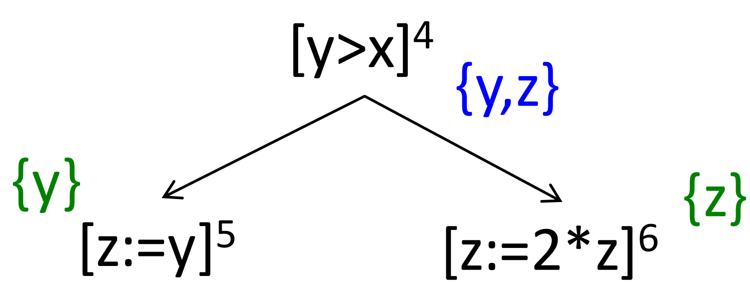


"assigned" – variable that gets assigned in the block

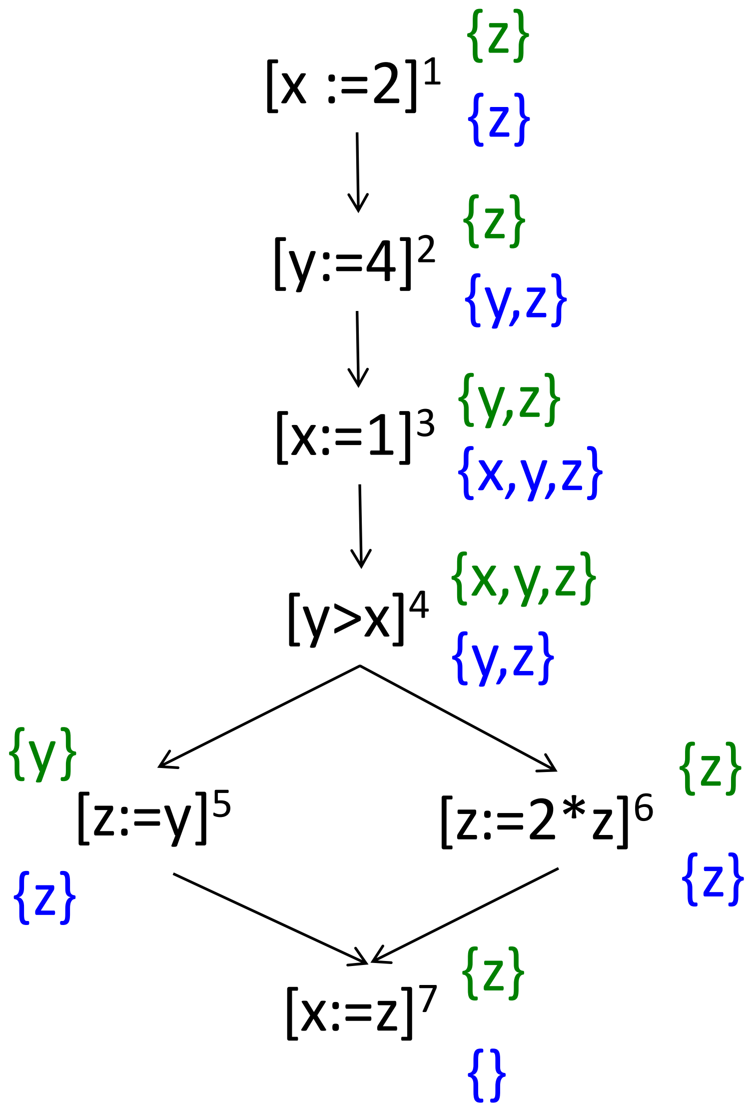
"used" – variables that are used in the block

(LV2) Edges:

$LV_{exit}(4) = LV_{entry}(5) \cup LV_{entry}(6)$



Example: Live variables analysis



Application: Dead code elimination

An assignment $[x := a]^l$ is dead if the value of x is not used before it is redefined.

Goal: Eliminate dead assignments from programs.

Example:

We know that $x \notin LV_{exit}(1) = \{z\}$, i.e. variable x is not used before it is redefined. Therefore block 1 is dead and can be eliminated:

~~$[x := 2]^1$~~; $[y:=4]^2$; $[x:=1]^3$;

if $[y>x]^4$ **then** $[z:=y]^5$ **else** $[z:=2*z]^6$ **end**; $[x:=z]^7$

How to formalize the analysis idea?

- (LV1) and (LV2) specify equations over a set of variables $LV_{entry}(l)$ and $LV_{exit}(l)$ for any label l.
- This equation system can be solved with standard algorithms (discussed later).
- The equation system itself can be specified more formally, as done on the next slide.

This specification consists of two parts:

1. The definition of the equation system.
2. Auxiliary functions kill and gen, which specify the analysis information removed (killed) and added (generated) when passing through an elementary block.

Formalization: Data flow equations

1. The data flow equations:

$$LV_{exit}(l) = \bigcup_{(l,\, l') \in CFG} LV_{entry}(l')$$

(and $LV_{exit}(l) = \{\}$ if l is the final label)

$$LV_{entry}(l) = (LV_{exit}(l) \setminus kill_{LV}(l)) \cup gen_{LV}(l)$$

2. The auxiliary kill and gen functions:

$kill_{LV}([x:=a]^l) = \{x\}$

$kill_{LV}([b]^l) = \{\}$

$gen_{LV}([x:=a]^l) = \{y \mid y \text{ is a free variable in } a\}$

$gen_{LV}([b]^l) = \{y \mid y \text{ is a free variable in } b\}$

Example: Equation system for LV analysis

$LV_{entry}(1) = LV_{exit}(1) \setminus \{x\}$

$LV_{entry}(2) = LV_{exit}(2) \setminus \{y\}$

$LV_{entry}(3) = LV_{exit}(3) \setminus \{x\}$

$LV_{entry}(4) = LV_{exit}(4) \cup \{x, y\}$

$LV_{entry}(5) = (LV_{exit}(5) \setminus \{z\}) \cup \{y\}$

$LV_{entry}(6) = (LV_{exit}(6) \setminus \{z\}) \cup \{z\}$

$LV_{entry}(7) = (LV_{exit}(7) \setminus \{x\}) \cup \{z\}$

$LV_{exit}(1) = LV_{entry}(2)$

$LV_{exit}(2) = LV_{entry}(3)$

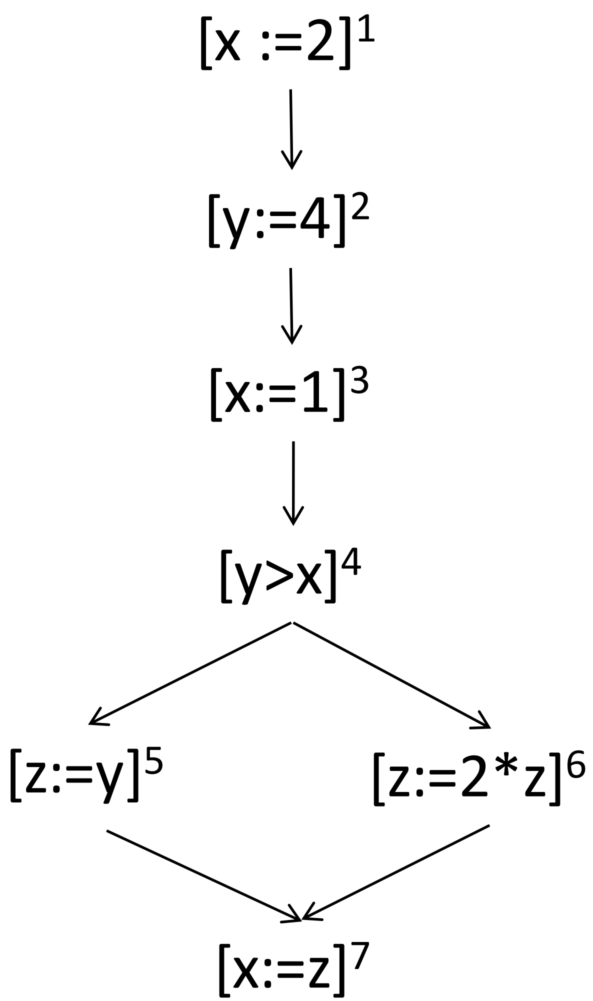
$LV_{exit}(3) = LV_{entry}(4)$

$LV_{exit}(4) = LV_{entry}(5) \cup LV_{entry}(6)$

$LV_{exit}(5) = LV_{entry}(7)$

$LV_{exit}(6) = LV_{entry}(7)$

$LV_{exit}(7) = \{\}$



Data Flow Analysis

Equation Solving

Fixed point solutions

- The equation system of the example defines the 14 sets

$$LV_{entry}(1),\ LV_{entry}(2),\ \ldots,\ LV_{exit}(7)$$

in terms of each other.

- When writing $\underline{LV}$ for the vector of these 14 sets, the equation system can be written as a function F where

$$\underline{LV} = F(\underline{LV})$$

- Using a vector of variables $\underline{X} = (X_1,\ldots, X_{14})$, the function can be defined as

$$F(\underline{X}) = (f_1(\underline{X}),\ \ldots,\ f_{14}(\underline{X}))$$

where for example

$$f_{11}(X_1,\ \ldots, X_{14}) = X_5 \cup X_6$$

- From the above equation it is clear that the solution $\underline{LV}$ we are looking for is the (least) fixed point of the function F.

Partially ordered sets

For any analysis, we are interested in expressing that one analysis result is "better" (more precise) than another.

In other words, we want the analysis domain to be partially ordered.

A partial ordering is a relation $\sqsubseteq$ that is

- reflexive: $\forall d : d \sqsubseteq d$
- transitive: $\forall d_1, d_2, d_3 : d_1 \sqsubseteq d_2$ and $d_2 \sqsubseteq d_3$ imply $d_1 \sqsubseteq d_3$
- anti-symmetric: $\forall d_1, d_2 : d_1 \sqsubseteq d_2$ and $d_2 \sqsubseteq d_1$ imply $d_1 = d_2$

A partially ordered set $(D, \sqsubseteq)$ is a set D with a partial ordering $\sqsubseteq$.

Least element: $a \in D$ s.t. $d \sqsubseteq a$ and $d \in D$ implies $d = a$.

Examples: Real numbers $(\mathbf{R}, \leq)$, power sets $(\wp(S), \subseteq)$, ...

Equation solving

- How can we obtain the least fixed point practically?
- For the least element $\perp \in D$ of a partially ordered set D we have

$$\perp \sqsubseteq F(\perp)$$

- Since F is monotone, we have by induction for all $n \in \mathbf{N}$

$$F^n(\perp) \sqsubseteq F^{n+1}(\perp)$$

- All elements of the sequence are in the domain D, and therefore, if D is finite, there exists an $n \in \mathbf{N}$ such that

$$F^n(\perp) = F(F^n(\perp))$$

- But this means that $F^n(\perp)$ is a fixed point! (And indeed a least fixed point.)

Chaotic iteration

- Implementing the iteration algorithm naively is computationally too expensive.
- More efficient algorithms exist, and are variants of the simplest scheme which is called chaotic iteration:

-- Initialization

$X_1 := \bot; \ldots; X_n := \bot$

-- Iteration

while $X_j \neq F_j(X_1, \ldots, X_n)$ for some j **do**

$X_j := F_j(X_1, \ldots, X_n)$

end

- A more advanced algorithm is the worklist algorithm, which keeps a list of edges of the control flow graph to indicate which items are in need of recomputation.

A worklist algorithm for solving the equations

Input:

A set of live variables analysis equations

Output:

The least solution to the equations: LV_{exit}

Data structures:

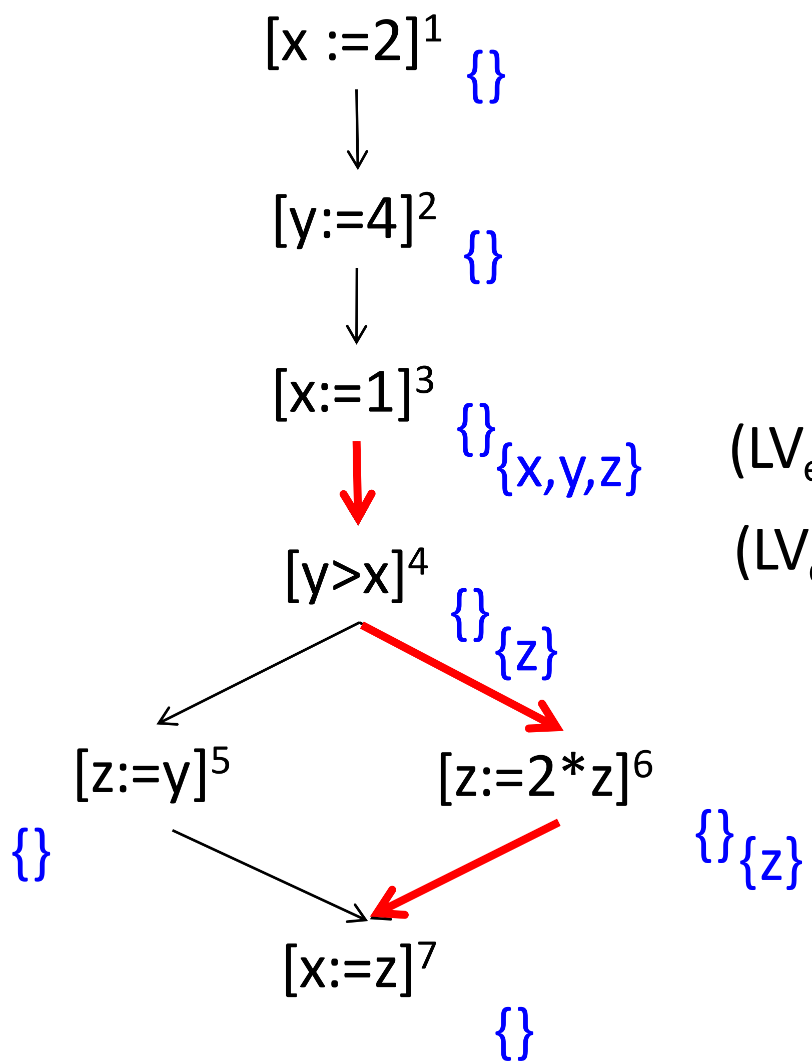
- The current analysis result for block exits: LV_{exit}
- The worklist W: A list of pairs (l, l') indicating that the current analysis result has changed at the entry to the block l' and hence the information must be recomputed for block l.

A worklist algorithm for solving the equations

```
-- Initialization
W := nil
for all (l, l') ∈ CFG do W := cons((l, l'),W) end
for all labels l do LV_exit (l) := {} end
-- Worklist loop
while W ≠ nil do
  (l, l') := head(W)
  W := tail(W)
  if (LV_exit (l') \ kill(l')) ∪ gen(l') ⊈ LV_exit (l) then
    LV_exit (l) := LV_exit (l) ∪ (LV_exit (l') \ kill(l')) ∪ gen(l')
  end
  for all l'' with (l'', l) ∈ CFG do W := cons((l'', l),W) end
end
```

Note: $(LV_{exit}(l') \setminus kill(l')) \cup gen(l') = LV_{entry}(l')$

Example: Working of the algorithm



W = (6,7),(5,7),...
W = (4,6),(5,7),...
W = (3,4),(5,7),...

$(LV_{exit}(7) \setminus \{x\}) \cup \{z\} \not\subseteq LV_{exit}(6)$

$(LV_{exit}(6) \setminus \{z\}) \cup \{z\} \not\subseteq LV_{exit}(4)$

$LV_{exit}(4) \cup \{x,y\} \not\subseteq LV_{exit}(3)$

Data Flow Analysis

Reaching Definitions Analysis

Reaching definitions analysis

Another example of a data flow analysis: reaching definitions (RD) analysis.

- The aim of the RD analysis is to determine

> "For each program point, which assignments may have been made and not overwritten, when program execution reaches this point along some path."

Note: The word "definition" is used for "assignment"

Example:

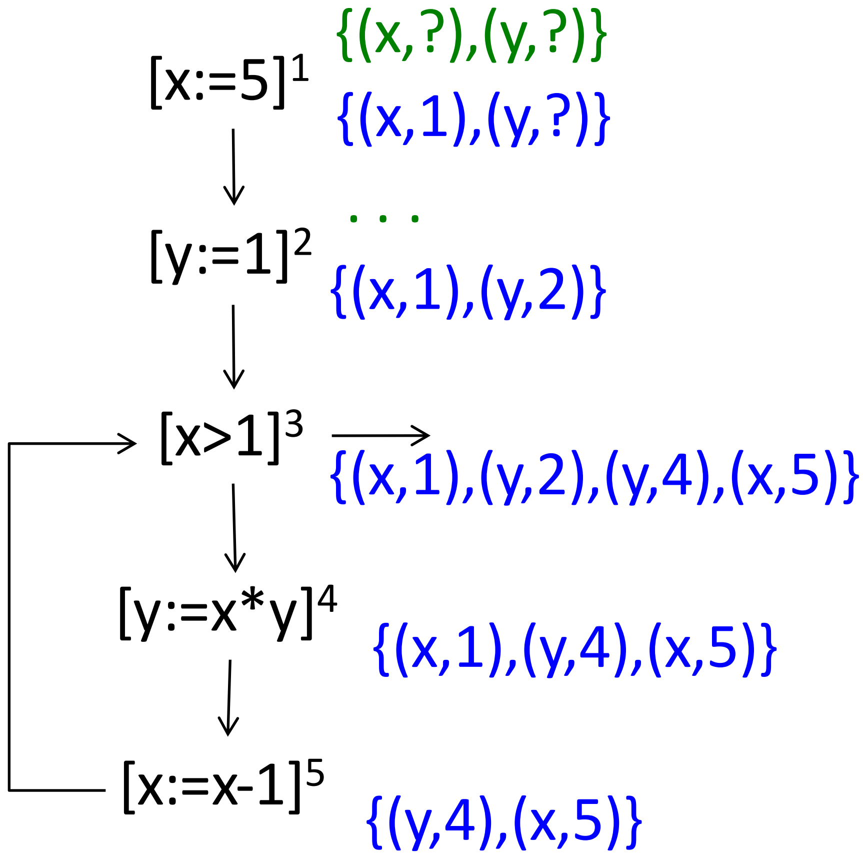
[x:=5]^1; [y:=1]^2; **while** [x>1]^3 **do** [y:=x*y]^4; [x:=x-1]^5 **end**

Which assignments may reach program point 5?

Reaching definitions analysis

Idea: analysis domain $\wp(Var_* \text{ x } Lab_*)$, work forward

- We write (x, l) to describe a definition of x in block l
- We write (x, ?) to describe that x is uninitialized



Formalization: Data flow equations

The reaching definitions analysis can be specified similarly to the scheme for LV analysis.

$$RD_{entry}(l') = \bigcup_{(l, l') \in CFG} RD_{exit}(l)$$

(and $RD_{entry}(l) = \{(x,?) \mid x \text{ is a free variable in the program}\}$ if l is the initial label)

$$RD_{exit}(l) = (RD_{entry}(l) \setminus kill_{RD}(l)) \cup gen_{RD}(l)$$

$$kill_{RD}([x:=a]^l) = \{(x,?)\} \cup \{(x,l') \mid \text{block } l' \text{ assigns to } x\}$$

$$kill_{RD}([b]^l) = \{\}$$

$$gen_{RD}([x:=a]^l) = \{(x,l)\}$$

$$gen_{RD}([b]^l) = \{\}$$

Use-Definition and Definition-Use chains

Sometimes it is convenient to directly link statements that produce values to statements that use them and vice versa

- Use-Definition chains (UD chains): each **use** of a variable is linked to all assignments that may reach it

[x:=0]^1; **[x:=3]^2**; (**if** [z=x]^3 **then** [z:=0]^4 **else** [z:=**x**]^5 **end**); [y:=**x**]^6; [x:=y+z]^7

- Definition-Use chains (DU chains): each assignment to a variable is linked to all **uses** of it

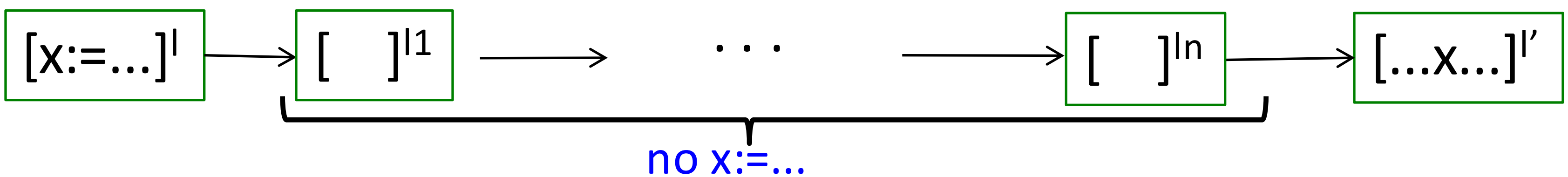
[x:=0]^1; **[x:=3]^2**; (**if** [z=x]^3 **then** [z:=0]^4 **else** [z:=x]^5 **end**); [y:=x]^6; [x:=y+z]^7

Definition of UD chains

- $UD(x, l')$ returns all the labels where an occurrence of x at l' may have obtained its value.

$$UD(x, l') = \{l \mid [x:=a]^{l} \text{ and } clear(x,l,l')\} \cup \{? \mid clear(x, l_{init}, l)\}$$

where the predicate $clear(x,l,l')$ describes a definition clear path: none of the intermediate blocks on a path from l to l' contains an assignment to x but block l assigns to x and block l' uses x.



- Can be computed with Reaching Definitions:

$UD(x,l) = \{l' \mid (x, l') \in RD_{entry}(l)\}$ if x is used in block l, else {}

Definition of DU chains

- DU(x, l) returns all the labels where the value assigned to x at l may be used.

- Can be computed from UD chains:

$$DU(x, l) = \{l' \mid l \in UD(x, l')\}$$

Reading

Textbook:

Flemming Nielson, Hanne Riis Nielson, Chris Hankin: *Principles of Program Analysis*, Springer, 2005.

Chapter 1: Sections 1.1-1.3, 1.7

Chapter 2: Sections 2.1, 2.3, 2.4